CREDIT-WORTHINESS TESTS AND INTERBANK COMPETITION

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This paper analyzes a competitive credit market where banks use imperfect and independent tests to assess the ability of a potential creditor to repay credit. The banks compete by announcing interest rates at which they will provide credit to those applicants who pass the banks' tests. The proportion of applicants who pass the test of at least one bank increases with the number of banks providing credit, so the average credit-worthiness decreases. It is then shown that in a situation where all banks charge the same interest rate, a bank always has the incentive to undercut in order to improve the average credit-worthiness of its own clientele. This feature represents the major difference from the situations in standard Bertrand and Bertrand-Edgeworth models.

KEYWORDS: Credit market, multiple and independent testing, auction games, Bertrand-Edgeworth competition.

1. INTRODUCTION

In this paper, we consider a credit market where banks face an adverse selection problem. The adverse selection problem is due to the existence of two types of firms which apply for a loan of fixed size. These two types differ in their ability to repay the loan. The banks' problem is then to assess an applicant's ability to repay the credit. In the absence of self-selection or signalling devices, such as collateralization or credit rationing, banks have to rely on active monitoring when they decide on the firm's application.

To determine the future ability of a firm to repay credit, which we call the credit-worthiness of the applicant, the bank's credit department will use any relevant information it has available to assign the applicant to a certain risk class. It will be assumed that for any given applicant, a bank might make judgment errors, i.e. that a risk class may contain more than one type of applicants. Furthermore it will be assumed that for a given applicant the judgment errors will be independent among banks.

Given this procedure for reducing but not totally eliminating the adverse selection problem, the banks will compete with each other by setting interest rates conditioned on their own test results. We are interested in the equilibria that will emerge in the credit market under Bertrand competition and multiple and independent testing for credit-worthiness. The tests are assumed to be imperfect and costless.

To be more specific, suppose that there is a market in which two banks face a continuum of firms. Let there be two types of firms, $a$ and $b$, which differ in their

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ability to repay a loan and suppose that the type b firms are more likely to be able to repay a credit. Each bank is endowed with a credit-worthiness test which has two possible outcomes, A and B. For low risk firms the probability that B will occur is higher than for firms of type a. This means that the risk class B contains a higher proportion of low risk firms than A. Now when a firm applies for a loan at these two banks, both banks will use their tests independently of each other and will assign the firm to one of the two risk classes. To simplify the exposition let us assume that a bank will not provide credit to firms of class A. Now it might well happen that a high risk firm is assigned to B by one or even by both banks. The banks now have to specify the interest rates under which they will provide credit to firms assigned to B. The problem which is caused by the multiple and independent testing is the following: Suppose that bank 1 charges a lower interest rate than bank 2. Firms which are accepted by bank 1 will sign the credit contract irrespective of a possible offer by bank 2. Consequently bank 1’s clientele is determined solely by its own credit-worthiness test. This is not true for bank 2. Its clientele consists of all those firms which are rejected by bank 1 but are accepted by bank 2. It will therefore contain a high ratio of high risk firms. In fact, depending on the parameters, bank 2 will face expected losses. This means that banks face externalities caused by the interest rates and the rejection decisions of the other banks.

We will consider two specifications, a one-stage game and a two-stage game. In the one-stage game there is a given number of potentially active banks. These banks simultaneously decide whether they will provide credit, and if so, at which interest rate. In this game interbank competition resembles a Bertrand-Edgeworth type of competition. Given its clientele, a bank would like to raise its interest rate to raise its profit. On the other hand, there are externalities due to interest rate differentials. Lowering the interest rate below that of a competitor has the advantage that the composition of its clientele becomes more favorable, which may entail a rise in its profit. Although these externalities imply that no equilibrium in pure strategies exists, the one-stage game does possess a unique equilibrium in mixed strategies with a continuous distribution function over interest rates. Comparative static results will be derived. It turns out that the probability with which a bank remains inactive, i.e. does not provide any credit, increases with the number of banks which are potentially active. The individual bank’s strategy with respect to the choice of interest rates converges uniformly to a continuous distribution function over the set of interest rates when the number of banks tends to infinity. In the limit, the expected number of banks that are active in the market is bounded away from zero. Furthermore, the limit distribution function over interest rates is approximately equal to the equilibrium strategy of the oligopoly with the corresponding number of banks. This means that even in the limit there is some degree of oligopolistic competition. This contrasts with the findings for classical Bertrand-Edgeworth competition. The model can also be interpreted as a first-bid auction with common values (see Milgrom and Weber (1982)). If the banks face a single applicant they can make bids by offering this firm certain interest rates. However the analogy is not
complete. If there are many banks in the market there does not exist a bid for a firm assigned to $B$, where a bank makes zero (expected) profit. The reason is that whenever a firm accepts the least favorable interest rate, at which it will make zero expected profit, the firm has been rejected by all other banks. Therefore it is a very bad risk on average. The bank will incur losses even at the highest possible interest rate. This is a "winner's curse" type implication of market participants (i.e. banks) who perform tests. It is basically due to this that the probability with which a bank remains inactive increases with the number of banks.

In the second specification we will consider the effects of the externalities described above in a two-stage game where banks can react to the interest rates charged by the other banks. In the first stage all banks announce an interest rate. Having observed the interest rates charged by the other banks, they will decide whether to leave the market or whether to provide credit to firms assigned to $B$ under the prespecified conditions. When there are only two banks a proposed refinement of the Nash equilibrium concept narrows down the set of equilibria. Along the equilibrium path both banks announce the interest rate under which a single bank would make zero profit. Then either both banks leave the market, or just one bank leaves the market and the other one provides credit at this particular interest rate. Due to the externalities it is impossible for both banks to provide credit at this interest rate. For example, for three banks the proposed refinement of the Nash equilibrium concept does not even suffice to eliminate equilibria where one bank makes positive expected profit. In the example that will be considered, bank 3 has a rather weak position when it competes with the other two banks. Whenever bank 3 charges the same interest rate as one of the other banks it will leave the market. The competition between banks 1 and 2 is symmetric: Whenever they charge the same interest rate, if bank 3 does not undercut them, banks 1 and 2 apply the mixed equilibrium at the second stage. Given that bank 3 charges the highest possible interest rate, Bertrand competition between banks 1 and 2 drives them down to zero profit, because the action taken by the third bank does not really affect them. Despite its weak position bank 3 can make positive profit. On restricting the two-stage game to a finite set of admissible interest rates it turns out that this equilibrium is perfect in the sense of Selten (1975).

The externalities which govern the interbank competition are due to the multiple and independent testing by the banks and the rejection decisions of the banks. In particular, they would also occur if the firms do not know their types. The offer of a certain interest rate influences the average ability to repay credit but it does not affect the behavior of the borrowers (see Stiglitz and Weiss (1981)). Also we do not consider collateral as a means of self-selection (Bester (1985), Stiglitz and Weiss (1981)). In fact, the assessment of the value of collateral by banks can itself be described as multiple and independent testing. Guasch and Weiss (1980a, 1981) and Nalebuff and Scharfstein (1987) consider other means of self-selection, such as application fees, in job markets with testing. Application fees are interpreted as opportunity costs, such as reduced wages during the trial hiring period. In our model information received after the
investment has taken place cannot influence the banks' strategies. When discussing the results (Proposition 2.2) we will comment briefly on the introduction of application costs into the model.

This paper is organized as follows: In Section 2 the one-stage game is considered. Non-existence of pure strategy equilibria as well as the existence of mixed strategy equilibria and comparative static results are derived. Section 3 contains a discussion of the two-stage game. The notion of a (perfect) limit equilibrium as a refinement of the Nash equilibrium concept is proposed in Section 3.2. For the duopoly all equilibrium paths that correspond to perfect limit equilibria are determined. In this case all banks make zero expected profit, whereas for more than two banks the existence of perfect limit equilibria where one bank makes positive profit is demonstrated. Section 4 contains some remarks on the robustness and possible applications of the model to other markets. The proofs are collected in Section 5.

2. THE ONE-STAGE GAME

2.1. Imperfect Monitoring

We consider a credit market with a continuum $[0, 1]$ of risk-neutral firms, each of which is seeking a loan $L = 1$. Having received a loan, a firm invests it in a project which will yield a return $X$ if it succeeds and 0 if it fails. We assume that a firm receives at most one credit. There exist two types of firms, the types differing in the success probability of their projects. The success probability of firms of type $a$ and $b$ are $p_a$ and $p_b$ respectively, $0 < p_a < p_b < 1$. Each firm knows its own type. The proportion of type $a$ firms $1, 0 < l < 1$, is common knowledge. The firms which undertake the project will use the return $X$ to repay the credit if they succeed. Therefore the firms will also differ in their ability to repay the loan.

Let there be $N$ risk-neutral banks in the market, $N \geq 2$; these banks can raise an arbitrary amount of funds at a unit gross interest rate. In turn the bank can supply credit to firms at an interest rate $r \geq 1$. If $a$ type $a$ and $b$ type $b$ firms receive a loan, then the bank's payoff will be $a(p_ar' - 1) + b(p_br' - 1)$, where $r' = \min(X, r)$.

