A Model of Competition in Banking: Bank Capital vs Expertise

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This paper presents a model of competition in the banking industry based upon the interplay of two factors: the level of capitalization of banks and their ability to monitor different types of projects (i.e., their expertise). In a setting of moral hazard with limited liability, banks must receive some rents to induce them to monitor projects diligently. The rents are decreasing in the banks’ expertise and in the amount of capital that banks are able to commit to a project. This leads to a trade-off between capital and expertise. The analysis shows how shocks to bank capital and interest rates, and technological shocks can affect competition and monitoring efficiency in the banking sector.

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Abrupt changes in financial services are transforming the banking industry and redefining its boundaries. Both in the United States and in Europe, deregulation has, on the one hand, eliminated geographic restrictions on the operations of banks, and, on the other hand, opened up competition from nonbank intermediaries. The elimination of geographic limits expands the loan markets available to banks and hence their incentives to develop their abilities to monitor different types of loans, whereas increasing competition from nonbanks may have very different effects. Moreover, how different banks respond to different types of deregulation is likely to depend on their capital levels since bank capital may affect much of what banks do, including their incentives to monitor loans and their ability to

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compete. Thus, it is far from obvious how deregulation affects banks or the loan market.2

In this paper I build a model for investigating the effects that deregulation is producing on the banking industry, particularly on the way that banks compete. Two features play a key role in the analysis: One is the bank’s capital; the other is its expertise, which I define as the bank’s cost of monitoring a borrower. This cost of monitoring depends on the bank’s specialization decision; in the formal analysis, this is a decision about how far from the borrower to locate the bank. The major question addressed with this framework is the following: How do specific forms of deregulation affect banks, and how do these effects differ across high-capital and low-capital banks? The different forms of deregulation I consider here are those that result in an increase in capital requirements, in an increase in interest rates, and in the lifting of geographic restrictions on the operations of banks.

Although the model is highly stylized, I interpret the regulatory shocks as a rebalancing of the optimal capital–expertise balance for banks in order to provide answers to this question about the effects of deregulation. First, the model shows that deregulation is not neutral. Specifically, it shows that highly capitalized banks benefit when competing with poorly capitalized ones after regulatory shocks that decrease capital requirements or otherwise cause the capital at low- and high-capital banks to increase proportionally, increase the riskless interest rate (through, say, a tightening of monetary policy), or eliminate geographic restrictions. The analysis also shows that low-capital banks have stronger incentives for financial specialization than banks with more financial strength.

An analysis based on the interaction between bank capital and expertise requires a model in which both elements play relevant economic roles. Building upon the work of Holmstrom and Tirole (1997), I consider a model with three classes of agents—investors, entrepreneurs, and bankers—which I enrich by adding a spatial component to differentiate banks.3 This model features a double moral-hazard problem. First, there is moral hazard between investors and entrepreneurs that can be alleviated by the monitoring services provided by banks. Second, there is moral hazard in the provision of these monitoring services. I focus on how the two dimensions in which banks differ, capital and expertise, affect the magnitude of the moral-hazard problem inherent in monitoring. The model exploits the idea that scarcity of capital may preclude some banks from taking full advantage of their monitoring expertise.4

When limited liability exists in a moral-hazard situation, the agents, for incentive reasons, enjoy informational rents. These rents may create a problem

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3 As explained below, the distance between a bank and a borrower relates to the cost that the bank must incur to monitor such a borrower.

4 The assumption that monitoring expertise varies across lines of business seems consistent with casual empirical observation. For example, Steiner and Teixeira (1990) refer to the fundamentally different skills that consumer lending and corporate lending require.
of transferability that, however, may preclude positive-net-present-value investments. In the model, banks compete for informational rents which are required to induce banks to monitor projects diligently. Both bank expertise—by reducing the moral-hazard problem—and bank capital—by reducing the problem of limited liability—decrease the size of such rents. Therefore, capital and expertise become substitutes that ameliorate the problem of transferability: Banks with less expertise must commit more capital to the businesses they monitor. Without scarce bank capital, banking expertise alone determines the match between banks and firms. However, capital scarcity interferes with efficient monitoring in the credit market and generates competitive advantages for high-cap banks.

These competitive advantages by high-cap banks are intensified by shocks that increase bank capital proportionally, such as easing of capital requirements, or that make bank capital relatively more valuable, such as an increase in the riskless interest rate. In addition, lifting geographic restrictions benefits high-cap banks by allowing them to access borrowers in new markets. In a related vein, the analysis shows that, to compensate for their relative disadvantage with respect to high-cap banks, low-cap banks have stronger incentives to develop expertise and specialize.

The two central elements of this analysis, bank capital and bank expertise, have certainly received uneven attention in the literature. Previous research has extensively analyzed the role of bank capital and suggested at least four distinct roles it can play. First, bank capital can ameliorate an excessive tendency by banks to take risks. Second, bank capital can serve as a cushion against solvency problems. Third, bank capital can signal the risk preferences of a bank. Fourth, bank capital can help a bank accommodate monetary shocks. I examine a new role to bank capital as a tool that allows a bank to offer lower loan rates without affecting its incentives to monitor.

In contrast, the role of bank expertise has been analyzed less. Financial intermediation theory (Diamond, 1984; Ramaskrishnan and Thakor, 1984; Boyd and Prescott, 1986) argues that diversification across borrowers makes intermediaries

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5 A few examples of models in which bank capital play an important role are Bernanke and Gertler (1987), Holstrom and Tirole (1997), and Diamond and Rajan (2000). See Winton (1995) for a model in which capital can substitute size (i.e., diversification) as the source of competitive advantage for intermediaries. See Berger et al. (1995) for a review of the reasons why bank capital matters.

6 Although some authors argued that capital requirements do not deter bank risk taking (Kahane, 1977) and can even exacerbate it (Besanko and Kanatas, 1996).

7 The U.S. regulators have used the CAMELS system as an early warning system to identify problems in banks. Capital adequacy is one of the elements used in that system. See Dewatripont and Tirole (1994, p. 66) for details.

8 Hughes and Mester (1998) present a model that develops this intuition.

9 According to the lending view of monetary policy balance sheet effects of banks can have macroeconomic effects. See Kashyap and Stein (1995) for an extensive treatment of the lending channel of monetary policy, Bernanke and Lown (1991) for an empirical investigation of the effects of capital crunch in the 1990–1991 recession in the United States, and Thakor (1996) for additional evidence on the macroeconomic effects of capital requirements.
more effective as monitors or screeners. In reality, however, banks tend to concentrate their portfolios by either region,\textsuperscript{10} industry, or demographic strata, suggesting a crucial advantage to expertise development (i.e., specialization). To highlight these expertise considerations, my model does not assume advantages of diversification and instead examines a case in which increasing bank expertise decreases the cost of monitoring, thus helping banks perform their intermediary function more efficiently.\textsuperscript{11} Boot and Thakor (2000) have analyzed a bank’s incentives to develop expertise, specifically how such incentives are affected when either interbank or capital market competition increases.\textsuperscript{12} In contrast, I consider a spatial model to analyze interbank competition in the context of the moral-hazard problem described above. Furthermore, in addition to examining the incentives for a bank to develop expertise, I examine how such development is affected by a bank’s level of capital.

The model is also related to models of spatial competition in banking. Following the classic spatial models by Hotelling (1929) and Salop (1979), most of the spatial models in banking (e.g., Repullo, 1990) have been used to analyze the banking industry in the absence of asymmetries of information. These models view banks as classical firms (ones that buy deposits and sell loans) rather than as monitoring agents in imperfect capital markets as I do here.\textsuperscript{13}

This paper is organized as follows. Section 1 develops the basic model. Section 2 analyzes a simplified version of the model without bank competition. Section 3 brings bank competition into the analysis. Section 4 investigates the competitive effects of several types of shocks. Section 5 extends the basic model in order to examine the expertise choice by banks. Section 6 offers conclusions about the results of the study. Proofs and most technical derivations may be found in the Appendix.

1. THE MODEL

Building on Holmstrom and Tirole (1997), I consider a model with the following elements.\textsuperscript{14}

\textsuperscript{10}To be sure, regulatory limitations have contributed, especially in the United States, toward the tendency of the banks to concentrate in certain geographical areas.

\textsuperscript{11}See Winton (2000) for an analysis of the trade-off faced by banks between specialization and diversification.

\textsuperscript{12}In particular, Boot and Thakor (2000) examine how increases in competition from other banks and by capital markets affect a bank’s incentives to specialize in a borrower sector, its decision of how much lending service capacity to build (i.e., its scale of operations), and its decision to allocate such lending capacity across relationship lending and transaction lending.

\textsuperscript{13}An exception is Besanko and Thakor (1992) who, in a context with asymmetric information, use a spatial competition model to examine the welfare effects of relaxing entry barriers and changing capital standards on borrowers’ and savers’ welfare.

\textsuperscript{14}Specifically the model introduces the basic structure of Holmstrom and Tirole (1997) in a spatial model to examine competition among heterogenous lenders.
1.1. Agents

There are three kinds of risk neutral agents—investors, entrepreneurs, and banks. Investors own a large aggregate endowment of uninformed capital, which can be invested in a safe technology that yields a gross return of $\gamma$ per unit of investment. Entrepreneurs lack capital but are endowed with a project that requires $I$ units of funds to generate a stochastic payoff, which may take one of two values, 0 or $R > 0$. The probability of a payoff $R$ depends on an unobservable action taken by the entrepreneur. If action $a_h$ (alternately, $a_l$) is undertaken, the probability of success is $p_h(p_l)$, where $p_h > p_l$. A moral hazard problem arises because the entrepreneur will enjoy a private benefit $B(B > 0)$ from choosing $a_l$ rather than $a_h$. Finally, I consider two competing banks, indexed by $i = \{0, 1\}$, that are endowed with capital and have access to a monitoring technology: If a bank incurs a private cost $C$, it will reduce the private benefit to the entrepreneur from $B$ to $b$; this reduces the entrepreneur’s incentives to choose action $a_l$.

The following parameter restrictions motivate the demand for banking:

**Assumption 1.** Excluding private benefits, projects have positive-net-present-value only if the high probability action, i.e., $a_h$, is chosen:

$$p_h \cdot R > I \cdot \gamma > p_l \cdot R.$$  

(1)

**Assumption 2.** No feasible financial contract between entrepreneurs and investors would allow a project to be undertaken:

$$p_h \cdot \left( R - \frac{B}{\Delta_p} \right) < I \cdot \gamma,$$

(2)

where $\Delta_p \equiv p_h - p_l$.

