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Overconfidence and delegated portfolio management ☆

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ABSTRACT

We study the impact of overconfidence on investment decisions by financial institutions. These institutions are characterized by the delegation of investment decisions to portfolio managers and the design of contracts that aim at aligning managers' incentives with those of the institution. We show that when rational and overconfident agents acquire information of the same precision, overconfident agents trade *lower* quantities than rational agents. However, overconfidence also generates incentives to overinvest in information acquisition. In such cases, overconfident agents trade larger quantities and take more risk than rational agents. The direct consequence of these results is that, as far as delegated portfolio management is concerned, overconfidence generates high trading volumes only through over-acquisition of information. Based on psychological evidence that overconfidence is generated by a self-attribution bias, our results are consistent with recent empirical evidence about mutual fund managers' portfolio-rebalancing patterns and changes in mutual funds' advisory contracts.

1. Introduction

It is widely acknowledged that some financial investment decisions are difficult to reconcile with fully rational behavior. Accordingly, the analysis of financial markets in the presence of irrational agents has received increasing attention over the last fifteen years. One form of irrational behavior studied extensively is overconfidence (see Kyle and Wang, 1997; Benos, 1998; Odean, 1998; Wang, 1998, 2001; Daniel et al., 1998, 2001; Caballé and Sákovics, 2003). A common feature of these models

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is that market participants are individual investors maximizing their expected utility of wealth given their beliefs. Furthermore, the common result is that overconfident investors hold riskier position/trade larger quantities than rational investors.

However, a large fraction of company stocks are held by financial institutions. As highlighted by Lewellen (2008), institutions held 68% of the US equity market at the end of 2007. Furthermore, (i) their investment strategies differ from those of individual investors (see Cohen, 1999; Grinblatt and Keloharju, 2000; Cohen et al., 2002; Ekholm and Pasternack, 2008); and (ii) recent empirical and experimental studies provide evidence of overconfidence on the side of professional investors (see, for example, Gort, 2007; Pütz and Ruenzi, 2008; Choi and Lou, 2008).[1] As a consequence, in order to analyze the impact of overconfidence on financial asset prices, one needs first to study how overconfidence influences the investment strategies of institutional investors.

A fundamental characteristic of financial institutions is the delegation of investment decisions to professional money managers. As a consequence of this delegation, agency conflicts may arise. Therefore, the institution will design a compensation contract aimed at mitigating conflicts of interest.

Faced with overconfident managers, the institution must also deal with the biases overconfidence may generate.[2] Hence, the institution will design a compensation contract that deals with both agency problems and biases associated with overconfidence.

In the end, the investment strategy of the manager will depend on his compensation contract. Therefore, in order to analyze how overconfidence influences institutional investments, one must endogenize compensation contracts and study investment strategies resulting from these endogenous contracts.

To address this issue, we analyze a delegated portfolio management problem in which a risk-averse financial institution (the principal) hires a risk-averse money manager (the agent) who may be of two types: rational or overconfident. By exerting effort, the agent acquires private information about the value of a risky asset. If the agent is rational, he updates his beliefs in a Bayesian fashion. However, if overconfident, the agent over-estimates the precision of his private signal. Based on his updated beliefs, the agent then makes an investment decision.

In this situation, the moral hazard problem faced by the principal has two aspects, overconfidence having an impact on each of them. First, the principal must provide the agent with incentives to exert effort and acquire information. Second, if the principal and the agent have different levels of risk aversion (as we will assume), the principal must calibrate the risk-taking incentives the contract provides to the agent.

In order to disentangle the impact of overconfidence on each aspect of the agency problems, we consider two different cases regarding the acquisition of information. First, we assume that effort is a binary choice. It implies that if rational and overconfident agents acquire information, it is of the same precision. As a consequence, the effect of overconfidence on the acquisition of information is neutralized, and only the effect of overconfidence on the portfolio allocation is at work. In this environment, we show that overconfident agents trade *lower* quantities than rational agents do. A direct implication of this result is that those obtained in the case of private investors (i.e., excessive trading) may not extend to the case of delegated portfolio management. When considering institutional investment, overconfidence may not generate excessive return volatility if the appropriate compensation contract is used.