The expected profit of a firm is given by $p_c(X - r')$, $c = a, b$, if it has received a loan and 0 otherwise. No bankruptcy costs are involved. It is obvious that a firm of any type prefers a loan contract with a low interest rate to one with a high interest rate. To simplify the exposition we assume that a firm accepts a credit contract under $r = X$ if it has no cheaper alternative.

We are interested in the competitive equilibrium in a situation where banks are not able to perfectly monitor types. We assume that no self-selection devices are available. Instead let each bank $i$ be endowed with a costless but imperfect test $T_i$ to determine the credit-worthiness of an applicant firm. The test $T_i$ randomly assigns the applicant to one of the two categories $A$ and $B$. Let $q(C|c)$ denote the probability that a firm is assigned to $C = A, B$ on the condition that it is of
type \( c = a, b \). The test \( T_i \) is then completely described by

\[
q(A|a) = q_a, \quad q(B|a) = 1 - q_a,
q(A|b) = q_b, \quad q(B|b) = 1 - q_b.
\]

Let us consider just one bank applying this test of credit-worthiness to the whole population \([0,1]\) of firms. Applying the law of large numbers we obtain that \( lq_a \) \( a \)-type firms are assigned to \( A \) and the remaining \( l(1 - q_a) \) \( a \)-type firms are assigned to \( B \). We proceed in a similar manner for firms of type \( b \). This means that \( lq_a + (1 - l)q_b \) firms are assigned to \( A \), \( lq_a \) being of type \( a \). Therefore the proportion of \( a \)-type firms in the population of all those firms assigned to \( A \) is \( l(a|A) = lq_a/(lq_a + (1 - l)q_b) \). The conditional probability of a firm being of type \( a \) given that it is assigned to the risk class \( B \) is \( l(a|B) = l(1 - q_a)/(l(1 - q_a) + (1 - l)(1 - q_b)) \). These conditional probabilities can be used to compute the average success probabilities of the risk classes \( A \) and \( B \), \( \tilde{p}_A \) and \( \tilde{p}_B \) respectively, as

\[
\tilde{p}_A = l(a|A)p_a + (1 - l(a|A))p_b,
\tilde{p}_B = l(a|B)p_a + (1 - l(a|B))p_b.
\]

The assumption

\[
(A1) \quad 0 < q_b < q_a < 1
\]

implies that \( \tilde{p}_A < \tilde{p} < \tilde{p}_B \), where \( \tilde{p} \) denotes the average success probability of all firms. Therefore the firms assigned to \( B \) are more likely to be able to repay their loans than firms assigned to \( A \).

Furthermore, the parameters \( q_a \) and \( q_b \) are identical for all banks and the tests \( T_i \) are stochastically independent for any given applicant. This assumption will facilitate the analysis. In any case the qualitative results remain valid as long as the tests are not stochastically identical.

### 2.2. The Model

In this section we will model interbank competition in the following way: The banks which provide credit simultaneously announce two interest rates, one charged to those firms assigned to class \( A \) and the other conditioned on the assignment to class \( B \). Furthermore, a bank has the opportunity to leave the market and not to provide any credit at all. All firms then apply to all those banks which stay in the market. These banks carry out their tests of credit-worthiness and truthfully report their test results to the applicants. Banks do not inform each other about their results and they do not observe the offers made by their competitors. A firm is then free to sign a credit contract with any bank under the prespecified conditions. Later on we will assume that the average
success probability of firms in class $A$ is so low that it is never optimal for a bank to provide credit to those firms. Furthermore, we will fix the behavior of firms which are indifferent between loan contracts. All this specifies a one-stage game where banks either make binding announcements of interest rates for firms assigned to $B$ or leave the market immediately.

A particular feature of this model of interbank competition is that the sets of firms of class $B$ are not identical for any two banks: The potential clientele of the banks is merely overlapping. It will be demonstrated that under certain conditions on the parameters there does not exist an equilibrium in pure strategies. However there is an equilibrium in mixed strategies.

Next we introduce the following two assumptions:

\[(A2) \quad \bar{p}_B X > 1 \quad \text{and} \]
\[(A3) \quad \bar{p}_A X < 1.\]

Assumption (A2) implies that at least one bank does not make losses if it charges $r = X$ to firms of class $B$. Assumption (A3) states that no matter what interest rate is charged to a firm of class $A$, the expected repayment amount will still be less than the amount of the loan itself. Provision of credit to these firms always entails expected losses. The assumption justifies the restriction of the analysis to cases where each bank announces a single interest rate which applies only for those firms that the bank assigns to $B$.

We now define the game. We only consider parameters $q_a, q_b, p_a, p_b, l$ which satisfy assumptions (A1)--(A3). The banks simultaneously announce whether or not they will provide credit. If a bank provides credit it also has to specify an interest rate $r \in [0, X]$, which only applies to those firms which the bank has assigned to $B$. Banks which leave the market receive zero profit. The payoff functions of the banks that stay in the market are derived below.

Let there be $N$ banks in the market and suppose that bank $i$ announces the interest rate $r^i$. The probability that a firm is accepted by bank $i$ is $1 - q_a$ if it is of type $a$ and $1 - q_b$ otherwise. If it is accepted the firm is free to sign the contract with bank $i$ under the interest rate $r^i$; bank $i$ cannot reject the firm after it has passed the test, i.e. after it has been assigned to $B$. We assume that any firm chooses among those banks which offer the lowest interest rate with equal probability. Let $C \subset \{1, \ldots, N\}$ be the set of banks which make an offer. Let $r$ be the vector of announced interest rates and $m(r, i)$ be the number of banks announcing the same interest rate as bank $i \in C$ and let $n(r, i)$ be the number of banks charging a lower interest rate. Consider now firms of type $b$. Clearly any such firm prefers low interest rates to high ones and thus only those firms which are rejected by all the banks that charge less than $r^i$ will constitute the potential clientele of bank $i$. The total number of these firms is $(1 - l)q_b^{m(r, i)}$. Of these a fraction $1 - q_b^{m(r, i)}$ will receive an offer from at least one of the banks charging $r^i$. Due to the assumption that firms distribute themselves uniformly among these banks, the number of type $b$ firms signing a contract with bank $i$ is precisely $(1 - l)q_b^{m(r, i)}(1 - q_b^{m(r, i)})/m(r, i)$. A similar formula holds for firms of type $a$. 
This implies that the total payoff to bank $i$ from the offer $r^i$ is

$$U(r, i) = \frac{q^n(r, i)(1 - q^n(r, i))}{m(r, i)} (p_a r^i - 1) + (1 - l) \frac{q^n(r, i)(1 - q^n(r, i))}{m(r, i)} (p_b r^i - 1).$$

This completes the definition of the one-stage game.

We can associate with these payoff functions a sequence of zero-profit interest rates $r(k), 1 \leq k \leq N$. Suppose that $k$ banks provide credit at the same interest rate $r$ and that no other bank provides credit: $r(k)$ is then defined to be that interest rate $r$ at which the banks make zero-profit. To compute $r(k)$ one has to solve $U(r, i) = 0$ for the $k$-tuple $r = (r, \ldots, r)$. It follows that

$$r(k) = \frac{l(1 - q^k_a) + (1 - l)(1 - q^k_b)}{p_a l(1 - q^k_a) + p_b(1 - l)(1 - q^k_b)}.$$

Under the assumption (A1) this sequence has the property that

$$1/p_b < r(1) < \cdots < r(N).$$

For $q_a = 1$ we obtain $1/p_b = r(1) = \cdots = r(N)$, because the risk class $B$ only contains firms of type $b$. But in general the sequence of zero-profit interest rates is strictly increasing in the number of banks offering it. This is because of the stochastic independence of the credit-worthiness tests: The more banks in the market the lower the average success probability of firms which eventually receive an offer. Suppose that there are two banks, $k = 2$. The number of type $a$ firms which are accepted by at least one bank is $l(1 - q^2_a)$ and the corresponding number of type $b$ firms is $(1 - l)(1 - q^2_b)$. Then the conditional probability that a firm of type $a$ is assigned to $B$ by at least one bank is given by

$$l'(a|B) = l(1 - q^2_a)/(l(1 - q^2_a) + (1 - l)(1 - q^2_b)).$$

It follows that $l'(a|B) > l(a|B)$, so the average success probability for $k = 1$ is strictly greater under (A1) than for $k = 2$. This argument applies in general. In fact the same phenomenon would occur if the applicants do not know their types. It should be noted that the zero-profit interest rate for one bank, $r(1)$, reflects the average success probability $\bar{p}_B$: $r(1) = 1/\bar{p}_B$. Therefore an equivalent formulation of (A2) is $r(1) < X$. It follows that a single bank does not make losses at $r \in [r(1), X]$. In equilibrium any announced interest rate $r$ must belong to this set.

This one-stage game is linked to two important classes of games: models of Bertrand-Edgeworth competition and first-bid auctions with common values. The former similarity is due to the fact that the potential clientele of two banks, i.e. the firms assigned to risk class $B$, do not coincide. Also the credit market model can be interpreted as an auction if one assumes that the banks face a single
applicant. Then the bid of a bank is either the offer of a certain interest rate if the firm passes the bank’s test, or no offer at all.