The moral hazard problem (unobservable action and private benefit $B$) implies that the entrepreneur must be guaranteed at least $\frac{B}{\Delta_p}$ in case of success.\(^{16}\) Thus, without any additional institution (i.e., banks), Assumptions 1 and 2 imply that no project would be financed.

**Assumption 3.** Under bank monitoring, there may exist a feasible investment contract between the entrepreneur and the investor:

$$p_h \cdot \left( R - \frac{b}{\Delta_p} \right) > I \cdot \gamma.$$  

(3)

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\(^{15}\) I follow the standard view that banks can exert tighter control on managerial discretion than public lenders. See Diamond (1991) or Holmstrom and Tirole (1997).

\(^{16}\) If $R_e$ denotes the amount that the entrepreneur keeps in case of success, then the entrepreneur chooses $a_h$ only if the inequality $p_h \cdot R_e \geq p_l \cdot R_e + B$ holds (i.e., if $R_e \geq \frac{B}{\Delta_p}$).
Assumption 3 (that considers bank monitoring but ignores monitoring costs) would permit the financing of the project. For example, a payment of $R_e = \frac{b}{h_1}$ to the entrepreneur would leave $p_b \cdot (R - \frac{b}{h_1})$ to the investors, which could cover the opportunity cost of financing the project, $I \cdot \gamma$. Bank monitoring costs are introduced next.

1.2. The Monitoring Technology

A central intuition for the model to capture is that banks have different kinds of expertise and that the cost of monitoring a project is reduced if a bank has strong expertise in the line of business of a project. To model this in a simple way, I assume that projects that differ in their technology can be represented as points on a two-unit technological circumference, whereby projects are positioned in relation to each other in the circumference based on similarity on technology. Further, I assume that banks’ expertise can also be represented as points in the circumference and that the resources spent by a bank on monitoring increase with the distance between the position of the bank and that of the project.\(^\text{17}\)

To simplify, I assume that banks are located at opposite points on the technological circumference (say at 0 and 1)\(^\text{18}\) and that the cost of monitoring is linear in the distance between banks and projects to be monitored. For instance, in order to monitor a project at $x$, the bank at 0 would incur a private cost of $c \cdot x$ while the bank at 1 would incur a cost of $(1 - x) \cdot c$. These assumptions on location, summarized in Fig. 1, simplify computations while maintaining the basic assumption required for the analysis—that banks are heterogeneous in their ability to monitor different projects.

1.3. The Stochastic Structure of the Economy

As discussed below, the joint distribution of projects’ returns affects the optimal financial contracts signed by agents. In order to emphasize the importance of bank capital I assume the following:

**Assumption 4.** Projects monitored by the same bank become perfectly correlated.

As shown in Lemma 2, banks must finance part of an investment with their own capital $K_i$. What Assumption 4 guarantees is that the amount that a bank finances in a project does not depend on other projects financed by the bank and thus that the size of a particular bank, measured by its own capital, determines only the scale of

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\(^{17}\) Although the spatial features of the model have been motivated in terms of technological distance, they could have been interpreted purely in terms of physical distance: A bank closer to a geographical area will be better informed about the business in that area. Indeed, Petersen and Rajan (2000) argue that the expertise of a lender on a project and the physical distance are closely related. Further, they argue that the increase in physical distance between small firms and their lenders that has occurred during the past few years in the United States can be related to developments in information technology.

\(^{18}\) Bank location is endogenized in Section 5.
FIG. 1. Locations. This figure represents the spatial features of the model. Two banks are located at opposite points in the two-unit length circumference, points 0 and 1, and a continuum of projects are located uniformly along the circumference. Point \( x \) represents the location of a generic project.

its operations. The opposite assumption, i.e., that project returns are independent (as argued in Diamond, 1984), would imply that agency costs for banks diminish by increasing the number of projects that a bank monitors, which would make the banking sector a natural monopoly where capital would not play any role. Such conclusions are clearly counterfactual. There are many banks in the economy and the importance of bank capital is evident. One simple way to reconcile the analysis with these facts is to assume a certain degree of correlation among the projects handled by a bank. I choose to consider the extreme case of perfect correlation for its tractability.

2. ANALYSIS OF THE MODEL: ONE BANK CASE

In order to illustrate the main forces operating in the model, I first consider a simplified version of the model with only one bank located in a large circumference of projects. In fact, this case can be analyzed as if the bank is located at the extreme of a segment which extends indefinitely; under this assumption I will proceed.

2.1. The Optimal Contract

An equilibrium should determine the projects undertaken, the source of their financing (the mix between bank capital and investor capital) and the prices of informed and uninformed capital. Consider the optimal financing contract that an entrepreneur separated \( x \) units from the bank offers to his or her financiers.\(^{19}\) The unobservability of \( b \) and \( C \) (and of the actions taken by the bank and the

\(^{19}\) Throughout the paper, \( f \) refers to investors (financiers), \( e \) to entrepreneurs, and \( b \) to banks.
entrepreneur) requires contracts to depend on realized cash flows. In fact, a contract consists of the share of the investment and the share of the cash flow corresponding to the bank and the uninformed investors. Formally, a contract is the solution of the problem

$$\max_{S, P} \quad R_e(x)$$

subject to constraints\(^{20}\)

$$I_f(x) + I_b(x) \geq I$$

$$R_e(x) + R_b(x) + R_f(x) \leq R$$

$$p_h \cdot R_f(x) \geq I_f(x) \cdot \gamma$$

$$p_h \cdot R_b(x) - C(x) \geq I_b(x) \cdot \beta$$

$$R_e(x) \geq \frac{b}{\Delta p}$$

$$R_b(x) \geq \frac{C(x)}{\Delta p},$$

where \(S \equiv [I_b(x), I_f(x)]\) and \(P \equiv [R_b(x), R_f(x), R_e(x)]\) represent, respectively, the investments in the project and the shares in the payoff if the project is successful, and \(\beta\) represents the return on bank capital.

Throughout this paper I assume that agents behave competitively.\(^{21}\) Hence the entrepreneur takes as given both the cost of uninformed capital, \(\gamma\), and the return on bank capital, \(\beta\), that is an endogenous variable that plays a central role in the analysis below.

The interpretation of the constraints in program (4) is straightforward: Eq. (5) refers to how the investment must be shared between the two financiers, while Eq. (6) refers to how the cash flow must be shared among the three parties. Equation (7) (Eq. (8)) describes the participation constraint for investors (the bank) while Eq. (9) (Eq. (10)) describes the incentive compatibility constraint for the entrepreneur (the bank).

The following lemmas characterize the optimal contract:\(^{22}\)

**Lemma 1.** A project is financed if and only if the monitoring cost of the project is below a cut-off level, \(C^*\), defined by:

\(^{20}\) All the variables are nonnegative unless otherwise noted.

\(^{21}\) Bank competition, introduced in the next section, will make this assumption better justified.

\(^{22}\) All proofs are in the Appendix unless otherwise noted.
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\[ C^* = \frac{R - \frac{I \cdot \gamma}{p_h} - \frac{b}{\Delta p}}{\frac{1}{\Delta p} \left( 1 - \frac{b \cdot \gamma}{p_h \cdot \beta} \right)}. \]  \hfill (11)

The monitoring cost cutoff \( C^* \) increases with the net present value of the projects, \( R - (I \cdot \gamma / p_h) \), and decreases with the prices of capital, \( \beta \) and \( \gamma \), and the size of the potential private benefit after monitoring \( b \).

**Lemma 2.** The optimal financial contract consists of an amount of bank financing, \( I^*_b(x) \), and of investor financing, \( I^*_f(x) \), and of their corresponding shares in the project’s payoff, \( R^*_b(x) \) and \( R^*_f(x) \), given by:

\[ I^*_b(x) = \frac{p_l \cdot C(x)}{\beta \cdot \Delta p}, \quad R^*_b(x) = \frac{C(x)}{\Delta p}, \]  \hfill (12)

\[ I^*_f(x) = I - \frac{p_l \cdot C(x)}{\beta \cdot \Delta p}, \quad R^*_f(x) = \left( 1 - \frac{p_l \cdot C(x)}{\beta \cdot \Delta p} \right) \cdot \frac{\gamma}{p_h}. \]  \hfill (13)

Four observations emerge from Lemma 2: (i) The entrepreneur reduces the participation of the bank to the minimum level at which monitoring is credible, \( I^*_b(x) = (p_l \cdot C(x)) / (\beta \cdot \Delta p) \), and finances the remaining investment with cheaper uninformed capital, \( I^*_f(x) \). (ii) Uninformed investors participate in the project only when sufficient informed capital is present. (iii) All projects require bank participation, but they differ in their bank monitoring costs. (iv) Projects with larger bank monitoring costs require more bank capital.

Financing positive-net-present-value projects can be infeasible for two reasons. First, even after bank monitoring, the agency problem remains too severe. Second, the bank must receive monitoring rents. Bank rents, i.e., \( W(x) \), which can be computed as \( W(x) = p_h \cdot R^*_b(x) - \gamma \cdot I^*_b(x) - C(x) = \frac{p_h}{\beta}(p_l \cdot C(x)) / \Delta p \), are a consequence of the scarcity of bank capital, and they will play a crucial role in the analysis of bank competition.