The second case we consider is such that effort is a continuous variable. We show that overconfident agents acquire more precise information and trade larger quantities than rational agents do. The riskiness of investment strategies is then influenced in two ways. First, it is decreased through a lower risk level per unit of investment, and second it is increased through larger risky-asset holdings. We

[1] Though this paper concentrates on the miscalibration of private information, empirical studies suggest that overconfidence can take several forms, including thinking of being better than others and a self-attribution, this latter bias leading to miscalibration of private information.

[2] Barber and Odean (2002) provide a review of psychological literature on overconfidence. As they explain, "overconfidence is greatest for difficult tasks, for forecasts with low predictability... Selecting common stocks that will outperform the market is a difficult task. Predictability is low; feedback is noisy. Thus stock selection is the type of tasks for which people are most overconfident."

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show that the latter effect always dominates the former implying that overconfidence always increases the level of risk undertaken.

One could find this result very intuitive since more precise information implies higher expected returns and lower risk per unit traded, hence providing room to trade larger quantities. However, this is not the full picture. The reason is that the return taken into account by the principal when designing a compensation contract is the portfolio return *net* of compensation expenses, and the cost of providing incentive to exert effort is different for overconfident and rational agents. We show that the principal provides more incentives to exert effort to overconfident agents. As a consequence, for a given portfolio-return volatility, the net-return volatility is smaller if the agent is overconfident than if the agent is rational. It then follows that the principal is willing to provide more trading incentives to an overconfident agent.

Hence, comparing the results obtained in the binary and the continuous effort cases, we show that in the case of delegation, overconfidence generates higher trading volume only through over-acquisition of information.

Based on psychological evidence that overconfidence is generated by a self-attribution bias, i.e., agents credit themselves for past good performance but blame others for failure, (see, for example, Gervais and Odean, 2001), our results are consistent with several pieces of empirical evidence. First, our results are consistent with those of Pütz and Ruenzi (2008) and Choi and Lou (2008) who show that fund managers trade more after a good performance. Second, in our model institutions acquire more information than individual investors. As a consequence, they realize better performances than individual investors, implying that institutions are the market participants who suffer from the stronger confidence reinforcement. Using this result in the market model of Daniel et al. (1998, Section III), we deduce that institutions are the market participants who follow momentum strategies. This is consistent with the findings of Grinblatt and Keloharju (2000) who provide evidence that the most sophisticated agents, foreign funds in their sample and professional money managers in our model, are those following momentum strategies. Third, our results suggest that, as a consequence of overconfidence reinforcement, the incentive component of the compensation contract increases. Therefore, we expect an increase in mutual fund management fees following good performances and no variation in case of bad performances. This is consistent with the findings of Warner and Wu (2005) on the changes in mutual fund advisory contracts.

The organization of the paper is as follows. Section 2 reviews the related literature. Section 3 presents the model. Sections 4 and 5 study optimal contracts and investment strategies in the binary and continuous effort cases, respectively. Finally, Section 6 discusses the results and their consistency with empirical evidence, while Section 7 presents our conclusion. All the proofs are contained in the Appendix.

2. Related literature

Our article bridges the literature on delegated portfolio management and overconfidence in financial markets.

Bhattacharya and Pfleiderer (1985) were the first to study delegated portfolio management in a principal-agent framework. However, their model is one of hidden information rather than hidden action since the principal can verify the level of risk taken by the agent.[3]

Cohen and Starks (1988), Admati and Pfleiderer (1996), Diamond (1998), Palomino and Prat (2003) study delegated portfolio management with moral hazard on both effort and risk. Cohen and Starks (1988) derive conditions under which the manager exerts more effort but chooses a riskier portfolio than investors prefer. Admati and Pfleiderer (1996) look at the impact of benchmarking on behavior. They show that, in general, benchmarking is inconsistent with obtaining the optimal portfolio and tends to decrease incentives to exert effort. Diamond (1998) shows that if the control space of the agent has full dimensionality (i.e., the principal has fewer degrees of freedom in setting the incentives than the agent has degrees of freedom in responding), then as the cost of effort shrinks, the optimal contract converges

[3] This type of problem is also analyzed in Stoughton (1993).

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to a linear contract. Palomino and Prat (2003) consider the case in which the agent has limited liability. They show that there exists an optimal contract which takes the form of a bonus contract.