2.3. Results

PROPOSITION 2.1: Under the Assumptions (A1)–(A3) the one-stage game does not possess an equilibrium in pure strategies.

PROOF: There does not exist a Nash equilibrium in pure strategies where no bank provides credit, since in this situation any bank would have an incentive to supply credit at the interest rate \( r = X \). Now suppose that just one bank provides credit at the interest rate \( r \). A necessary condition for this to be an equilibrium is \( r = X \), for otherwise, the bank could raise its profit by slightly raising the interest rate, but then one of its competitors could make positive profit by charging \( r(1) < r' < X \). Finally suppose that \( K \) banks, \( K \geq 2 \), provide credit. We distinguish two cases: First let us suppose that the lowest interest rate is charged by just one bank. Clearly this cannot be an equilibrium because this bank can raise its profit by raising its interest rate. Second let us suppose that the lowest interest rate is charged by \( L \geq 2 \) banks and let this interest rate be \( r^* \). We have to show that at least one of these \( L \) banks has an incentive to set an interest rate \( r \) below \( r^* \). First notice that \( r^* > r(L) \). Now consider for one of the banks charging \( r^* \) the effect of slightly lowering the interest rate:

\[
\lim_{s \to r^*} U(s, r^*) - U(r^*, r^*)
\]

\[
= l(1 - q_a)(p_a r^* - 1) + (1 - l)(1 - q_b)(p_b r^* - 1)
- l(1 - q_a L^{-1})L^{-1}(p_a r^* - 1) - (1 - l)(1 - q_b L^{-1})L^{-1}(p_b r^* - 1).
\]

This term has a positive derivative with respect to \( r^* \). By definition of \( r(L) \) this term is positive for \( r^* = r(L) \) and therefore for all \( r^* > r(L) \). It follows that \( r^* \) cannot constitute an equilibrium. Q.E.D.

The basic feature of the interbank competition which leads to the nonexistence result is that the sequence of zero-profit interest rates is strictly increasing with the number of banks that offer it. Consider the case where two banks provide credit. Given its clientele, a bank has an incentive to raise its interest rate. On the other hand the bank’s profit is not determined solely by the terms of its loan contracts: Its profit is also influenced by the average success probability of its clientele, which in turn also depends on the other banks’ interest rates. Suppose that bank 1 charges a higher interest rate than its competitor. It follows that bank 1 provides credit to those firms which are rejected by bank 2 and which are assigned to class \( B \) by bank 1. This population of firms has a high ratio of firms of type \( a \) and therefore a low average success probability. On the contrary bank 2 is not affected by bank 1’s announcement: Its clientele is exclusively determined by bank 2’s test of credit-worthiness and consequently its clientele has the high average success probability \( \bar{p}_B = 1/r(1) \). On reversing this argument it follows that any situation where both banks charge the same interest rate cannot
be an equilibrium: A bank has the incentive to lower the interest rate slightly and thereby increase its profit due to a jump in the average success probability. To summarize, we observe that there are two forces which govern interbank competition: On the one hand a bank wants to raise its interest rate given its clientele, but on the other hand a bank wants to lower the interest rate in order to improve the average probability that its clientele will repay the loan. We have already noted that this kind of competition resembles Bertrand-Edgeworth competition. The classical description of Bertrand-Edgeworth competition is based on capacity constraints and on increasing marginal cost. In our specification a bank could serve the whole market and the tests of credit-worthiness are costless. The similarity is solely due to the externalities caused by interest rate differentials and the rejection decisions of the banks. Considering the interpretation of the model as an auction game, these externalities give rise to the winner's curse: If the banks bid according to their own test results $B$, the winning bank will overestimate the credit-worthiness of the firm on average. It is also very important to notice that these externalities generate a difference to the usual first-bid auctions with common values: For a bank there might not exist a bid such that the bank will make zero-profit whatever its competitors do—in the usual auction models this is the zero bid. Here we observe that however large we choose $X$ we can find a number of banks, $N$, such that if a firm accepts a bank’s offer $r = X$, then the bank will make expected losses, provided that all banks are active. To guarantee itself zero profit a bank has to leave the market, i.e. it does not make any offer at all.

We will now establish the existence of equilibria in mixed strategies and derive comparative static results. In fact we will give an explicit representation of symmetric mixed strategy equilibria in the proof of the next Proposition. Any such equilibrium can be written as $(\pi, P)$, where $\pi$ denotes the probability with which a bank leaves the market and $P$ is a distribution function over interest rates in $[0, X]$, which a bank uses to determine the interest rate it announces if it stays in the market. In the following let $C^0$ and $C^\infty$ denote the space of continuous and infinitely continuously differentiable functions respectively.

**Proposition 2.2:** Let there be $n + 1$ banks in the market. For any $n \geq 1$ there exists a unique symmetric equilibrium $(\pi^{(n)}, P^{(n)})$ which satisfies $P^{(n)} \in C^0([0, X])$. These equilibria have the following properties: There exists a number $n^* \in \mathbb{R}^+$ and interest rates $r^{(n)} \geq r(1)$ such that:

1. if $n \leq n^*$ then
   1.1. $\pi^{(n)} = 0$,
   1.2. $P^{(n)} \in C^\infty([r^{(n)}, X])$ and $P^{(n)}(r) > 0 \iff X \geq r > r^{(n)}$,
   1.3. all banks make positive expected profit if $n < n^*$
   
2. if $n > n^*$ then
   2.1. $0 < \pi^{(n)} < 1$
   2.2. $P^{(n)} \in C^\infty([r(1), X])$ and $P^{(n)}(r) > 0 \iff X \geq r > r(1)$,
   2.3. all banks make zero expected profit.
The proof is given in the Appendix and explicit representations of \( n^* \), \( r^{(n)} \), \( \pi^{(n)} \), \( P^{(n)} \) and the corresponding profits are also derived there. In particular it should be mentioned that \( n^* \) can take any positive value depending on the choice of parameters \( q_a, q_b \).

The critical number of banks in the market, \( n^* + 1 \), is defined so that a bank can make nonnegative expected profit in equilibrium if it charges the highest possible interest rate \( r = X \). It is then clear that no bank will leave the market, i.e. \( \pi^{(n)} = 0 \), and that the bank would realize the equilibrium profit if it announced \( X \) given the mixed strategies of the other banks. In particular it follows that no symmetric zero-profit equilibrium exists if the distribution function over interest rates is continuous. If there are more than \( n^* + 1 \) banks in the market, setting \( X \) against the strategies of the opponents entails expected losses. Banks no longer stay in the market with probability 1, i.e. \( a^{(n)} > 0 \). In part 1 of the Proposition, \( r^{(n)} \) denotes the lower bound of the support of \( P^{(n)} \). It can be shown that \( r^{(n)} > r(1) \) if \( n < n^* \).

The results are (partly) robust to borrowers having a small cost (disutility) of applying for a loan at each bank. At least for certain values of the parameters, e.g. \( n < n^* \), all applicant firms have positive expected profit and so a sufficiently small cost will not alter the equilibrium behavior of banks and firms. However fixing the cost and then increasing the number of banks will change the results, because there is now a positive probability that an (arbitrarily) large number of banks is active. Then a firm will not apply to all banks—possibly even not to all low interest rate banks. In the extreme case where application costs are very large, it might happen that firms apply to at most one bank. Given the different probabilities of being detected, there will be parameter values where bad risk firms will not apply at all and where each good risk firm will apply to just one bank. This will be true under the assumption that banks still only serve firms which pass the bank's test. We will then obtain similar sorting results as those of Guasch and Weiss (1980a, 1981) and Nalebuff and Scharfstein (1987).

We have already noted that the results obtained for auction models (Milgrom and Weber (1982)) are not directly applicable: Even if the firm accepts the highest possible interest rate, \( X \), it might not be able to repay its credit on average. The average ability to do so depends on how many banks have rejected the firm at lower interest rates. This is the basic reason for the different results obtained in parts 1 and 2 of the Proposition. However the results on auctions suggest that if the estimates of the credit-worthiness of the firm are drawn according to a density function we obtain pure strategy equilibria.

We will now present the results for the behavior of the equilibrium strategies when the number of firms tends to infinity.

**PROPOSITION 2.3:** There exists a number \( n^\infty > 0 \) and a distribution function \( P^\infty \) over \([r(1), X] \) which is \( C^\infty \) on this interval such that:

1. \( n(1 - \pi^{(n)}) \to n^\infty \), and
2. \( P^{(n)} \to P^\infty \) uniformly on \([r(1), X] \).
The proof is given in the Appendix.

Because \( n \) denotes the number of competitors of a given bank, \( n^\infty \) should be interpreted as the limit of the expected number of competitors of that bank. \( P^\infty \) is then the limit of the interest rate strategies that the competitors of the bank apply. Therefore it seems natural to relate this to the oligopoly with \( n^\infty + 1 \) banks. It can be shown that \( n^\infty \) is approximately equal to \( n^* \) and that \( P(n^*) \) is approximately equal to \( P^\infty \). The conclusion that should be drawn from this observation is that if the number of banks increases, the equilibria resemble the oligopolistic equilibrium with \( n^* + 1 \) banks in the market. In particular even in the limit there is some degree of oligopolistic competition: The limit of the product of the number of banks and the probability of staying in the market (which in turn is chosen so as to yield zero expected profit to the banks) remains finite and is positive.