**2.2. Equilibrium with One Bank**

In order to characterize the equilibrium fully, it remains to compute the cost of bank capital, \( \beta^* \), and to find out the marginal project \( m^* \) to be financed. This requires Eqs. (14) and (15) to be solved:

\[ \frac{C(m^*)}{\Delta p} + \frac{b}{\Delta p} + \frac{(I - I^*_b(m^*)) \cdot \gamma}{p_h} = R, \]  \hfill (14)

\[ \int_0^{m^*} I^*_b(x) \, dx = K_0, \]  \hfill (15)

where \( I^*_b(x) = (p_l \cdot C(x)) / (\beta^* \Delta p) \) comes from the solution of program (4).

\[ ^{23} \text{This assumes that } \beta \geq \gamma, \text{ which in equilibrium will be the case.} \]
Replacing \( I_b(x) \) in Eqs. (14) and (15), we obtain:

\[
\frac{C(m^*)}{\Delta p} + b\left(\frac{1 - \frac{\gamma}{\beta^*} \cdot \frac{c(m^*)}{\Delta p}}{p_b}\right) \cdot \gamma = R, \tag{16}
\]

\[
\int_0^m p_t \cdot C(x) \cdot \frac{\beta^*}{\Delta p} \cdot dx = K_0. \tag{17}
\]

Equations (16) and (17) are best interpreted separately. For a given \( \beta^* \), Eq. (16) represents the sharing of profits of the marginal project—the project in which the entrepreneur keeps the minimum payoff consistent with incentive compatibility (i.e., \( \frac{b}{\Delta p} \)). Equation (17) is the resource constraint for bank capital. For a given \( m^* \), \( \beta^* \) adjusts to employ all informed capital on the projects. Solving we get:

\[
m^* = \frac{R - \frac{I \cdot \gamma}{p_b} - \frac{b}{\Delta p}}{\sqrt{\left(R - \frac{I \cdot \gamma}{p_b} - \frac{b}{\Delta p}\right)^2 + \frac{8 \cdot \frac{c}{p_b} \cdot K_0}{\Delta p}}} + \frac{\sqrt{2 \cdot \frac{c}{p_b}}}{\Delta p}.
\tag{18}
\]

\[
\beta^* = \frac{c \cdot \frac{p_t \cdot (m^*)^2}{\Delta p}}{2 \Delta p \cdot K_0}.
\tag{19}
\]

Equation (18) shows that the distance of the marginal project to the bank, i.e., \( m^* \), is composed of terms related to the net present value of the project net of agency rents, i.e., \( R - \frac{I \cdot \gamma}{p_b} - \frac{b}{\Delta p} \); the amount of bank capital, i.e., \( \frac{8 \cdot \frac{c}{p_b} \cdot K_0}{\Delta p} \); and the minimum amount compatible with bank’s incentives, i.e., \( 2 \frac{c}{p_b} \). Propositions 1 and 2 offer the main comparative statics.

**Proposition 1.** The distance of the marginal project to the bank, \( m^* \), increases with the net present value of the projects \( R - I \cdot \gamma / p_b \), and with the amount of bank capital \( K_0 \), and decreases with the monitoring cost \( c \), the interest rate \( \gamma \), and the size of the private benefit \( b \).

**Proposition 2.** The cost of capital, \( \beta^* \), increases with the net present value of the projects \( R - I \cdot \gamma / p_b \) and decreases with the bank capital \( K_0 \), the interest rate \( \gamma \), the monitoring cost \( c \), and the size of the private benefit \( b \).

The prior computation of the equilibrium is valid as long as \( \beta^* \geq \gamma \) (i.e., that bank capital is scarce). If this condition does not hold (i.e., \( \beta^* < \gamma \)), the equilibrium can be computed by imposing \( \beta = \gamma \) (the bank can always obtain the return of uninformed capital) and substituting it into (18). This gives the marginal project, \( m_\gamma \), in case that bank capital is not scarce, i.e., \( K_0 > K_{m_\gamma} \):

\[
m_\gamma = \frac{R - \frac{I \cdot \gamma}{p_b} - \frac{b}{\Delta p}}{\left(1 - \frac{p_t}{p_b}\right) \cdot \gamma}.
\tag{20}
\[ K_{m_y} = \frac{p_h \cdot (R - \frac{I \cdot \gamma}{p_h} - \frac{b}{\Delta p})^2}{2 \cdot (1 - \frac{p_h}{p_0}) \cdot \frac{c}{\Delta p}}. \]  

(21)

Proposition 3 summarizes the previous discussion:

**Proposition 3.** If bank capital, \( K_0 \), is scarce, i.e., \( K_0 \leq K_{m_y} \), then the equilibrium cost of bank capital and the equilibrium marginal project are \( \beta^* \) and \( m^* \), respectively. If bank capital is not scarce, i.e., \( K_0 > K_{m_y} \), they are instead \( \gamma \) and \( m_Y \).

3. ANALYSIS OF THE MODEL: BANK COMPETITION

In order to study the effects of bank capital on bank competition, I now solve the model with two banks that (i) own different amounts of capital, and (ii) are located at opposite points on the circumference. I denote the point at which the high-cap bank is located as 0 and the point at which the low-cap bank is located as 1, and denote their respective amounts of capital as \( K_0 > K_1 \).

An important assumption that I maintain throughout the analysis is that bank capital is fixed and exogenously endowed. This assumption is justified if bank capital represents inside equity and wealth constraints impede insiders from widening their bank’s capital base. Furthermore, as long as private monitoring costs are incurred by insiders, the ability to raise outside equity will not help solve the agency problem between banks and their lenders.\(^{24}\) More generally, Myers and Majluf (1984) demonstrate that issuing equity, an informationally sensitive security, can have substantial costs.\(^{25}\) This suggests that differences in banks’ capitalization will persist, and thus, that a model that takes the amount of capital owned by banks as fixed can offer reasonable implications.

For bank competition to occur, the bank at 0 ought to be able to finance projects at the opposite half of the circumference. In what follows I assume that the monitoring cost parameter is small enough so that the bank at 0 can feasibly monitor a project located at 1. Formally, I assume that condition (22) holds:

\[ c \leq \left( R - \frac{I \cdot \gamma}{p_h} - \frac{b}{\Delta p} \right) p_h. \]  

(22)

3.1. The Definition of Equilibrium

The model features four regimes depending on the amount of capital owned by banks. When the capital of the two banks is very scarce (regime 1), no bank interaction occurs. When bank capital is plentiful (regime 4), banks get no rents and

\(^{24}\) Undoubtedly, a number of interesting issues arise if the equity issuance process is endogenized. See footnote 31.

\(^{25}\) A contrasting argument is presented in Boot and Thakor (1993), who show that by stimulating informed trading, issuing informationally sensitive securities can end up reducing the cost of capital for the issuer.
FIG. 2. Two-bank competition. This figure depicts the matching between banks and projects with two-bank competition. Banks are located at 0 and 1, and projects uniformly along the circumference. The bank at 0 finances the projects located between 0 and \( y^* \) and between \( (2 - y^*) \) and 2 (i.e., 0). The rest of the projects are financed by the bank at 1.

therefore no monitoring distortion would occur. An intermediate situation occurs only if the low-cap bank gains scarcity rents (regime 3). Finally, both banks can interact and simultaneously obtain rents for their capital (regime 2).

I focus on regime 2 next and offer a brief analysis of the other three regimes in the appendix. Regime 2 is the most interesting because it allows for analysis of bank competition when rents are being generated, and thus, as shown in Section 4, it elicits the most interesting comparative statics results.26

As in the one-bank case, an equilibrium should determine the balance between bank and investor capital invested in each project. However, in the case of more than one bank involved, the matching between investors and banks also needs to be considered. Fortunately, the analysis is greatly simplified by the assumption that banks in the circumference are located opposite of the same diameter such that the interaction between the banks from both sides of the circumference is symmetrical.27 In fact, the equilibrium is fully defined by a triple \((\beta_0^*, \beta_1^*, y^*)\), which contains the costs of capital of the banks at 0 and 1, i.e., \(\beta_0^* \) and \(\beta_1^* \), and the location that separates the banks’ portfolios on the semicircumference between 0 and 1, i.e., \(y^* \in (0, 1).\)28 Moreover, \(2y^* \) is also a measure of the size of the portfolio of the bank at 0. Figure 2 represents the equilibrium with two banks.

3.2. Analysis of the Model

In order to compute the equilibrium, I must solve for the agents’ optimal decisions. Consider first the entrepreneurs’ financing decisions. If they finance their

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26 Also see Proposition 13 in the Appendix for the amounts of capital that separate regimes.
27 In Section 5, I endogenize bank location and examine the general case in which banks interact asymmetrically in the two halves of the circumference.
28 By symmetry, \(2 - y^* \) corresponds to the location that separates banks’ portfolios at the semicircumference between 1 and 2.
projects from the bank at 0 (at 1), they would use an amount of bank capital given by Eq. (23) (Eq. (24)):

\[ I_{b0}^*(x) = \frac{p_l \cdot c}{\beta_0 \cdot \Delta p} x, \]  
\[ I_{b1}^*(x) = \frac{p_l \cdot c}{\beta_1 \cdot \Delta p} (1 - x). \]  

(23)  
(24)

In contrast to the one-bank case, entrepreneurs must compare the financial conditions offered by the two banks. In equilibrium, the entrepreneurs can be separated between those who use the bank at 0 (Eq. (23)) and those use the bank at 1 (Eq. (24)). Further, there is a unique entrepreneur indifferent between banks.\(^\text{29}\) Let \( R_e(x, i) \) be the residual cash flow in case of success for an entrepreneur located at \( x \) financed by a bank located at \( i \). Hence:

\[ R_e(x, 0) = R - \frac{c \cdot x}{\Delta p} - \frac{(I - I_{b0}^*(x)) \cdot \gamma}{p_h}, \]  
\[ R_e(x, 1) = R - \frac{c \cdot (1 - x)}{\Delta p} - \frac{(I - I_{b1}^*(x)) \cdot \gamma}{p_h}. \]  

(25)  
(26)

For the indifferent entrepreneur, it must be \( R_e(y^*, 0) = R_e(y^*, 1) \), or, expressed differently:

\[ \frac{c \cdot y^*}{\Delta p} - \frac{I_{b0}^*(y^*) \cdot \gamma}{p_h} = \frac{c \cdot (1 - y^*)}{\Delta p} - \frac{I_{b1}^*(y^*) \cdot \gamma}{p_h}. \]  

(27)

Consider next the behavior of banks. Idle capital is incompatible with perfect competition and obtaining rents from capital use. Consequently, banks employ all of their capital in project financing. In equilibrium:

\[ \int_0^{y^*} I_{b0}^*(x) \, dx = K_0 \]  
\[ \int_{y^*}^1 I_{b1}^*(x) \, dx = K_1. \]  

(28)  
(29)

Combining (27) with (28) and (29) yields Eq. (30), which relates the equilibrium market share \( y^* \) with the amounts of bank capital \( (K_0, K_1) \):

\[ \frac{K_0}{y^*} - \frac{K_1}{1 - y^*} = \frac{p_h \cdot c}{2 \Delta p \cdot \gamma} (2y^* - 1). \]  

(30)

\text{This is formally stated in Proposition 4 and proven in the Appendix.}
FIG. 3. Bank competition with scarce capital. This figure depicts the equilibrium relationship between the distance from a project to a bank and the amount of bank financing used in such project. The figure displays the use of bank financing in the semicircumference between 0 and 1, where banks are located at 0 and 1 and projects uniformly along the segment line. The bank at 0 finances projects located between 0 and \( y^* \) and the bank at 1 those between \( y^* \) and 1.