The influence of overconfidence on decisions in the context of delegation has been studied by Gervais et al. (2006) and Hackbarth (2008). Gervais et al. (2006) consider a capital budgeting problem faced by a risk-averse overconfident manager. They find that a risk-neutral principal may be better off hiring an overconfident agent than a rational one. The main difference between their study and ours is that in our model the size of the investment (i.e., the amount invested in risky assets) is endogenous while in their model the amount invested and the possible final values of the project are exogenous. This implies that overconfidence has only positive aspects from the principal view-point: it reduces the difference in risk-taking incentives. Conversely, in our model, given that the agent chooses the size of investment, the risk-averse principal faces the risk that an overconfident agent over-invests (i.e., invests more than the optimal amount from the principal's view-point).

Hackbarth (2008) studies the impact of overconfidence on capital structure. He shows that managers with growth and/or risk perception biases tend to choose higher debt levels and issue new debt more often compared to otherwise identical unbiased managers. The main difference with our article is that Hackbarth assumes that managers act in the interest of investors, i.e., aim at maximizing the perceived value of the firm.

The consequences of overconfidence in financial markets in the case of agents trading for their own account have been studied in the context of both perfectly and imperfectly competitive markets. Under the assumption of perfect competition, Odean (1998, Section III.A) studies a market in which all informed agents are overconfident about the precision of their information. He shows that as overconfidence increases, trading volume and price volatility increase and the expected utility of overconfident agents is lower than if their beliefs are properly calibrated. Studying price reactions to public and private information, Daniel et al. (1998) show that overconfidence increases price volatility around private signals, and that price moves resulting from the arrival of private information are on average partially reversed in the long run. Wang (2001, Section III) studies population dynamics in the presence of rational and overconfident agents. He shows that if overconfident agents are moderately overconfident and, initially, represent a sufficiently large fraction of the population, then overconfident agents as a group will dominate the economy, in the long run. Finally, Daniel et al. (2001) derive an asset pricing model taking into account agents' overconfidence. They show that in an economy in which agents are risk averse with negative exponential utility, and uncertainty is normally distributed, prices overreact to private signals and true expected returns decompose additively into a risk premium and components arising from mispricing.

In imperfectly competitive markets, Odean (1998, Section III.B) shows that overconfidence can lead to market breakdowns and, that when a market equilibrium exists, expected volume, market depth, price volatility and the level of informational efficiency increase as the insider's overconfidence increases. Kyle and Wang (1997) and Wang (2001, Section II) show that in a market with two informed agents, overconfidence acts as a commitment to trade aggressively. As a consequence, an overconfident informed agent may earn a higher expected utility than a rational one and overconfident agents may dominate the economy, in the long run. Finally, Caballé and Sákovics (2003) differentiate between private self-confidence (the self-confidence of the speculators) and public self-confidence (the self-confidence they attribute to their competitors). They show that public self-confidence and private self-confidence have different effects (sometimes opposite) on trading volume, price volatility, informational efficiency and expected profits.

3. The model

We consider the following 2-state economy. There is one risk-free asset with return normalized to 0, and one risky asset with return v being the realization of a random variable $\tilde{v}$. With probability 1/2, state 1 of the economy is realized and $v = 1$, and with probability 1/2, state 2 of the economy is realized and $v = -1$.

Portfolio management is delegated by a principal to an agent endowed with a mean-variance utility function. Through the exertion of effort, the agent can acquire information about the value of the risky asset. The distribution of $\tilde{v}$ conditional on exerting effort and observing a private signal is described below.

Denote W_i the final wealth of the agent in state i ($i = 1, 2$), and q the probability of obtaining W_2 given his information. Then, the utility of the agent is assumed to be

$$U(W_1, W_2, q) - C(e) = (1-q)W_1 + qW_2 - \gamma q(1-q)(W_2 - W_1)^2 - C(e)$$

where e represents the effort level, $C(.)$ the effort cost function, and $\gamma > 0$ a coefficient of absolute risk aversion.