The results of Proposition 2.3 should be contrasted to the results of Allen and Hellwig (1986). Here the limit of Bertrand-Edgeworth oligopolies is considered when the number of players increases with a corresponding decrease in the market power of an individual player due to a decrease in capacity. Then the equilibrium price distributions converge to the competitive price. In our specification the individual bank could still serve the whole market. A reduction of market power could therefore only come about through increased competition among banks. In any case, the competitive equilibrium is not defined in our context. Comparing our limiting results with those of Rosenthal and Weiss (1984), where a job market with signalling is considered, the main difference is the following: In their model the individual player’s mixed strategy converges to a pure strategy, while the overall probability distribution for pure strategies that are observed in the market remains nondegenerate, whereas in our one-stage model even the individual strategies converge to a continuous distribution function. For auction models it was demonstrated that, in the limit, the market efficiently aggregates the private information of the bidders, if they can distinguish the event that the common value equals \( v \) from the event that it is less than \( v \). See Milgrom (1979), R. Wilson (1977), and also Nalebuff and Scharfstein (1987). With just finitely many possible realizations of the common value and the banks’ estimates, the efficient aggregation of information is not possible. Also in the usual auction models all bidders will stay in the market.

3. THE TWO-STAGE GAME

3.1. The Model

We will now model interbank competition as a two-stage game. In the first stage all \( N \) banks simultaneously announce a probability measure over the set of interest rates, the realizations of which they observe at the end of stage one. Given the \( N \)-tuple of interest rates, the banks simultaneously decide whether they will serve the market under the prespecified conditions. In particular it should be
THORSTEN BROECKER

noted that the banks do not reveal their test results because the interest rate offers are conditioned on the firm passing the test. All firms will apply to all those banks which stay in the market. Given that a firm has passed the test of a bank, i.e. has been assigned to category B, it is free to sign a contract with this bank. Payoffs are then defined as in the one-stage game. Only subgame-perfect Nash equilibria are considered. We will demonstrate that for certain parameter values pure strategy equilibria do, now, exist. Properties of these equilibria will be derived.

As for the one-stage game we will impose Assumptions (A1)–(A3). We first define some notation. The space of pure strategies in the first stage is given by [0, X]. A mixed strategy of bank i, \( \mu_i \), is therefore a probability measure on this interval. The space of pure strategies in the second stage consists of the alternatives of staying in the market or leaving, conditional on the observed interest rates \( r = (r^1, \ldots, r^N) \). The second stage strategy of bank i is then a function \( \phi_i : [0, X]^N \to [0,1] \), where \( \phi_i(r) \) denotes the probability with which the bank stays, given \( r \). A full strategy of bank i is thus a vector \( \sigma^i = (\mu^i, \phi^i(\cdot)) \). Let \( \sigma = (\sigma^1, \ldots, \sigma^N) \) be an N-tuple of strategies, let \( r \) be a realization of interest rates according to \( (\mu^i) \), and let \( C(r) \) be the set of banks staying in the market according to \( (\phi^i(r)) \). Then bank i’s payoff is given by \( U(r, i) \), \( i \in C(r) \), and 0 otherwise, where \( r_{C} = (r^j)_{j \in C(r)} \). It should be noted that we do not model firm behavior explicitly as a separate stage of the game. Instead we will come back to the assumption that firms choose with equal probability among contracts to which they are indifferent when we discuss the results.

We denote this two-stage game by \( G(q_a, q_b) \). \( G(1,0) \) then stands for the corresponding game when perfect discrimination of types is possible. For \( G(1,0) \) we implicitly assume that \( q_a = 1, q_b = 0, p_a X < 1, p_b X > 1 \).

Depending on the parameters, the game \( G(q_a, q_b) \) possesses subgame-perfect Nash equilibria in pure strategies. Let us first consider the case of two banks: Suppose that both banks announce \( r(1) \) but that only one bank provides credit. Suppose further that whenever one of the two banks charges a higher interest rate the other bank will stay in the market with probability 1. To establish that this constitutes an equilibrium we need to show that a deviating bank does not make positive profit even at \( r = X \). If this condition does not hold then there does not exist a pure strategy equilibrium. For more than two banks the situation is more complex: Again assume that a bank cannot make positive profit even at \( r = X \) if another bank provides credit at a lower interest rate. Then all banks announcing \( r(1) \) but only one bank providing credit is an equilibrium path. However there may exist other pure-strategy equilibria. Consider the following strategy combination for \( N = 3 \) banks: Banks 1 and 2 announce \( r(1) \) but leave the market, whereas firm 3 announces \( r = X \) and stays in the market. Whenever bank 1 or bank 2 deviates in the first stage, bank 2 or bank 1 will stay and bank 3 will leave. Whenever bank 3 deviates the other banks will leave the market. This constitutes a subgame-perfect equilibrium even if bank 2 stays in the market, as long as bank 1 cannot make positive profit at an interest rate \( r(1) < r < X \). We will now
formalize this condition:

\[(A4) \quad \text{Let } \hat{l} := \frac{lq_a(1-q_a)}{(lq_a(1-q_a) + (1-l)q_b(1-q_b))} \text{ and define } \hat{p}_{AB} := \hat{l}p_A + (1-\hat{l})p_B. \text{ Then } \hat{p}_{AB} X < 1.\]

Suppose that two banks provide credit at interest rates \( r_1 > r_2 \). Then bank 1 serves all those firms which are rejected by bank 2 but are accepted by bank 1. \( \hat{l} \) is the conditional probability of a firm being of type \( a \) if it belongs to this population. \( \hat{p}_{AB} \) is the corresponding average success probability. On average this population will not be able to repay the loan. This argument also applies when a bank is undercut by more than one competitor. Two remarks should be made: First \( (A4) \) implies \( (A3) \). Second given \( 0 < p_a < p_b, 0 < l < 1, 0 < q_a < 1 \) such that \( p_a X < 1 \), one can find \( q_b > 0 \) such that \( (A1)-(A4) \) hold. The assumptions therefore cover the important possibility that the tests are rather accurate in detecting firms of type \( b \). Condition \( (A4) \) will not give pure-strategy equilibria in the one-stage game, because the banks have to commit themselves simultaneously to whether or not they will provide credit. We will assume \( (A1)-(A4) \) throughout the remaining analysis.

In the next section we will introduce a refinement of the concept of subgame-perfect Nash equilibrium. We thereby eliminate many of the above equilibria. In fact for \( G(1,0) \) the equilibrium path where all banks supply credit at \( r = 1/p_b \) will be the only one. The additional requirement introduced is that an equilibrium of \( G(q_a, q_b) \) can be approximated by subgame-perfect Nash equilibria of finite versions of the game independently of the chosen sequence of approximating games. Basically, we will exclude equilibria which occur in the degenerate cases when the zero-profit interest rate \( r(1) \) belongs to the strategy spaces of the finite approximating games.

### 3.2. Equilibrium Concepts

Let \( D \) be a finite subset of \([0, X] \). Denote by \( G(D, q_a, q_b) \) the finite version of \( G(q_a, q_b) \) which is obtained by taking \( D \) as the set of admissible interest rates. A strategy of bank \( i \) is denoted by \( s^i = (p(i, D, \cdot), f(i, D, \cdot)) \), where \( p(i, D, \cdot) \) is a probability distribution over \( D \) and where \( f(i, D, \cdot) : D^N \rightarrow [0,1] \) is the probability that bank \( i \) stays in the market. Payoffs are then defined as they are for a game with a continuum of interest rates.

**Definition 3.1—Weak limit of strategies:** Let \( (D_n) \) be a successively finer approximation of \([0, X] \) by finite sets \( D_n \subset [0, X] \). Let \( s_n = (s^i_n) = (p(i, D_n, \cdot), f(i, D_n, \cdot)) \) be an \( N \)-tuple of strategies of \( G(D_n, q_a, q_b) \).

We say that an \( N \)-tuple of strategies \( \sigma = (\sigma^i) = (\mu^i, \phi^i) \) of \( G(q_a, q_b) \) is the weak limit of \( s_n \), \( s_n \rightarrow \sigma \) (weak), if

\[
(a) \quad \forall i, \forall h \in C([0, X]) \quad \sum h(r) p(i, D_n, r) \rightarrow \int h(r) d\mu^i,
\]
and

(b) \( \forall i, \forall h \in C([0, X]^N) \)

\[ \sum h(r) f(i, D_n, r) \prod p(j, D_n, r^j) \to \int h(r) \phi^i(r) \, d\mu^1(r^1) \ldots d\mu^N(r^N). \]

**Definition 3.2—Limit equilibrium:** 1. A \( N \)-tuple of strategies \( \sigma \) of \( G(q_a, q_b) \) is called a limit equilibrium (LE) if (a) \( \sigma \) is a subgame-perfect Nash equilibrium of \( G(q_a, q_b) \) and if (b) for any successively finer finite approximation \( D_n \to [0, X] \) there exists a sequence \( s_n \) of subgame perfect Nash equilibria of \( G(D_n, q_a, q_b) \) such that \( \sigma \) is the weak limit of \( s_n \).