Once \( y^* \) is obtained, the equilibrium returns on bank capital follow:

\[
\begin{align*}
\beta^*_0 &= \int_0^{y^*} \frac{\left(\frac{c}{\Delta p} - c\right)x}{K_0} \, dx = \frac{c \cdot p_1 \cdot y^{*2}}{2\Delta p K_0}, \\
\beta^*_1 &= \int_{y^*}^{1} \frac{\left(\frac{c}{\Delta p} - c\right)(1-x)}{K_1} \, dx = \frac{c \cdot p_1 \cdot (1 - y^*)^2}{2\Delta p K_1}.
\end{align*}
\]

(31)

(32)

Figure 3 offers a graphical representation of the equilibrium,\(^{30}\) while Proposition 4 summarizes its main properties.

**Proposition 4.** The equilibrium features separate intervals. Both banks earn rents, i.e., \( \gamma < \beta^*_0 < \beta^*_1 \), and the high-cap bank finances an excessive number of projects relative to first best: \( y^* > \frac{1}{2} \).

A few observations emerge from this analysis: (i) The financial advantages of a high-cap bank allow it to finance some projects that a low-cap bank could monitor at lower cost. (ii) The monitoring efficiency of the credit market (i.e., the amount of monitoring costs that are dissipated in the industry) can be measured by \( y^* \), the market share of the high-cap bank. Larger values of \( y^* \) correspond to a poorer matching between projects and banks. (iii) Two prices for bank capital (\( \beta^*_0 \) and \( \beta^*_1 \)) can coexist in equilibrium without creating arbitrage opportunities. An entrepreneur does not necessarily choose the cheapest source of bank capital, because the quantity of capital is also relevant to his or her decision. (iv) The possibility of transferring bank capital between banks does not eliminate the role of

\(^{30}\) The segment (0, 1) represents the semicircumference between 0 and 1.
bank capital. Due to moral hazard, monitoring only occurs if a bank keeps a residual share for itself, and keeping a residual share impedes adequate compensation for the bank that transfers the capital. Hence, a competitive interbank market that is subject to the informational asymmetries considered here would not alleviate the problem of credible monitoring by banks.

3.3. Discussion of the Basic Trade-off

Suppose an entrepreneur is located at $x > \frac{1}{2}$, and assume that both banks offer, respectively, the amounts of funds $I_0$ and $I_1$ to participate in his or her project. In order to attract uninformed capital, monitoring must be credible, which requires the entrepreneur at $x$ to give up from his or her payoff in case of success, either $\frac{c \cdot x}{\Delta p}$ to the bank at 0 or $\frac{c \cdot (1-x)}{\Delta p}$ to the bank at 1. Hence, the entrepreneur chooses the bank at 1 (his or her natural bank) only if

$$I_0 \cdot \gamma - \frac{c \cdot x}{\Delta p} p_h \leq I_1 \cdot \gamma - \frac{c \cdot (1-x)}{\Delta p} p_h,$$

or, if one rearranges the equation:

$$(2x - 1) \cdot \frac{c \cdot p_h}{\Delta p} \geq (I_0 - I_1) \cdot \gamma. \quad (33)$$

The left-hand side of inequality (33) reflects the cost differences between banks (including monitoring costs and rents), while the right-hand side captures the cost savings of raising capital from external sources. The interpretation of this trade-off is straightforward. Because minimum payments to banks are fixed for incentive compatibility reasons, banks compete by offering capital to entrepreneurs. A bank with more capital can increase the amount it offers, while a bank with more expertise can reduce credibly the ex-post amount of cash flow. This trade-off between capital and expertise drives the results of the model.

4. FINANCIAL SHOCKS, GEOGRAPHIC RESTRICTIONS, AND BANK COMPETITION

In this section, I use the model to examine two issues. First, I analyze the effect of changes in financial conditions (shocks to bank capital and interest rates) on bank competition. Second, I illustrate the effects on bank competition of lifting geographic restrictions on banking. In both cases, I restrict the analysis to the case of bank competition with rents (regime 2).
4.1. Shocks in Financial Conditions

4.1.1. Shocks to Bank Capital

Suppose both banks experience a positive multiplicative shock to their capital of size \((1 + dK_0)\) and \((1 + dK_1)\), respectively. For simplicity, consider the case in which, after the shocks, the bank at 0 remains the high-cap bank:

\[
K_0 \cdot (1 + dK_0) > K_1 \cdot (1 + dK_1). \tag{34}
\]

Proposition 5 thus considers the effects of capital shocks on competition:

**Proposition 5.** The high-cap bank's market share \(y^\ast\) increases (decreases) whenever the ratio of shocks in capital (i.e., \(dK_0/dK_1\)) is larger (smaller) than a cut-off, \(G^\ast(K_0, K_1)\), such that:

\[
G^\ast(K_0, K_1) = \frac{K_1}{K_0} \cdot \frac{y^\ast}{1 - y^\ast}. \tag{35}
\]

An interesting case is the case of proportional shocks to bank capital:

**Corollary 1.** If bank capital increases proportionally, then the high-cap bank gains market share (i.e., \(G^\ast(K_0, K_1) < 1\)).

A narrow interpretation of the previous results would consider only shocks that affect bank capital directly (e.g., changes in regulatory capital requirements or depletion of the capital base due to loan losses). A broader interpretation would include other events that affect the availability of financial resources to banks or the use of those resources in the intermediation process. For example, securitization and other disintermediation activities can liberate capital by helping to mobilize the financial resources of banks more effectively. Proposition 5 suggests that these financial innovations are not neutral but have effects on bank competition: High-cap banks benefit from financial innovations that expand capital or facilitate its use.

Changes in the monitoring parameter \(c\) produce results parallel to proportional changes in bank capital. As Eq. (30) shows, a reduction in \(c\) (for example, due to technological progress) produces the same effects as a proportional increase in bank capital. It is remarkable that shocks to bank capital and shocks to monitoring costs have exactly the opposite effects. This again highlights the effects of the trade-off between capital and expertise and illustrates the likely effects of technological change on bank competition. Both financial and technical innovations tend to magnify the effect of bank capital on bank competition.\(^{31}\)

\(^{31}\) However, technological shocks could produce different effects in a model with endogenous bank capital. For instance, consider a variation of the model in which the amount of bank capital increases with its expertise in the marginal project financed. In such a model, technological shocks could affect the ability of different banks to raise capital and thus increase the market share of the low-cap bank (which has better knowledge of its borrowers) at the expense of the high-cap bank.
4.1.2. Interest Rate Shocks

The second exercise considers changes in $\gamma$, the return of an alternative technology that represents the marginal productivity of capital and therefore the opportunity cost of investors’ uninformed capital.\(^{32}\) Here I relate such changes to the effects of monetary policy and use the model to examine the effects of monetary policy on bank competition. Further, I assume that money is nonneutral so that changes in nominal rates translate into changes in real ones.

**Proposition 6.** An increase in the riskless interest rate (and hence the bank’s expected cost of funding) will favor the high-cap bank over the low-cap bank (i.e., $y^*$ moves away from $\frac{1}{2}$). Furthermore, an increase in this interest rate will reduce the spread in bank capital costs: $d(\beta_0^* - \beta_1^*)/d\gamma < 0$.

Proposition 6 shows that an increase in interest rates makes the high-cap bank gain market share and the low-cap bank lose market share. The reason is the following. When $\gamma$ increases, the optimal balance of external to internal funds changes for entrepreneurs. Because the ex-post payoff to banks, which is related to monitoring costs, remains fixed, increases in $\gamma$ make bank capital more valuable. Therefore, tighter monetary conditions make the relative financial strength of banks more valuable and their relative lack of expertise less important.

That financial considerations become more important in an environment of high interest rates is unsurprising and should be concluded from any reasonable model in which financial constraints play a meaningful role. However, the analysis also shows that financial considerations gain relevance at the expense of the operating efficiency in the credit market. Higher interest rates worsen the matching between borrowers and lenders and, overall, represent a waste of resources due to a less efficient monitoring in the banking industry.\(^ {33}\)

Proposition 6 illustrates a novel role of bank capital in monetary policy. If capital is heterogeneously divided among banks, monetary shocks (i.e., liquidity contractions or expansions) will produce competition effects. These effects complement other effects stressed in the literature on the lending channel effects of monetary policy (e.g., Kashyap and Stein, 1995, 2000). There, it is argued that the tightening of monetary conditions has a strong effect on bank-dependent firms, because bank lending declines after the monetary contraction and borrowers cannot substitute toward other sources of finance. Here, I suggest that low-capitalized, highly specialized intermediaries will suffer disproportionately from a monetary contraction and that these effects should be particularly important in market segments with acute financial asymmetry among banks.\(^ {34}\)

\(^{32}\) Implicitly, I am considering that assumptions 1, 2, and 3 still hold when $\gamma$ changes.

\(^{33}\) However, a caveat similar to the one in footnote 31 applies. In a model with endogenous bank capital, the interest rate shock could affect both the ability of banks to raise capital and the cost of the capital raised.

\(^{34}\) See also Thakor (1996), who discusses other potential effects of monetary policy on bank lending policies. By examining interbank competition for loans in the presence of equilibrium credit rationing,
4.2. The Effects of Lifting Geographic Restrictions

After more than six decades of geographic limitations in the American banking industry, the Riegle–Neal Interstate Banking and Branching Efficiency Act of 1994 permitted interstate branching. A model with a spacial structure seems appropriate to illustrate the major competitive implications of this regulatory change.

For simplicity, I compare two scenarios. In the first, banks are limited to lend to projects no further than half of the distance from their competitor. The second scenario is simply the equilibrium identified above. Proposition 7 compares them:

**Proposition 7.** Lifting geographic restrictions results in an increase (decrease) in market share for the high-cap (low-cap) bank and an increase (decrease) in its profits.

Not surprisingly, Proposition 7 asserts that the high-cap bank benefits and the low-cap bank loses from the lifting of geographic restrictions, a prediction made previously in the banking literature. For example, Abrams and Settle (1993) refer to the rent-seeking theory (first referred to in Sprague, 1903) and assert that the basic conflict resulting from a relaxation of branching restrictions is that large banks would out-compete small banks for the rents they had previously enjoyed. Furthermore, the empirical evidence seems to support rent-seeking effects. For instance, consistent with the mechanism explored in this model, Flannery (1984) documents that geographic restrictions enhance small-bank profits.