2. A limit equilibrium is said to have a property \( P \) if for any sequence of approximating games \( G(D_n, q_a, q_b) \) there exists a sequence of approximating equilibria having this property.

3. A limit equilibrium path (LEP) is an equilibrium path of \( G(q_a, q_b) \) which is induced by a limit equilibrium. A limit equilibrium path is said to have a property \( P \) if an inducing limit equilibrium has this property.

Definition 3.2 is a refinement of the Nash equilibrium concept for games with a continuum of pure strategies. First, it requires that a given Nash equilibrium can be approximated by equilibria of the game when it is restricted to a finite subset of pure strategies. Second, it requires that the approximation be possible independently of the chosen sequence of finite subsets of pure strategies approximating the continuum of pure strategies. This concept should be distinguished from the concepts of stability, stated by Kohlberg and Mertens (1986), or perfectness, stated by Selten (1975), which are directly applicable only for games with finite strategy spaces. Here it is required that equilibria of a given game can be approximated by equilibria of slightly perturbed games.

In Definition 3.2 the idea of equilibria being approximated by equilibria of finite versions of the game is used to carry over concepts such as perfectness to the situation where we have an interval of pure strategies. Loosely speaking, an equilibrium \( \sigma \) is said to have such a property \( P \), if for any finite strategy space close to the full one we can find an equilibrium which has property \( P \) and which is close to \( \sigma \).

**3.3. Results**

We will now present the results for the two-stage game, starting with the case of perfect monitoring. The proofs will be given in the Appendix.

**Proposition 3.1:** Under perfect monitoring the unique limit equilibrium path of \( G(1, 0) \) is given by all banks providing credit at the zero-profit interest rate \( r = 1/p_b \) to firms of type \( b \). This limit equilibrium path is strong.
This Proposition states that the other subgame-perfect equilibria of $G(1,0)$ cannot be approximated by the equilibria of any given sequence of finite versions of the game. It furthermore states that any finite game $G(D,1,0)$, where $D$ is sufficiently close to $[0,X]$, possesses a strong equilibrium close to the given one. This result remains valid if all the banks which charge the lowest possible interest rate in the given finite strategy space receive a set of firms of positive measure. Suppose the extreme case that at this interest rate all firms choose the same bank. Then the situation where only this bank provides credit at the lowest interest rate constitutes a limit equilibrium path.

When we turn to the case of imperfect and independent testing the situation changes drastically. It was pointed out in Section 2.2 that there is no longer a single zero-profit interest rate but in fact a strictly increasing sequence $r(l)$, $1 \leq l \leq N$. It is obvious that the equilibrium of the type specified in Proposition 3.1 is impossible. It cannot happen that at an interest rate close to $r(1)$ more than one bank could be active with probability 1. We obtain the following proposition.

**Proposition 3.2:** Let there be two banks competing in the market, i.e. $N = 2$ and assume that $(A1)-(A4)$ hold. Any limit equilibrium path of $G(q_a, q_b)$ can be characterized by both banks charging $r(1)$ with probability 1; at most only one bank will actually provide credit. If a bank is active it will serve the market with probability 1. Both banks make zero profit.

The Proposition states that there are two possible market structures. Either both banks are inactive and do not provide any credit at all, or duopolistic competition leads to a zero-profit “monopoly” situation. The fact that both banks can be inactive even though one bank could provide credit and still make zero profit is due to our equilibrium selection. This situation is the limit of the symmetric equilibrium where both banks charge with positive probability $\theta$ the lowest possible interest rate $r > r(1)$ of a finite set $D$. However if $r \to r(1)$ we obtain $\theta \to 0$. The other two subgame perfect equilibria where one bank provides credit at $r = r(1)$ with probability 1 and the other bank is inactive give, in the limit, the provision of credit at $r(1)$ by just one bank.

We therefore have the following implication: It is not possible to approximate the LE of $G(1,0)$ by LE of $G(q_a, q_b)$ for any sequence $(q_a, q_b) \to (1,0)$. Again, it should be noted that this statement also holds if firms which are indifferent between contracts choose according to some strictly positive probability distribution over them. The above discontinuity disappears if we first let $(q_a, q_b) \to (1,0)$ for a given finite strategy space and then let the number of admissible interest rates tend to infinity, i.e. if we reverse the limit processes.

Proposition 3.2 motivates the following proposition.

**Proposition 3.3:** Under $(A1)-(A4)$ any limit equilibrium path of $G(q_a, q_b)$ where all banks set $r(1)$ and where just one bank actually provides credit is perfect.

The results demonstrate that in a competitive credit market operating with independent imperfect monitoring devices “monopolistic” market structures can
emerge. "Monopoly" here means that credit is provided by just one bank. In fact for two banks this is the only limit equilibrium path. It should be noted that introducing a small cost (disutility) of applying for a loan does not alter the result: In equilibrium there is just one bank active.

It will now be shown that for 3 banks the equilibrium described in Section 3.1 where one bank makes positive profit, is in fact a perfect limit equilibrium. Before stating the result we must define some notation. Let there be three banks in the market and let \( D = \{ r^1, \ldots, r^N \} \subset [r(1), X] \) be a finite set of interest rates, \( r(1) < r^1 < \cdots < r^N = X \). Suppose that two banks charge the same interest rate \( r \) and that the third bank charges a higher interest rate \( r' > r \). It is clear that the equilibrium strategies of the first two banks in the subgame defined by \( (r, r, r') \) are not affected by \( r' \). Denote by \( \theta(r) \) the corresponding mixed equilibrium strategy according to which the first two banks stay in the market. Applying \( \theta(r) \), the first two banks make zero expected profit, but the equilibrium strategy of the third bank is affected by \( \theta(r) \). If this value is too high, the third bank would have expected losses when providing credit at \( r' \). To formalize this, denote by \( K(r') \) the highest index \( k \) such that bank 3 does not make expected losses when it stays in the market in the subgame \( (r^k, r^k, r') \), \( k < K(r') \), assuming that banks 1 and 2 apply \( \theta(r^k) \). If no such \( k \) exists, we assume \( K(r') = 0 \).

We will now introduce the following two additional assumptions:

\[
(A5) \quad r(1) < X < r(2), \\
(A6) \quad K(r^N) \geq 2.
\]

Assumption (A5) says that if two banks charge the same lowest interest rate and apply the mixed equilibrium strategy at the second stage this equilibrium strategy will be strictly mixed and they make zero expected profit. Assumption (A6) implies that if two banks charge \( r^1 \) and stay with probability \( \theta(r^1) \), the third bank makes positive expected profit by charging \( X \). We can now state the result:

**Proposition 3.4:** Let \( D = \{ r^1, \ldots, r^N \} \subset [r(1), X] \) be a finite set of interest rates, \( r(1) < r^1 < \cdots < r^N = X \). Assume that (A1)–(A6) hold. Then \( G(D, q_a, q_b) \) possesses a perfect equilibrium which has the following equilibrium path: Banks 1 and 2 charge \( r^1 \) and stay in the market with probability \( \theta(r^1) \). Bank 3 provides credit at \( r = X \) and makes positive expected profit, whereas the other two banks make zero expected profit.

The proof is given in the Appendix.

By Proposition 3.4 it is apparent that \( G(q_a, q_b) \) also possesses a perfect limit equilibrium where one bank makes positive profit.

5. **Conclusions**

In this section we will comment on the robustness and the possible extensions of the model and its application to other markets.
The result of Proposition 3.3 has a broader interpretation: If each of the tests $T_i$ is used by $n_i$ banks, $i = 1, \ldots, N$, there is a perfect equilibrium of this game where banks of one of these groups supply credit at the zero-profit interest rate $r(1)$. Going one step further, one could extend the game by introducing a preceding stage in which banks can choose the test they will apply. Suppose that the space of pure strategies of this first stage consists of the choice of one of the independent but equally reliable credit-worthiness tests. Having chosen the test, the game will proceed as described above. We then immediately obtain a result similar to Proposition 3.1: The unique type of LE is given by all banks choosing the same test and providing credit at the zero-profit interest rate. The reason is that for any finite version of the game, where the set of interest rates does not contain $r(1)$, the following constitutes an equilibrium: All banks choose the same test and supply credit at the lowest interest rate. Clearly all banks will make positive profit. We can refer to this as the tendency towards standardized judgment procedures in how to assess the credit-worthiness of firms.

The result that just one bank will provide credit at the zero-profit interest rate is not only robust in the game-theoretic sense of being perfect. In the following we will argue that this kind of market equilibrium also emerges under certain modifications of the basic game. It is quite clear that a bank with a more reliable or less costly test than those of the other banks will be the only bank that stays in the market. In fact this bank will make positive profit. Furthermore our assumption that firms distribute themselves equally among those banks which charge the same interest rate will not change the result. This kind of firm behavior is of no relevance when credit is provided at the zero-profit interest rate $r(1)$. Now suppose that banks face capacity constraints, so that a bank can accept only a proportion $c$ of applying firms. If two banks provide credit, the corresponding zero-profit interest rate is strictly larger than $r(1)$. This in turn implies that no more than one bank will serve the market. For more than one bank to operate in the market it is necessary that their market shares be either disjoint or totally coincide. Assuming that tests are independent, the latter is only possible if banks exchange their test results, which induces incentive problems. The former is possible if firms will not receive credit if they have applied to more than one bank. In this case market equilibria will emerge where all banks provide credit at $r(1)$, provided that all banks face the same ratio of good and bad risks. Therefore if the firms' choices among contracts with the same interest rate induce different ratios of good to bad risks for different banks, only the bank with the most favorable ratio will stay in the market.

It is also worth noting that there is an advantage of being first, because of the negative externalities caused by the rejection decisions of the other banks: In a multi-period model, a bank will face a bad risk pool if other banks have selected their clientele in previous periods. This is precisely the same effect as when banks charge increasing interest rates. For the reverse effect see Guasch and Weiss (1980a) and also Nalebuff and Stiglitz (1982).

The basic setting of multiple and independent testing can also be applied to other markets in which the uninformed agents set prices and evaluate products.
Examples are the evaluation of collateral in credit markets, interviews of applicants in job markets, or the assessment of risks in insurance markets. One reason that self-selection and sorting devices are not always used is that the applicants do not know their own types. Our model is applicable to this situation. It might also be possible to combine self-selection and sorting devices with a preclassification of risks.

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APPENDIX

PROOF OF PROPOSITION 2.2: Without loss of generality, we restrict the admissible interest rates to \([r(1), X]\). Let there be \(n + 1\) banks in the market. Let \(\pi\) denote the probability with which a bank will leave the market and denote by \(P\) a distribution function over \([r(1), X]\) that generates a bank's announcement of interest rates if it stays. We will assume that \(P\) is continuous.

Given the data \(\pi\) and \(P\), the profit of bank 1 at the interest rate \(r\) is

\[
V(\pi, P(\cdot), n, r) = \sum_{k=0}^{n} \pi^{n-k}(1-\pi)^{k} \sum_{i=0}^{k} \left[ (1-P(r))^{k-i} P(r)^{i} \right] \left[ q_a(1-q_a)(p_a r-1) + (1-l) q_b(1-q_b)(p_b r-1) \right].
\]

\(\pi^{n-k}(1-\pi)^{k} \binom{n}{k}\) is the probability that \(k\) of the \(n\) competitors of bank 1 stay in the market. 

\((1-P(r))^{k-i} P(r)^{i} \binom{k}{i}\) is then the probability that \(i\) of these \(k\) banks undercut bank 1. The above expression can be simplified to

\[
V(\pi, P(\cdot), n, r) = \left[ \pi + (1-\pi)(1-P(r)+P(r)q_a) \right]^{n} l(1-q_a)(p_a r-1) + \left[ \pi + (1-\pi)(1-P(r)+P(r)q_b) \right]^{n} l(1-q_b)(p_b r-1).
\]

First we will identify the critical value for the number of firms, \(n^* + 1 \in \mathbb{R}\), below which all banks will stay with probability one in the market. It holds in any equilibrium that the lowest interest rate \(r\) at which \(P(r) = 1\) is equal to \(X\). It is therefore sufficient to determine the maximum number of banks such that a bank does not make losses by setting \(r = X\).

**Lemma 1:** Let \(A \equiv (1-l)(1-q_b)(p_b X-1)/(l(1-q_a)(1-p_a X))\). Define \(n^*\) to be the solution of \(V(0,1, n, X) = 0\). Then \(n^* = \ln A/\ln q_a/q_b\). Furthermore for any equilibrium \((\pi, P)\) such that \(P \in C^0([r(1), X])\) it holds that \(\pi = 0 \iff n \leq n^*\).

The next Lemma contains the major step which is necessary to compute the equilibrium probability distribution \(P\):

**Lemma 2:** Define \(\alpha_a(P) \equiv \pi + (1-\pi)(1-P + Pq_a)\). Then the solution \(R = R(P)\) of \(V(\pi, P, n, R) = V \forall P \in [0, X]\), \(\pi < 1\) is given by the equation

\[
R(P) = \frac{V + \alpha_a(P)\ln(1-q_a) + \alpha_b(P)\ln(1-l)(1-q_b)}{p_a \alpha_a(P)\ln(1-q_a) + p_b \alpha_b(P)\ln(1-l)(1-q_b)}.
\]

\(R(P)\) has the following properties:

\[
R(P) \in C^\infty([0,1]), \quad R'(P) > 0, \quad R(0) > r(1) \iff V > 0.
\]
CREDIT-WORTHINESS

PROOF: We have to prove \(2\). The denominator of \(R'(P)/(l - n)n\) is

\[
\bar{V} \left[ p_a \alpha_a(P)^{n-1} (1 - q_a)^2 + p_b \alpha_b(P)^{n-1} (1 - l)(1 - q_b)^2 \right] + \left[ \alpha_a(P)^{n-1} \alpha_b(P)^{n-1} (1 - l)(1 - q_a)(1 - q_b) \right] 
\times (p_b - p_a)(\alpha_a(P)(1 - q_b) - \alpha_b(P)(1 - q_a)).
\]

The second factor of the second term is equal to \((p_a - p_b)(q_a - q_b)\) which is positive. This proves the Lemma. Q.E.D.

We will now simply apply Lemma 2 to the two cases \(n \leq n^*\) and \(n \geq n^*\) respectively.

**LEMMA 3**: Let \(n \leq n^*\). Then the one-stage game possesses a unique symmetric equilibrium \((\pi(n), P(n))\) under the constraint \(P(n) \in C^0([r(1), X])\). It has the properties

1. \(\pi(n) = 0\);
2. let \(r(n)\) be the highest interest rate such that \(P(r) = 0\), i.e. \(r(n) = R(0)\); then
   a. \(r(n) > r(1) \iff n < n^*\),
   b. \(P(n) \in C^\infty([r(n), X])\);
3. the banks make positive profit as long as \(n < n^*\).

**PROOF**: It is clear that the lowest interest rate \(r\) satisfying \(P(r) = 1\) has to be equal to \(X\). Therefore the banks’ profit will be at least \(V(0, 1, n, X) = \bar{V}\). Now apply Lemma 2 with \(V\) and \(\pi = 0\) and consider the trivial extension of the inverse of \(R = R(P)\). Call this distribution function \(P(\cdot)\). Then \(P(\cdot)\) satisfies the conditions \((2). (0, P)\) is an equilibrium because it does not pay to set an interest rate below \(r(n)\). Furthermore \(P\) is unique. This proves the Lemma. Q.E.D.

We will now consider the case \(n \geq n^*\). In this situation it is important to compute explicitly the probability \(\pi\) with which a single bank will leave the market. Banks will make zero profit. As we have already mentioned in any equilibrium \((\pi, P)\) the lowest interest rate \(r\) such that \(P(r) = 1\) is equal to \(X\). This implies that in any equilibrium a bank is indifferent between leaving the market and setting \(r(n)\). We will use this to determine the optimal \(\pi(n)\).

**LEMMA 4**: In any equilibrium \((\pi, P), P \in C^0([r(1), X])\), for \(n + 1\) banks, \(n \geq n^*, \pi(n)\) is the unique solution to \(V(\pi, 1, n, X) = 0\). It is given by

\[
\pi(n) = \frac{q_a - \frac{n}{\sqrt{A}}}{q_a - q_b \sqrt{A} + \frac{n}{\sqrt{A}} - 1},
\]

where

\[
A = \frac{(1 - l)(1 - q_b)(p_b X - 1)}{l(1 - q_a)(1 - p_a X)} > 1.
\]

\(\pi(n)\) has the property that \(\pi(n) > 0 \iff n > n^*\).

Using this value \(\pi(n)\), we can again apply Lemma 2:

**LEMMA 5**: Let \(n \geq n^*\). Then the one-stage game possesses a unique symmetric equilibrium \((\pi(n), P(n))\) under the constraint \(P(n) \in C^0([r(1), X])\). It has the following properties:

1. \(\pi(n) > 0 \iff n > n^*\) and \(\pi(n) \uparrow 1\);
2. \(P(n) > 0 \forall r > r(1)\) and \(P(n) \in C^\infty([r(1), X])\);
3. the banks make zero profit.

This completes the proof of the Proposition. Q.E.D.
PROOF OF PROPOSITION 2.3: It is common knowledge that 
\[ n^* := \lim_{n \to \infty} n (1 - \pi^{(n)}) = \frac{\ln A}{q_b - q_a}. \]
Using the logarithm it then follows that 
\[ \exp(-n^*P(1 - q_a)) = \frac{\alpha_{n^*}(P) n^*}{p_a \alpha_{n^*}(P) n^* i (1 - q_a) + p_a \alpha_{n^*}(P) n^* i (1 - q_a) + p_a \alpha_{n^*}(P) n^* i (1 - q_a)}. \]
This proves the Proposition. Q. E. D.