A full examination of the lifting of geographic restrictions as a regulatory measure requires considering some other effects. First, lifting geographic restrictions will improve banks’ ability to diversify risks. Second, it can affect the cost efficiency by banks in a number of ways. For instance, while economies of scale would increase such efficiency, a worsening of the matching between lenders and borrowers, as suggested by the analysis presented here, would diminish it. Third, the availability of credit to some of the firms can be affected. On the one hand, it can be reduced for firms that are far from the expertise of the high-cap bank. This will occur when the low-cap bank loses its economic viability and other banks, which lack information about the firm, fail to finance them. On the other hand, the availability of credit can increase if lifting geographic restrictions makes high-cap banks reach firms that were previously unable to obtain funds. See Berger et al. (1999) for further discussion and an empirical examination of these effects.

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He shows that, with risk-based capital requirements, the effect of monetary policy on bank lending policies depends on its effect on the term structure of interest rates and demonstrates that easing monetary policy could either reduce or increase bank lending.

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5. THE EXPERTISE CHOICE

This section considers the expertise choice (i.e., location) of banks. The purpose is twofold. The first is to offer foundations for the previous analysis: How reasonable is it to assume that banks are separated in the expertise dimension? The second is to explore the effects of bank capital on the incentives for intermediaries to specialize.

In terms of the model developed so far, the expertise choice refers to the addition of a preliminary stage in which banks choose their location in the expertise circumference. I consider the following timing of events. First, banks choose their locations in the circumference. Second, financial contracts among entrepreneurs, investors, and banks are signed. Third, actions by entrepreneurs and banks (e.g., monitoring) are taken. Fourth, payoffs are realized.

The analysis will proceed in two steps. First, I characterize the equilibrium for arbitrary locations of banks in the circumference. Second, I examine the choice of location by banks.

5.1. Equilibrium as a Function of Bank Locations

To examine the equilibrium as a function of bank locations, I consider the case in which competition among banks plays a central role. Specifically, I assume that the amount of capital owned by the high-cap bank is large enough, and/or the monitoring cost is small enough, such that, if standing alone, the high-cap bank could finance all of the projects in the market (i.e., the circumference) by itself. \[ m^a > 1 \]

In this case, irrespective of the location of the low-cap bank, meaningful bank competition will exist.

As a convention, I set the location of the high-cap bank at 0 (and given the circularity of the market, also at 2) and denote the location of the low-cap bank as \( d \). Notationally, I use the subindexes 0 and \( d \) respectively for the high- and low-cap bank. Given \( d \), the location for the low-cap bank, an equilibrium is a situation in which (i) entrepreneurs choose their cheapest financing sources, and (ii) banks employ all their capital to finance projects.

I will proceed as follows. First, I separate the possible equilibria in two classes and then offer the equations describing each class and a graphical illustration of each. I refer to the third part of the Appendix for the actual derivation of the equations and for a formal justification of the existence, uniqueness, and continuity of the equilibrium proposed. I restrict the analysis to examine the equilibrium locations of the low-cap bank between [1, 2] (i.e., \( d \in [1, 2] \)). The equilibrium for locations between 0 and 1 follows from the symmetry of the problem. Using the fact (proven in the Appendix) that, in equilibrium, bank portfolios are continuous in project locations, I identify the two projects \( y^*_L \) and \( y^*_R \) that separate bank portfolios.

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36 See Boot and Thakor (2000) for an alternative and thorough analysis of a bank’s incentives to specialize in the context of relationship banking.

37 Formally, I assume that \( m^a \), as defined in (18), is greater than 1.
and characterize the equilibrium fully. Projects $y_L^*$ and $y_R^*$ are marginal projects in the sense that they correspond to entrepreneurs that are indifferent in choosing between either bank as their financier. I denote by $y_L^*$ the marginal project to the left of $d$ and use $y_R^*$ to denote the project to its right (i.e., $y_L^* < d < y_R^*$).

According to the location of $y_L^*$, two equilibria can be defined: (i) far equilibrium if $y_L^* \leq 1$ and (ii) close equilibrium if $y_L^* > 1$. Proposition 8 establishes the existence and classes of equilibria:

**Proposition 8.** Location $d^*$ determines the class of equilibrium. Specifically, if $d < d^*$ ($d > d^*$), a unique far (close) equilibrium exists.

In the Appendix I characterize $d^*$. Here, I simply present the two sets of equations that describe the equilibria. Equations (36) to (39) characterize the far equilibrium:

\[
\left(1 - \frac{p_l \gamma}{p_h \beta_0^*}\right) \cdot y_L^* = \left(1 - \frac{p_l \gamma}{p_h \beta_d^*}\right) \cdot (d - y_L^*)
\] (36)

\[
\left(1 - \frac{p_l \gamma}{p_h \beta_0^*}\right) \cdot (2 - y_R^*) = \left(1 - \frac{p_l \gamma}{p_h \beta_d^*}\right) \cdot (y_R^* - d)
\] (37)

\[
\beta_0^* = \max \left\{ \gamma, \frac{p_c}{2 \Delta p K_0} (y_L^2 + (2 - y_R^*)^2) \right\}
\] (38)

\[
\beta_d^* = \max \left\{ \gamma, \frac{p_c}{2 \Delta p K_d} (d - y_L^2 + (y_R^* - d)^2) \right\}
\] (39)

and Eqs. (40) to (43) characterize the close equilibria:

\[
2 - d = \frac{p_l \gamma}{p_h \beta_0^*} \cdot (2 - y_L^*) - \frac{p_l \gamma}{p_h \beta_d^*} \cdot (d - y_L^*)
\] (40)

\[
\left(1 - \frac{p_l \gamma}{p_h \beta_0^*}\right) \cdot (2 - y_R^*) = \left(1 - \frac{p_l \gamma}{p_h \beta_d^*}\right) \cdot (y_R^* - d)
\] (41)

\[
\beta_0^* = \max \left\{ \gamma, \frac{p_c}{2 \Delta p K_0} (-y_L^2 + 4 y_L^* - 2 + (2 - y_R^*)^2) \right\}
\] (42)

\[
\beta_d^* = \max \left\{ \gamma, \frac{p_c}{2 \Delta p K_d} ((d - y_L^*)^2 + (y_R^* - d)^2) \right\}
\] (43)

Equation (36) ((40)) defines the marginal project to the left of the low-cap bank in the far (close) equilibrium. Similarly, Eq. (37) ((41)) defines the marginal project to the right of the low-cap bank in the far (close) equilibrium. Equations (38), (39), (42), and (43) correspond to the definition of the cost of bank capital for the high-cap and the low-cap banks in the far and close equilibrium, respectively. Variables

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38 More precisely, an equilibrium is described by the vector $(y_L^*, y_R^*, \beta_0^*, \beta_d^*)$, where $\beta_0^*$ and $\beta_d^*$ represent the returns on bank capital.
FIG. 4. Far equilibrium. This figure depicts the use of bank financing as a function of a project’s location in a far equilibrium. In a far equilibrium the low-cap bank, located at \( d \), finances projects located in both halves of the credit market (i.e., projects between \( y_L \) and \( y_R \)), and the high-cap bank, located at 0, finances the rest.

with an asterisk represent the equilibrium values of the endogenous variables of the system. Graphic illustrations (see Figs. 4 and 5) help to explain the differences among equilibria.39

As is apparent from the graphs, the main characteristic of the far equilibrium is the large separation among banks. That separation allows the low-cap bank to compete in both halves of the credit market. In contrast, in the close equilibrium, the short distance among banks ensures that the high-cap bank controls one half of the market.

5.2. The Location Equilibrium

I now can proceed to examine the location equilibrium. The location analysis consists of finding the location that maximizes and minimizes profits for both banks. Formally, in the case of close equilibrium, the low-cap solves40

\[
\text{Max } \beta_d^* \quad \text{subject to (36)--(39).}
\]

And in the case of far equilibrium, the low-cap solves

\[
\text{Max } \beta_d^* \quad \text{subject to (40)--(43). Proposition 9 characterizes the solutions.} 41
\]

PROPOSITION 9. Some separation (i.e., \( d^* = d^\circ \)) is the most (least) preferred location for the low-cap (high-cap) bank. In contrast, no separation (i.e., \( d^* = 2 \)) is the most (least) preferred location for the high-cap (low-cap) bank.

39 As in previous sections, for simplicity, I represent graphically the circumference in a segment line with the high-cap bank located at 0 and 2.
40 The problem of the high-cap bank could be similarly formalized.
41 In the Appendix, I present a numerical example that illustrates the equilibrium.
Proposition 9 implies that the low-cap bank minimizes its comparative financial disadvantage through specializing, i.e., separating itself from the high-cap bank. Specialization allows it to concentrate on a market niche while minimizing the effects of its financial weakness. Proposition 9 suggests that, for intermediaries, financial specialization substitutes for financial strength.\(^{42}\)

Finally, it remains to consider the equilibrium of the location game. Because banks have conflicting interests in location, the equilibrium of the game is sensitive to the order of moves. Proposition 10 states the three possibilities:

**Proposition 10.** *The equilibrium of the location game is the following:*

(a) If the low-cap chooses its location first, then no separation occurs.
(b) If the high-cap chooses its location first, then some separation occurs.
(c) If banks choose their locations simultaneously, then some separation occurs with probability one.

Note that in both (b) and (c), bank separation occurs in equilibrium. Importantly, both the equilibrium characterization and the comparative statics results derived in Sections 3 and 4 do not depend on full separation but hold also with some separation. Consequently, Proposition 10 offers foundations for the results derived before.

6. CONCLUDING REMARKS

I have presented a model of competition in the banking industry based on the interaction between bank capital and monitoring expertise. Expertise is associated with the technical knowledge that allows a bank to offer its lending services for less. Expertise, moreover, is a relative concept. Because different projects require

\(^{42}\) However, maximum separation, i.e., \(d = 1\), is not optimal for the low-cap bank. The reason is that bank rents increase with their distance from the monitored project, which induces banks to locate away from projects. So separation from the high-cap bank reduces the high-cap competition but also reduces the distance to the monitored projects, which, in turn, lowers bank rents.
different knowledge, expertise is always expertise in relation to particular projects or lines of business.