PROOF OF PROPOSITION 3.1: The strategy tuple where all banks provide credit at the interest rate 
\[ r = \frac{1}{p_b}, \]
constitutes an equilibrium of \( G(1,0) \).
Without loss of generality let \( D \subset [1/p_b, X] \). Let \( D \) be finite and denote the smallest interest rate in \( D \) not equal to \( 1/p_b \) by \( r^1 \). Then the strategy tuple where all banks provide credit at \( r^1 \) is a subgame-perfect Nash equilibrium of \( G(D,1,0) \). Obviously \( r = 1/p_b \) can be approximated by these equilibria and is therefore a LE.
To prove uniqueness, we have to show that there exists a sequence \( D_n \rightarrow [1/p_b, X] \) such that all banks providing credit at the corresponding interest rate \( r_n \) is the only subgame-perfect Nash equilibrium of \( G(D_n,1,0) \).
Consider the following set \( D = \{r^1, \ldots, r^n\} \) of interest rates \( r^j = \frac{1}{p_b} + j(X - 1/p_b)/n, \ j = 1, 3, \ldots, n, \ r^n = \frac{1}{p_b} + (X - 1)/p_b/5n \). In any equilibrium all banks will make positive profit because each bank could choose \( r^j \). Now let \( p = (p_1, \ldots, p^n) \) be the first stage part of such an equilibrium. Define \( k \) to be the highest index such that \( p'(r^k) \neq 0 \) for all \( i \). This means that \( r^k \) is the highest interest rate which will not be undercut with probability 1 by at least one bank. The property that \( p' \) is part of an equilibrium implies \( p'(r^k) = 0 \) for all \( i \) and for all \( i > k \). We will now demonstrate that \( k \geq 2 \) is impossible by proving that any bank strictly prefers \( r_{k-1} \) to \( r_k \), contrary to the definition of \( k \).
Let \( q \) be the probability that all banks but bank 1 announce \( r_k \), i.e. \( q = p^2(r_k) \cdots p^N(r_k) \neq 0 \). The payoff accruing to bank 1 in case it announces \( r_k \) is given by
\[ U(r_k) = q p_b \left( r_k - \frac{1}{p_b} \right) (1 - l)/N. \]
Notice that if all the other banks do not also announce \( r_k \), bank 1’s profits will be zero. The profit at \( r_k-1 \) is given by
\[ U(r_k-1) = q p_b \left( r_k - 1/p_b \right) (1 - l) + \text{nonneg. terms}. \]
Due to the construction of \( D \) it holds that
\[ N > \left( r_k - \frac{1}{p_b} \right) / \left( r_k - 1/p_b \right), \quad \forall N \geq 2, \]
which proves the assertion. Q. E. D.

PROOF OF PROPOSITION 3.2: The proof contains three steps. First we will characterize all subgame-perfect Nash equilibria of \( G(q_a, q_b) \) where at least one bank announces \( r(1) \) with probability 1. Second we will show that any LE must have this property. And finally we have to establish which of the equilibria determined in the first step are in fact LE.
Without loss of generality we only consider the set of interest rates \( [r(1), X] \). Suppose that in equilibrium bank 1 announces \( r(1) \) with probability 1. Clearly it will make zero profit and therefore it is indifferent between any interest rate given the strategy of bank 2. This implies that bank 1 is not able to undercut its opponent with positive probability and thereby to make positive profit. This is only possible if bank 2 also charges \( r(1) \) with probability 1 and threatens to stay in case bank 1 sets a higher interest rate. Therefore the first stage part of the equilibrium is that both banks set \( r(1) \) and threaten to supply credit if the other bank sets a higher interest rate. It remains to determine the probabilities \( \theta_1, \theta_2 \) with which banks 1 and 2 resp. will stay in the market if both announce \( r(1) \). We have the following possibilities:
\[ \theta_1 = 0 \Leftrightarrow \theta_2 = 0, \quad \theta_1 > 0 \Leftrightarrow \theta_2 = 0, \]
and vice versa. This characterizes all equilibria of \( G(q_a, q_b) \) where at least one bank announces \( r(1) \) with probability 1.
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The above are also equilibria of the finite games $G(D, q_1, q_2)$ provided that $r(1) \in D$. We will now consider finite approximations such that $r(1) \notin D$. Let $D = \{r^1, r^2, \ldots, r^n\}$, let $p^1, p^2$ be the first stage components of an equilibrium and let $r^k$ be the smallest interest rate satisfying $r^k \geq r(2)$. Denote by $r^1$ and $r^m$ the highest interest rates chosen with positive probability by banks 1 and 2 respectively.

Suppose $m > l$. Whenever $r^m$ occurs it is optimal for bank 2 to leave the market in stage two, because it would otherwise have losses due to (A4). Thus bank 2 makes zero profit. Following the same line of argument as in step 1, this implies that bank 1 sets $r^1$ with probability 1.

Suppose $m = l \geq k$. We will demonstrate that bank 1 strictly prefers $r^{m-1}$ to $r^m$. The profit accruing to bank 1 under $r^m$ is given by

$$U(r^m) = p^2(r^m)((1 - q_a)(p_a r^{m-1} - 1) + (1 - l)(1 - q_b)(p_b r^{m-1} - 1)),$$

whereas for $r^{m-1}$ the profit $U(r^{m-1})$ is at least

$$p^2(r^{m-1})((1 - q_a)(p_a r^{k-1} - 1) + (1 - l)(1 - q_b)(p_b r^{k-1} - 1)) > (1 - q_a)(p_a r^{k-1} - 1) + (1 - l)(1 - q_b)(p_b r^{k-1} - 1)/2.$$

It follows that $U(r^{m-1}) > U(r^m)$ if $D$ is sufficiently close to $[r(1), X]$.

If $1 < m = l < k$, at least one bank will make zero profit. But it could do better by charging $r^{m-1}$ and thereby undercut its opponent with positive probability.

So far we have shown that any equilibrium of $G(D, q_1, q_2)$ must involve $r^1$ being set with probability 1 by at least one bank. It follows that in any LE $r(1)$ will be set with probability 1 by at least one bank.

Now assume that both banks announce $r^1$. There are only two possibilities for their behavior in the second stage of the game: one bank stays and the other one leaves, or both stay with probability $\phi^*$, $0 < \phi^* < 1$. $\phi^*$ is the unique mixed strategy at which both banks make zero expected profit. $\phi^*$ tends to 0 if $r^1$ approaches $r(1)$. We have established that the equilibria characterized in the proposition are in fact LE.

Finally in the third step we will now show that all other equilibria of $G(q_1, q_2)$ are not LE. Without loss of generality, let bank 1 announce $r^1$ with probability 1. It will stay with probability 0, $\theta^*$, or 1 if its opponent also announces $r^1$. The LE strategy of bank 1 cannot involve a completely mixed strategy in stage 2: its limit strategy can only be $(r(1), 0)$ or $(r(1), 1)$. Let us now consider bank 2. If the LE strategy of bank 2 is given by setting $r(1)$ with probability 1 and staying with probability $\theta$, $0 < \theta < 1$, there has to exist a sequence of equilibrium strategies $(0, D, r^l, r^m)$ such that $0, D, r^l$ converges weakly to $(r(1), r(1))$ and such that

$$\forall h \in C([r(1), X]^2) \quad p(2, D, r^1)f(2, D, r^1, r^l, r^m)h(r^1, r^l) \to h(r(1), r(1)).$$

Observe that because of $f(2, D, r^l, r^1) \in (0, \phi^*, 1)$, the above can possibly hold only if $f(2, D, r^l, r^1) = 1$. This in turn implies that $p(2, D, r^1)$ converges to $\theta$. Therefore for any $D$ close to $[r(1), X]$ the probability with which bank 2 announces $r^1$ is bounded away from 0 and 1. But this cannot be part of an equilibrium strategy if bank 1 announces $r^1$ with probability 1 and stays with probability 0, $\theta$, or 1. This completes the proof.

Proof of Proposition 3.3: We have to show that there exists a LE inducing the given LEP such that for any $D$ close to $[r(1), X]$ there exists a perfect equilibrium of $G(D, q_1, q_2)$ close to the LE. We will first define for $[r(1), X]$ an equilibrium $\sigma$, a restriction of which to any finite version of the game will approximate $\sigma$.

Let $r = (r^1, \ldots, r^N)$, $\ell_{\min} = \min(r')$, $C(r) = \{ j/r^j = r_{\min} \} = \{ j_1, \ldots, j_x \}$. Consider the following strategies: In the first stage all banks set $r(1)$ with probability 1; in the second stage they apply

$$\phi^*(r) = \begin{cases} 0 & \text{if } j \notin C(r), \\ 0 & \text{if } j = j_1, l > \max \{ z/\ell_{\min} \geq r(z) \}, \\ 1 & \text{otherwise.} \end{cases}$$

Provided that (A4) holds it is optimal for any bank to leave the market if it is undercut by a
competitor. Furthermore if the lowest interest rate which is charged in the market is greater or equal to \( r(l) \), at most \( l \) banks can provide credit at this interest rate. The above strategy therefore constitutes a subgame-perfect Nash equilibrium of \( G(q_a, q_b) \).

Let \( D \subset [r(1), X] \) be finite. A restriction of the above strategies to \( G(D, q_a, q_b) \) is given by charging the lowest interest rate \( r^l \in D, r^l \neq r(1) \) and by \( f(i, D, r) = \phi(r) \) for \( r \in D^N, f(i, D, r) = 0 \) if \( r^l = r(1) \). Clearly the above is a LE inducing the LEP, described in the Proposition under the assumption that bank 1 stays.