One of the main insights of this paper is that as banks compete to lend, their capital and expertise are substitutes. This happens because, first, in an environment of moral hazard and limited liability, banks must be guaranteed ex-post rents in order to monitor diligently, and, second, both expertise and capital affect the size of those rents.

The joint consideration of capital and expertise leads to some novel conclusions about efficiency. The banking industry will be more efficient the more important expertise is relative to capital. In addition, this analysis has provided several comparative statics results that can be translated into testable empirical implications. Specifically, highly capitalized banks should benefit when competing with those that are poorly capitalized after (i) a decrease in capital requirements or some others regulatory shocks that causes both banks’ capital to increase proportionally, (ii) technological improvements that reduce monitoring costs by intermediaries, (iii) an increase in the interest rate due to a tightening of monetary policy, and (iv) a lifting of geographic restrictions. Shocks in the opposite direction favor lower-capitalized banks. Finally, the analysis of bank expertise has suggested another empirical implication: (v) poorly capitalized banks have stronger incentives for financial specialization than banks with more capital.

The previous implications are subject to two important caveats that motivate future research. The first caveat concerns the entrepreneurial character of banks. I have assumed that the amount of capital severely limits the expansion of a bank, but I have ignored agency considerations inside the bank. However, in practice equity owned by managers (i.e., inside equity) can represent only a small portion of the total equity. Therefore, in a strict sense, the analysis must either be interpreted in terms of inside equity or explicit agency considerations should be included. The second caveat refers to treatment of bank capital as an exogenous variable. While this may be valid in some circumstances, a fully satisfactory treatment of long-term effects would require a model of bank capital accumulation. This task remains a challenge left for future research.

**APPENDIX**

The Appendix consists of three parts. The first part considers the analysis of the other regimes of competition omitted in Section 3. The second part offers the proofs of the propositions and results obtained in the text. Finally, the third part deals with technical derivations related to the expertise choice in Section 5.

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43 In this analysis bank capital and bank size (i.e., the amount that a bank can invest) are perfectly correlated. However, in practice, banks with the same level of capital may feature different capital to asset ratios. So in a strict sense, the empirical implications may refer to bank size rather than to bank capitalization.
A.1. Other Regimes of Competition

A.1.1. Regime 1: No Bank Interaction

The analysis of regime 1 is identical to the analysis with one bank. With bank competition, regime 1 is present only if (i) the amount of bank capital of both banks is small and (ii) \( c \) is not too small. To see the importance of (ii) define \( m_0 \) as the marginal project financed by a bank without any capital,

\[
\frac{c \cdot m_0}{\Delta p} + \frac{b}{\Delta p} + \frac{I \cdot \gamma}{p_h} = R,
\]

or

\[
m_0 = \left( R - \frac{I \cdot \gamma}{p_h} - \frac{b}{\Delta p} \right) \frac{\Delta p}{c}. \tag{46}
\]

Regime 1 can occur only if \( m_0 < \frac{1}{2} \). In terms of the monitoring cost:

\[
c \geq \left( R - \frac{I \cdot \gamma}{p_h} - \frac{b}{\Delta p} \right) 2 \Delta p. \tag{47}
\]

The computations of \( \beta^*_0 \) and \( \beta^*_1 \) and the marginal projects in regime 1 are like those performed in Section 2 and are therefore omitted.

A.1.2. Regime 3: Only the Low-cap Bank Earns Rents

Regime 3 corresponds with a situation in which only the low-cap bank earns rents. Formally regime 3 would occur for the set of pairs \((K_0, K_1)\) for which the restriction \( \beta^*_0 \geq \gamma \) binds. To compute the equilibrium in regime 3 we must identify the indifferent entrepreneur

\[
R_e(y^*, 0) = R_e(y^*, 1) \tag{48}
\]

and consider the incentives for banks to finance projects. Formally:

\[
\int_{y^*}^{1} I^{s1}_b(x) \, dx = K_1 \tag{49}
\]

\[
\int_{0}^{y^*} I^{s0}_b(x) \, dx \leq K_0. \tag{50}
\]

Idle capital for the bank at 0 (expression (50)) implies:

\[
\beta^*_0 = \gamma. \tag{51}
\]

\[44\] The precise conditions are given in Proposition 13.
Equations (48), (49), and (51) characterize the equilibrium. Combining them we find:

$$\frac{K_1}{1 - y^*} = \frac{c \cdot p_h}{2 \Delta p} \left[ 1 - \left( 2 - \frac{p_l}{p_h} \right) y^* \right].$$  

(52)

Once $y^*$ is obtained, $\beta_1^*$ can be computed as:

$$\beta_1^* = \int_{y^*}^{1} \frac{(\frac{c \cdot p_h}{\Delta p} - c)(1 - x)}{K_1} dx = \frac{c \cdot p_l \cdot (1 - y^*)^2}{2 \Delta p K_1}.$$  

(53)

Proposition 11 presents the properties of regime 3:

**Proposition 11.** In regime 3, (i) the equilibrium features separate intervals, (ii) $y^* > \frac{1}{2}$, and (iii) only the low-cap bank at 1 earns rents: $\beta_1^* > \beta_0^* = \gamma$.

A.1.3. Regime 4: Nonscarce Bank Capital

In regime 4, expertise considerations determine the matching between entrepreneurs and banks. Proposition 12 summarizes regime 4.

**Proposition 12.** In regime 4, $y^* = \frac{1}{2}$ and no bank earns rents: $\beta_0^* = \beta_1^* = \gamma$.

A.1.4. Equilibrium Relationships between Bank Capital and Regimes

The amount of bank capital determines the regime that governs the competition among banks. Proposition 13 partitions the $(K_0, K_1)$ plane into four regions corresponding to the four regimes of competition.

**Proposition 13.** If $K_0 > K_1$, four regions can be distinguished in the space $(K_0, K_1)$:

(a) If $K_1 \leq f(K_0)$ and $K_0 \in [0, K_0^0]$, then regime 1 holds.
(b) If $f(K_0) \leq K_1 \leq g(K_0)$ and $K_0 \in [K_0^0, K_1^0]$, then regime 2 holds.
(c) If $g(K_0) \leq K_1 \leq K_1^2$ and $K_0 > K_1^0$, then regime 3 holds.
(d) If $K_1 \geq K_1^2$, then regime 4 holds.

The function $f(.)$ relates the level of capital $K_0$ with the minimum level of capital $K_1$ required to finance all projects. The function $g(.)$ relates the level of capital $K_0$ with the minimum level of capital $K_1$ that makes the restriction $\beta_0 \geq \gamma$ exactly binding. The functions $f(.)$ and $g(.)$ are both continuous and decreasing. They, together with the values $K_0^0$, $K_1^0$, and $K_1^2$, are explicitly defined in the proof of Proposition 13 in the second part of this Appendix.
A.2. Proofs

Lemmas 1 and 2

To find the optimal contract \((R^*_b(x), R^*_f(x), R^*_e(x), I^*_b(x), I^*_f(x))\) impose equality on the restrictions and solve the linear system formed by (5), (6), (7), (8), and (10). If such solution satisfies (9), then it is the solution of the problem. Otherwise the choice set is empty and the problem does not have a solution (i.e., the project will not be funded). The marginal project satisfies (9) with equality:

\[
\frac{C(x)}{\Delta p} + \frac{b}{\Delta p} + \frac{(I - I_b(x)) \gamma}{p_h} = R; \tag{54}
\]

substituting \(I_b(x)\) for the optimal value \(I^*_b(x)\) gives the cut-off \(C^*\).

Propositions 1 and 2

I focus on the signs that are not obvious from a glance at (18) and (19). Totally differentiating (14) and (15) gives:

(a) \(m^*\) with respect to \(\gamma\)

\[
\frac{c}{\Delta p} \frac{dm^*}{d\gamma} + \left( \frac{I}{p_h} - \frac{p_l \cdot c \cdot m^*}{\Delta p \cdot \beta^* \cdot p_h} \right) \frac{d\gamma}{d\gamma} - \left( \frac{c}{\Delta p} \cdot \frac{p_l \cdot \gamma}{p_h \cdot \beta^*} \right) \frac{dm^*}{d\gamma} + \frac{p_l \cdot c \cdot m^*}{\Delta p \cdot \beta^*} \frac{d\beta^*}{d\gamma} = 0
\]

therefore:

\[
\frac{dm^*}{d\gamma} = \left( -\frac{\frac{I}{p_h} + \frac{p_l \cdot c \cdot m^*}{\Delta p \cdot \beta^* \cdot p_h}}{\frac{c}{\Delta p} \cdot (1 - \frac{p_l \cdot \gamma}{p_h \cdot \beta^*})} \cdot \frac{\Delta p \cdot \beta^* \cdot p_h}{\Delta p \cdot \beta^* \cdot p_h} - \frac{\frac{p_l \cdot c \cdot m^*}{\Delta p \cdot \beta^*}}{\frac{p_l \cdot c \cdot m^*}{\Delta p \cdot \beta^*}} \right) = (+) < 0. \tag{55}
\]

(b) \(\beta^*\) with respect to \(K_0\)

\[
\frac{c}{\Delta p} \cdot dm^* - \left( \frac{c}{\Delta p} \cdot \frac{p_l \cdot \gamma}{p_h \cdot \beta^*} \right) \frac{dm^*}{d\gamma} + \frac{p_l \cdot c \cdot m^*}{\Delta p \cdot \beta^* \cdot p_h} \frac{d\beta^*}{d\gamma} = 0
\]

\[
\frac{p_l \cdot c \cdot m^*}{\Delta p \cdot \beta^*} \frac{dm^*}{d\gamma} - \frac{p_l \cdot c \cdot (m^*)^2}{2 \Delta p \cdot (\beta^*)^2} \frac{d\beta^*}{d\gamma} - dK_0 = 0
\]

therefore:
\[
\frac{d\beta^*}{dk_0} = \left(\frac{c}{\Delta p} \left(1 - \frac{p_l \cdot \gamma}{p_h \cdot \beta^*}\right) \frac{p_l \cdot c \cdot m^*}{\Delta p \cdot \beta^*} - \frac{p_l \cdot c \cdot m^*}{\Delta p \cdot (\beta^*)^2} \right) \frac{\gamma}{p_h} = (+) < 0.
\]

(c) \(\beta^*\) with respect to \(c\)

\[
\frac{c}{\Delta p} \frac{dm^*}{dc} + \frac{m^*}{\Delta p} dc - \left(\frac{m^*}{\Delta p} \frac{p_l \cdot \gamma}{\beta^*} \right) dc
\]

\[
- \left(\frac{c}{\Delta p} \cdot \frac{p_l \cdot \gamma}{p_h \cdot \beta^*}\right) dm^* + \frac{p_l \cdot c \cdot m^*}{\Delta p} \cdot \beta^* \cdot dc - \frac{p_l \cdot c \cdot (m^*)^2}{2 \Delta p \cdot (\beta^*)^2} \cdot db^* = 0
\]

therefore:

\[
\frac{d\beta^*}{dc} = \left(\frac{c}{\Delta p} \left(1 - \frac{p_l \cdot \gamma}{p_h \cdot \beta^*}\right) \frac{p_l \cdot c \cdot m^*}{\Delta p \cdot \beta^*} \cdot \frac{c}{\Delta p} \frac{p_l \cdot (m^*)^2}{\Delta p \cdot (\beta^*)^2} \right) = (+) < 0.
\]

\[\text{PROPOSITION 4.}\]

(a) Existence and uniqueness

From (30), consider \(\Phi(y)\) as a function of \(y\):

\[
\Phi(y) = \frac{K_0}{y} - \frac{K_1}{1 - y} - \frac{c \cdot p_h}{2 \Delta p \cdot \gamma} (2y - 1).
\]

\(\Phi()\) is continuous in \((0, 1)\), \(\Phi'(y) > 0\), \(\lim_{y \to 0} \Phi(y) = +\infty\), and \(\lim_{y \to 1} \Phi(y) = -\infty\). By Bolzano's theorem, there exists a unique \(y^*\) such that \(\Phi(y^*) = 0\).

(b) Properties

(b.1) \(y^* > \frac{1}{2}\) occurs if \(\Phi(\frac{1}{2}) > 0\), but \(\Phi(\frac{1}{2}) = K_0 - K_1 > 0\).

(b.2) Continuity of banks' portfolios.

It is enough to show that for all \(y > y^*\) \((y < y^*)\), the entrepreneur finds the bank at \(1\) more (less) desirable than the bank at \(0\). Take \(y^+ = y^* + \delta\) with \(\delta > 0\) (the case \(\delta < 0\) is symmetric). Then:

\[
R_c(y^+, 0) = R - \frac{c y^+}{\Delta p} - \frac{(1 - \frac{p_l \cdot c}{\Delta p} y^+) \gamma}{p_h}
\]

\[
< R - \frac{c(1 - y^+)}{\Delta p} - \frac{(1 - \frac{p_l \cdot c}{\Delta p} (1 - y^+)) \gamma}{p_h} = R_c(y^+, 1).
\]
After simplifying:

\[- \frac{c \delta}{\Delta p} + \frac{p_0 c \gamma}{\Delta p} < \frac{c \delta}{\Delta p} - \frac{p_1 c \gamma}{\Delta p}, \]  
(60)

\[\frac{p_1 \gamma}{p_h \beta^*_0} + \frac{p_1 \gamma}{p_h \beta^*_1} < 2, \]
(61)

\[\text{(b.3) } \beta^*_1 > \beta^*_0 > \gamma. \]

The project at 1/2 satisfies

\[- \frac{c \cdot \frac{1}{2}}{\Delta p} + \frac{c \cdot p_1 \cdot \gamma \cdot \frac{1}{2}}{p_h \beta^*_0} > \frac{c \cdot (1 - \frac{1}{2})}{\Delta p} + \frac{c \cdot p_1 \cdot \gamma \cdot \frac{1}{2}}{p_h \beta^*_1}, \]
(62)

which implies \[\beta^*_1 > \beta^*_0. \] Proposition 13 below gives the conditions on bank capital for \[\beta^*_0 > \gamma. \]

**Proposition 5 and Corollary 1**

Assume that the shocks are such that \(K'_0 = K_0(1 + dK_0)\) and \(K'_1 = K_1(1 + dK_1)\). Substituting in (30) we get:

\[\frac{K_0(1 + dK_0)}{y^*} - \frac{K_1(1 + dK_1)}{1 - y^*} = \frac{(2y^* - 1)c \cdot p_h}{2\Delta p \cdot \gamma}. \]
(63)

The equilibrium of the system does not change as long as the right-hand side of the new equation does not change with respect to (30). This occurs provided that

\[\frac{dK_0}{dK_1} = \frac{K_1}{K_0} \cdot \frac{y^*}{(1 - y^*)}; \]
(64)

therefore, that ratio defines \(G^*(K_0, K_1)\):

\[G^*(K_0, K_1) = \frac{K_1}{K_0} \cdot \frac{y^*(K_0, K_1)}{(1 - y^*(K_0, K_1))}. \]
(65)

If \[\frac{dK_0}{dK_1} > (K_0/K_1) \cdot (y^*/(1 - y^*))\) then \(y^*\) must increase to preserve the equilibrium; the opposite occurs if the inequality is reversed. Finally, to show that \(G^*(K_0, K_1) < 1\), note that in (30)

\[\frac{K_0}{y^*} - \frac{K_1}{1 - y^*} > 0, \]
(66)

which directly proves the proposition. □
Proposition 6

It is enough to show that \( y^* \) has increased. Differentiating in (30):

\[
\frac{dy^*}{dy} = \frac{(2y^* - 1)e}{\Delta_p y} + \frac{2yK_0}{n_0 y^2} + \frac{2yK_1}{n_0(1-y^2)} > 0.
\]

Proposition 7

With geographic restrictions, the bank at 0 can lend to at most half of the projects; without them it can extend its market share beyond the half point (see Proposition 4). Therefore the proposition follows.

Proposition 8

Value \( d' \) simply corresponds to the distance such that \( y^*_L = 1 \) in the far equilibrium. If \( d > d' \), then when computing the far equilibrium, \( y^*_L > 1 \), which implies that the close equilibrium rules. Alternatively if \( d < d' \), then when computing the far equilibrium, \( y^*_L < 1 \); therefore the far equilibrium rules. The existence and uniqueness of the equilibrium is discussed in the third part of this Appendix.

Proposition 9

We proceed in two parts:

(a) \( d' \) dominates any other close equilibrium location for the low-cap bank while \( d = 2 \) does so for the high-cap bank.

Suppose that the low-cap bank is located on \( d' > d' \). A movement to \( d' - \epsilon \) (i.e., to the left), keeping prices (i.e., \( \beta_0 \) and \( \beta_d \)) constant, would make the low-cap bank gain \( \epsilon \) market share in the right side of the portfolio and lose \( \epsilon \) in the left side of it. However, such a movement would leave idle capital for the low-cap bank and make the high-cap bank lack enough capital to fund all the projects on the right side. To restore the equilibrium, prices will change, \( \beta_0 \) dropping and \( \beta_d \) increasing, and so will profits.

(b) \( d' \) dominates any other far equilibrium location for both the low-cap and the high-cap bank.

Consider an equilibrium with the low-cap bank at \( d' < d \) and suppose that the low-cap bank moves to \( d' + \epsilon \) instead (i.e., moves to the right). If \( \beta's \) will not change, that will make the low-cap bank gain \( \epsilon \) market share in the left side of the portfolio and lose \(-\epsilon \) in the right side of it. However, such movement will make both banks lack the capital to fund all the projects, which in turn will increase prices (both \( \beta_0 \) and \( \beta_d \)) and both banks’ profits.
Proposition 10

Proposition 10 follows immediately from Proposition 9.

Proposition 11

(i) See (b.2) in the proof of Proposition 4. (ii) $y^* = \frac{1}{2}$ can only be an equilibrium if $K_1 = (c \cdot p_0)/(8 \cdot \gamma \cdot \Delta p)$. But regime 3 requires $K_1 < (c \cdot p_0)/(8 \cdot \gamma \cdot \Delta p)$ (see the bounds in the proof of Proposition 13). Less capital for the low-cap bank gives $y^* > \frac{1}{2}$. (iii) is immediately implied by (ii). □

Proposition 12

Entrepreneurs are the residual claimants of the profits. When both sources of capital cost the same, profits can be expressed as $R^*_e(x) = R - (I \cdot \gamma / p_k) - C(x)$. To maximize profits, entrepreneurs minimize monitoring costs and therefore finance their projects from their closest banks. □

Proposition 13

(a) Derivation of $f(.)$

Let $(K_0, K_1)$ be a pair of capitals that make all projects be financed. According to (18), the marginal projects would be

$$m^*_0 = \frac{R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_k}}{2 \frac{c}{\Delta p}} + \sqrt{\left(R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_k}\right)^2 + \frac{8}{p_k} \gamma \cdot \frac{c}{\Delta p} K_0}$$

and

$$1 - m^*_1 = \frac{R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_k}}{2 \frac{c}{\Delta p}} + \sqrt{\left(R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_k}\right)^2 + \frac{8}{p_k} \gamma \cdot \frac{c}{\Delta p} K_1}.$$ 

If the market is just covered, $m^*_0 = 1 - m^*_1$. Hence $f(.)$ is defined by:

$$K_1 = \frac{\left(2 \frac{c}{\Delta p} - 2 \left(R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_k}\right) - \sqrt{\left(R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_k}\right)^2 + \frac{8}{p_k} \gamma \cdot \frac{c}{\Delta p} K_0}\right)^2 - \left(R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_k}\right)^2}{\frac{8}{p_k} \gamma \cdot \frac{c}{\Delta p}}.$$  

If $m^*_0|_{K_1=0} > (1 - m^*_1)|_{K_1=0}$ (i.e., marginal projects cross without bank capital), then regime 1 does not exist. All projects are financed regardless of the amount of capital in the system. As stated above, (47) guarantees the possibility of regime 1.
(b) Derivation of \( g(.) \)

The function \( g(.) \) can be constructed using (52), the constraint \( \beta_0 = \gamma \), and the definition of \( \beta_0 = (c \cdot p_i \cdot \gamma^2)/(2\Delta p K_0) \). From the last two, \( K_0 \cdot \gamma = (p_i \cdot c \cdot (\gamma^*)^2)/(2\gamma \Delta p) \), or explicitly, \( \gamma^* = \sqrt{(2 \cdot K_0 \cdot \gamma^2 \Delta p)/(p_i \cdot c)} \). Substituting \( \gamma^* \) in Eq. (52) and then solving \( K_1 \) as a function of \( K_0 \), \( g(.) \) obtains.