Let \( \delta' \) be a completely mixed strategy over \( D \) and let \( \delta \in (0, \delta), i = 1, \ldots, N \). We now look at the perturbed game which is defined by replacing a strategy \( p^l(i, D, \cdot) = p' \) by \( p'(r') = (1 - \delta')p^l + \delta'\pi' \). No trembling hand mechanism is involved at the second stage. We will show that for some \( \pi', \ldots, \pi^N \) and for any \( \delta \) sufficiently small \( (r'(r')) \) is an equilibrium of the perturbed game. In the following we will use the notation \( \pi'(r^0) = 0 \) if the interest rate \( r^0 = r(1) \) does not belong to \( D \).

Given that all other banks announce \( r'(r) \), \( i \geq 2 \), bank 1 strictly prefers \( r^l(r') \) to any \( r^l(r') \), \( i > 1 \), if \( \delta \) is sufficiently small, because the probability of being undercut at a higher interest rate will be close to 1.

Let \( U(r) = l(1 - q_a)(p_a r - 1) + (1 - l)(1 - q_i)(p_i r - 1) \) be the profit accruing to a bank which is the only one setting \( r \) and not being undercut by any other bank. Then the expected profit for bank \( k, k \geq 2 \) given \( t^k(r') \) is at least

\[
\sum_{m \neq l} \delta^l \pi^l m E(\pi^2) U(r^m) + (1 - \delta^k + \delta^k \pi^k) E(\pi^2) U(r^l), \quad \text{where}
\]

\[
E(\pi^2) = \prod_{j=1}^{k-1} \delta^j (1 - \pi^j - \pi^l).
\]

If bank \( k \) happens to announce \( r^2 \) it will make positive profit only if none of the banks \( 1, \ldots, k-1 \) charges \( r \leq r^2 \). If bank \( k \) happens to announce \( r^m, m \geq 3 \), it can make positive profit only if it is not strictly undercut by one of the banks \( 1, \ldots, k-1 \), the probability of which is smaller than or equal to \( E(\pi^2) \). Now one can find \( \pi^2 \) close to \( 1 - \pi^1 \) such that for all \( k, l \geq 2 \), the expression (1) is strictly greater than (2). Having shown that \( (r', \ldots, r^1) \) is an equilibrium of the game for some perturbations, where all banks make positive expected profit, we can easily relax the assumption that no mistakes be made at stage 2. This completes the proof.

\[ Q.E.D. \]

**PROOF OF PROPOSITION 3.4:** First, we give a complete definition of the equilibrium \( (p(i, \cdot), f(i, \cdot)) \).

We consider:

\[ p(1, r^1) = p(2, r^1) = 1, \]
\[ p(3, r^N) = 1, \]
\[ f(1, r^1, r^2, r^3) = \begin{cases} 1, & r^1 < r^2, r^1 < r^3, \\ \delta(r^1), & r^1 = r^2 \leq r^3, \\ 0, & r^1 > r^2 \text{ or } r^1 > r^3. \end{cases} \]
\[ f(2, r^1, r^2, r^3) = f(1, r^2, r^1, r^3), \]
\[ f(3, r^1, r^2, r^3) = \begin{cases} 0, & r^1 \leq r^3 \text{ or } r^2 \leq r^3 \text{ or } r^1 = r^2 = r^3, k > K(r^3), \\ 1, & \text{otherwise}. \end{cases} \]

By construction, bank \( i \) makes positive profit when \( (r^1, r^2, r^3) \) is announced if and only if \( f(i, r^1, r^2, r^3) = 1 \).

Let \( \delta, \varepsilon \in (0,1) \) and let \( \pi, \sigma \) denote probability distributions over \( D \).

We now look at the following perturbed game: In the first stage bank 1's and bank 2's pure strategies are \( s(r^k) = (1 - \delta)\pi^k + \delta \pi \) and bank 3's pure strategies are \( t(r^k) = (1 - \varepsilon)\pi^k + \varepsilon \sigma, k = \ldots, 3 \).
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1, ..., N. Let there be no trembling hand mechanism at the second stage, so that bank \( i \) applies \( f(i, r_1^i, r_2^i, r_3^i) \) whenever \((r_1^i, r_2^i, r_3^i)\) is announced at the first stage. The payoff of bank \( i \) under \( (s(r_1^i), s(r_2^i), t(r_3^i)) \), \( V(i, r_1^i, r_2^i, r_3^i) \), is the expected payoff of bank \( i \) under these probability distributions.

We have to find \( \delta, \epsilon, \pi, \sigma \) such that

\[
V(1, r_1^1, r_1^2, r_3^N) > V(1, r_N^k, r_1^1, r_3^N), \quad \forall k \geq 2,
\]

\[
V(3, r_1^1, r_1^2, r_3^N) > V(3, r_1^1, r_N^k), \quad \forall k \leq N - 1.
\]

If this is possible sufficiently small trembles at the second stage will not alter the result. We will then have established the perfectness of the equilibrium.

Suppose that banks 1 and 2 choose both \( s(r_1^i) \). Suppose that bank 3 happens to announce \( r_N^k \) according to the strategy it applies and that the announcements of the other two banks are \( r_1^i, r_2^i \). Bank 3 makes positive profit under \( (r_1^i, r_2^i, r_N^k) \) if \( r_N^k < r_1^i, r_2^i \), or if \( r_1^i = r_2^i = r_N^k, l \leq K(r_N^k) \). Denote by \( T(r_1^i, r_2^i) \) the function that has the value 1 if \( l \leq K(r_N^k) \) and 0 otherwise. Denote by \( W(r_N^k, r') \) the expected payoff of bank 3 if banks 1 and 2 charge \( r' \) and if bank 3 charges \( r_N^k \) and denote by \( U(r_N^k) \) the profit a single bank would make if it charges \( r_N^k \). Then the expected payoff of bank 3 under \( t(r_N^k) \) is

\[
V(3, r_1^1, r_1^2, r_3^N)
= \sum_{j=1}^{N-1} t_j(r_3^N)U(r') \left( \sigma_{j+1} + \cdots + \sigma_N \right)^2
+ \sum_{j=1}^{N} t_j(r_3^N) \sum_{l=1}^{j} W(r', r') T(r', r') s_l(r_3^N)^2.
\]

A similar formula holds for \( V(3, r_1^1, r_1^2, r_N^k), 1 \leq k \leq N - 1 \). It follows that

\[
\frac{V(3, r_1^1, r_1^2, r_N^k) - V(3, r_1^1, r_1^2, r_N^k)}{(1 - \epsilon)}
= \sum_{l=1}^{N} W(r_N^k, r') T(r_N^k, r') s_l(r_3^N)^2 - \sum_{l=1}^{k} W(r_N^k, r') T(r_N^k, r') s_l(r_3^N)^2
- U(r_N^k) \left( \sigma_{k+1} + \cdots + \sigma_N \right)^2.
\]

The difference between the first two terms is for all \( 1 \leq k \leq N - 1 \) greater or equal to the difference between these two terms when \( k \) is replaced by \( N - 1 \). This expression in turn is strictly positive and does not depend on \( \epsilon, \sigma \). The third term can be made arbitrarily small by choosing \( \pi_1 \) sufficiently close to 1. This proves that \( V(3, r_1^1, r_1^2, r_3^N) > V(3, r_1^1, r_1^2, r_N^k) \) for all \( 1 \leq k \leq N - 1 \).

Suppose now that bank 2 applies \( (1 - \delta) r_1^2 + \delta \pi \) and that bank 3 applies \( (1 - \delta) r_N^k + \epsilon \sigma \). Furthermore, let \( \pi_1 \) be sufficiently close to 1 so that the above inequality holds. The profit of bank 1 under \( (1 - \delta)r_1^2 + \delta \sigma \) is given by

\[
(1 - \delta + \delta \pi_1) \delta (1 - \pi_1) U(r_3^1)
+ \sum_{i=2}^{N-1} \delta \pi_i \delta (1 - \pi_1 - \cdots - \pi_i)(1 - \epsilon \sigma_1 - \cdots - \epsilon \sigma_{i-1}) U(r')
\]

Notice that bank 1 only makes positive profit if \( r_1^2 < r_3^2, r_1 < r_3^3 \). The profit of bank 1 under \( s(r_N^k) \) is given by the above formula where \( (1 - \delta) + \delta \pi_1 \) has to be replaced by \( \delta \pi_1 \) and \( \delta \pi_N \) to be replaced by \( (1 - \delta + \delta \pi_N) \). The difference of these two expressions is then

\[
(1 - \delta) \delta (1 - \pi_1) U(r_3^1)
- (1 - \delta) \delta (1 - \pi_1 - \cdots - \pi_k)(1 - \epsilon \sigma_1 - \cdots - \epsilon \sigma_{k-1}) U(r_N^k).
\]

One can find \( \pi_3 > 0 \) close to \( 1 - \pi_1 \) such that this difference is positive for any \( \epsilon, \sigma \).

This also demonstrates that the symmetric equilibrium for two banks is perfect. It should be noted that to establish perfectness it was only necessary to impose conditions on \( \pi_1 \) and \( \pi_2 \).

This proves the Proposition. 

Q.E.D.
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