(c) Cut-offs

\( K_1^2 \) is defined by the capital required to finance projects \((0, \frac{1}{2})\) if the cost of capital is \( \gamma \) (i.e., \( K_1^2 \equiv \int_0^{1/2} l_b^0(x) \, dx \) where \( l_b^0(x) = ((p_i \cdot c)/(\gamma \cdot \Delta p))x \): \( K_1^2 = (c \cdot p_i)/(\gamma \cdot 8\Delta p) \).

\( K_0^0 \) can be obtained as \( f^{-1}(0) \), solving the equation

\[
\frac{2}{\Delta p} - 2 \left( R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_h} \right) - \sqrt{\left( R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_h} \right)^2 + \frac{8c \cdot \gamma}{p_h \cdot \Delta p}} = \left( R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_h} \right),
\]

or explicitly:

\[
K_0^0 = \frac{9c \cdot p_h}{2\Delta p \cdot \gamma} + \frac{p_h \cdot \Delta p}{c \cdot \gamma} \left( R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_h} \right)^2 - \frac{9p_h}{2\gamma} \left( R - \frac{b}{\Delta p} - \frac{I \cdot \gamma}{p_h} \right).
\]

Finally \( K_0^1 \) can be constructed as:

\[
K_0^1 = \int_0^{1/2} \frac{p_i \cdot c}{2\Delta p \cdot \gamma} \cdot \frac{1}{1 + \Delta p} \, dx = \frac{p_i \cdot c}{4\Delta p \cdot \gamma} \cdot \left( \frac{1}{1 + \Delta p} \right)^2.
\]

A.3. Technical Derivations Omitted in Section 5

(a) Derivation of the far equilibrium equations, i.e., (36) to (39)

Entrepreneurs consider both the monitoring cost and the capital in order to choose a bank. In a far equilibrium the left marginal project is closer to 0 than to 2; therefore, the residual cash flow in case of success \( R_s(x, n) \) for an entrepreneur located at \( x \) after borrowing from a bank located at \( n \) is \( R_s(y_L, 0) = R - c \cdot y_L/\Delta p - (1 - I_b^0(y_L) \cdot \gamma/p_h) \) and \( R_s(y_L, d) = R - c \cdot (d - y_L)/\Delta p - (1 - I_b^d(x) \cdot \gamma/p_h) \). For the indifferent entrepreneur, \( R_s(y_L, 0) = R_s(y_L, d) \),

\[
\frac{c \cdot y_L}{\Delta p} - \frac{I_b^0(y_L) \cdot \gamma}{p_h} = \frac{c \cdot (d - y_L)}{\Delta p} - \frac{I_b^d(y_L) \cdot \gamma}{p_h},
\]
where \( I_b^{0^*}(y^*_L) = ((p_l \cdot c)/(\beta^*_0 \cdot \Delta p))y^*_L \) and \( I_b^{d*}(y^*_L) = ((p_l \cdot c)/(\beta^*_d \cdot \Delta p))(d - y^*_L) \).

Similarly, for the right marginal project, \( R_L(y_R, 2) = R - (c \cdot (2 - y_R)/\Delta p) - ((I - I_b^{0^*}(y_R)) \cdot \gamma / p_h) \) and \( R_R(y_R, d) = R - (c \cdot (y_R - d)/\Delta p) - ((I - I_b^{d*}(y_R)) \cdot \gamma / p_h) \). Again for the indifferent entrepreneur

\[
\frac{c \cdot (2 - y^*_R)}{\Delta p} - \frac{I_b^{2*}(y^*_R) \cdot \gamma}{p_h} = \frac{c \cdot (y^*_R - d)}{\Delta p} - \frac{I_b^{d*}(y^*_R) \cdot \gamma}{p_h},
\]

where now \( I_b^{2*}(y^*_R) = ((p_l \cdot c)/(\beta^*_0 \cdot \Delta p))(2 - y^*_R) \) and \( I_b^{d*}(y^*_R) = ((p_l \cdot c)/(\beta^*_d \cdot \Delta p))(y^*_R - d) \). To obtain the cost of capital (Eqs. (38) and (39)), simply consider the capital employed by banks to finance projects at both sides of their locations and the no arbitrage constraint \( \beta \geq \gamma \).

(b) Derivation of the close equilibrium equations, i.e., (40) to (43)

The left marginal project is now closer to 2 than to 0,

\[
\frac{c \cdot (2 - y^*_L)}{\Delta p} - \frac{I_b^{2*}(y^*_L) \cdot \gamma}{p_h} = \frac{c \cdot (d - y^*_L)}{\Delta p} - \frac{I_b^{d*}(y^*_L) \cdot \gamma}{p_h},
\]

where \( I_b^{2*}(y^*_L) = ((p_l \cdot c)/(\beta^*_0 \cdot \Delta p))(2 - y^*_L) \) and \( I_b^{d*}(y^*_L) = ((p_l \cdot c)/(\beta^*_d \cdot \Delta p))(d - y^*_L) \). For the right marginal project,

\[
\frac{c \cdot (2 - y^*_R)}{\Delta p} - \frac{I_b^{2*}(y^*_R) \cdot \gamma}{p_h} = \frac{c \cdot (y^*_R - d)}{\Delta p} - \frac{I_b^{d*}(y^*_R) \cdot \gamma}{p_h},
\]

where, as before \( I_b^{2*}(y^*_R) = ((p_l \cdot c)/(\beta^*_0 \cdot \Delta p))(2 - y^*_R) \) and \( I_b^{d*}(y^*_R) = ((p_l \cdot c)/(\beta^*_d \cdot \Delta p))(y^*_R - d) \). Equations (42) and (43) can be obtained as before.

(c) Algorithm to compute the equilibrium

The equilibrium can be computed solving the equations that characterize the far equilibrium (Eqs. (36) to (39)). As proved below, the solution exists, and it is unique. If \( y^*_L \leq 1 \), then the solution of the model is found; if \( y^*_L > 1 \), then the solution of the model is the solution of the equations that characterize the close equilibrium (Eqs. (40) to (43)). If in any of the steps \( \beta_0 < \gamma \) or \( \beta_d < \gamma \) (or both), they should be fixed at \( \gamma \) and proceed to compute the equilibrium marginal projects, ignoring the equations that define the cost of bank capital. This procedure is similar to the one used to solve regime 3.

(d) Properties of the equilibrium

(d.1) Existence and continuity

The proposed equilibrium consists of bank portfolios formed by the projects closer to each bank. In equilibrium, both banks finance at least some projects and employ all the capital they own. That bank portfolios are not empty (and therefore that marginal project exist) is guaranteed by (i) the fact that projects close enough to the banks are necessarily financed by those banks and (ii) the assumption that
the high-cap bank can cover the market (see footnote 37). The continuity of the banks’ portfolios can be established using an argument similar to the one in the proof of Proposition 4.

(d.2) Uniqueness

To prove the uniqueness of the equilibrium, we will proceed in two steps. First we will prove a property that relates the marginal projects for a given location. Next we prove that having more than one equilibrium would conflict with that property, which, by contradiction, proves the uniqueness of the equilibrium.

Fact 1 (Property). For a location \( d \leq d^* \) (far equilibrium), the marginal projects satisfy

\[
y^*_L = \frac{2}{2 - d}(2 - y^*_R) \tag{72}
\]

and for a location \( d > d^* \) (close equilibrium), the marginal projects satisfy

\[
y^*_L = \frac{4d - 2y^*_R - dy^*_R}{2 - 2y^*_R + d}. \tag{73}
\]

In both cases:

\[
y^*_L(y^*_R) < 0. \tag{74}
\]

To find Eq. (72), divide Eq. (36) by (37) and express \( y^*_L \) as a function of \( y^*_R \). To find Eq. (73) divide Eq. (40) by Eq. (41). Simple differentiation of the resulting expression gives (74).

To prove uniqueness, assume the opposite, and suppose that, for a given distribution of bank capital \( (K_0, K_d) \), there is more than one equilibria. Denote two of those potential equilibria as (1) \( (y^*_L, y^*_R, \beta^*_0, \beta^*_d) \) and (2) \( (y^*_L', y^*_R', \beta^*_0', \beta^*_d') \), where at least one of the four parameters in the vector is different. To focus on a relevant case, suppose that \( \beta^*_d > \gamma \) (otherwise \( \beta^*_0 = \beta^*_d = \gamma, y^*_L = 0.5, \) and \( y^*_R = 1.5 \) and the property is proved). Suppose w.l.o.g. that \( y^*_R > y^*_R' \) and \( y^*_L < y^*_L' \) (by the property found above, no other possibilities can exist), which is consistent with only one of the following two cases:

(a) \( \beta^*_d > \beta^*_d' \), which implies that the low-cap bank finances more projects and uses more capital in each of them in (2) than in (1). This is contradiction with the full employment of capital in (1).

(b) \( \beta^*_d < \beta^*_d' \) but \( \beta^*_0 < \beta^*_0' \). In this case the high-cap bank finances fewer projects and uses less capital in each of them in (2) than in (1). This is contradiction with the full use of capital by the high-cap bank in (1).

Finally, if \( y^*_L = y^*_L' \) and \( y^*_R = y^*_R' \) in both equilibria and the capital of the low-cap bank is used in both (1) and (2), then it immediately follows that \( \beta^*_d = \beta^*_d' \) and \( \beta^*_0 = \beta^*_0' \).
(e) Numerical Example

Consider a numerical example using the following parameters: $p_l = 25$, $p_h = 0.75$, $\gamma = 1$, $\beta_0^* = 1$, $c = 1$, $K_0 = 0.15$, and $K_d = 0.0375$. The location analysis consists of solving problem (45) for far equilibrium and solving (44) for close equilibrium.\(^{45}\) Table I summarizes these computations ($d = 1.859$ separates far from close equilibria). A glance at Table I shows that, as predicted by the formal analysis, the location that maximizes profits for the low-cap bank is $d = d^0 = 1.859$, while the location that maximizes profits for high-cap bank is $d = 2$ (i.e., no separation).

### REFERENCES


\(^{45}\) Banks’ profits are $K_0\beta_0^*$ and $K_d\beta_d^*$. 

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**Table I**

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Myers, S., and Majluf, N. S. (1994). Corporate financing and investment decisions when firms have information that investors do not have, *J. Finan. Econ.* 13, 187–221.