The principal is also endowed with a mean-variance utility function. If we denote W_i^p the final wealth of the principal in state i, and q the probability of obtaining W_2^p, then the expected utility of the principal is assumed to be

$$U_p(W_1^p, W_2^p, q) = (1-q)W_1^p + qW_2^p - \gamma_p q(1-q)\left(W_2^p - W_1^p\right)^2$$

with $\gamma_p < \gamma$, implying that the principal is less risk averse than the agent.

We also assume that the principal cannot acquire information at any cost. It implies that if the principal does not hire an agent, his expected utility is zero since his optimal portfolio is then one that is fully invested in the risk-free asset. In this environment, the principal faces a trade-off between the utility derived from market returns and the cost of obtaining such returns (i.e., providing the agent with incentive to exert effort and choose the appropriate portfolio).

Finally, as is standard in contract theory, it is assumed that the principal has all the bargaining power and makes a take-it-or-leave-it offer to the agent. If the agent rejects the contract proposed by the principal, his outside option is to trade for his own account.

4. The binary-effort case

There are two possible effort levels, $e = 0$ and $e = 1$. If choosing $e = 1$, the agent incurs a loss of utility $C(1) = c$ and receives a private signal about the risky asset. The observed signal is either $s = 1$ or $s = -1$. Conditional on signals, the distribution of the return of the risky asset is

$$\text{Prob}(v|s = v) = (1+k)/2$$
$$\text{Prob}(v|s \neq v) = (1-k)/2$$

with $k \in (0, 1)$.

We define overconfidence as follows. After receiving a signal s ($s = 1$ or $s = -1$), an overconfident agent believes that

$$\text{Prob}(v|s = v) = (1+K)/2$$
$$\text{Prob}(v|s \neq v) = (1-K)/2$$

with $K \in (k, 1)$. Hence, overconfidence means that the agent miscalibrates the precision of his information, perceiving it as being more precise than it really is.

4.1. Benchmark case: a private investor

As a benchmark, we consider the case of a private investor who first acquires information and then chooses the quantity (x) he trades/invests in the risky asset. We have the following result.

Proposition 1. *Assume that* $c \leqslant \frac{K^2}{4\gamma(1-K^2)}$. *Then, the agent acquires information and trades a quantity* $\hat{x}(K, \gamma, s) = \frac{Ks}{2\gamma(1-K^2)}$.

The proposition states that, for a given observed signal, the larger the level overconfidence, the larger the quantity traded by the agent. This result is similar to those of Odean (1998) and Wang (2001) in the context of perfectly competitive markets: for a given precision of information, overconfidence generates incentives to trade larger quantities than those traded by rational agents.

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4.2. Delegated portfolio management

When portfolio management is delegated, *if* the agent exerts effort, observes a signal s and acts in the interest of the principal, he trades a quantity $\hat{x}(k, \gamma_p, s)$ in the risky asset. In what follows, we will refer to such trading quantities as the first-best quantities.

As already mentioned, if the agent rejects the contract proposed by the principal, his outside option is to trade for his own account. However, it is assumed that in such a case, the information gathered by the agent is of lower precision than that obtained if working for the principal. The idea is that when working for the principal, the agent has access to a better information acquisition technology leading to more accurate predictions. Formally, we assume that a rational agent, working for an institution, who observes a signal s (correctly) believes that $\text{Prob}(v|s = v) = (1 + k)/2$ and $\text{Prob}(v|s \neq v) = (1 - k)/2$. If the same agent trades for his own account, he acquires less precise information and believes that $\text{Prob}(v|s = v) = (1 + \beta k)/2, \beta < 1$. Similarly, an overconfident agent, working for an institution believes that $\text{Prob}(v|s = v) = (1 + K)/2$, and if trading for his own account, believes that $\text{Prob}(v|s = v) = (1 + \beta K)/2$. We deduce from the benchmark case that the reservation utility of the agent is

$$\overline{U}(\beta, K) = \text{Max}\left(\frac{\beta^2 K^2}{4\gamma(1 - \beta^2 K^2)} - c, 0\right)$$

Before proceeding, it should be noted that in the delegation problem we analyze, the moral hazard problem faced by the principal is twofold. He must provide the agent with incentives to (1) exert effort and acquire information, and (2) trade the appropriate quantity. This moral hazard problem related to risk may be different depending on the agent's type (rational or overconfident). If the agent is rational, then for any strictly positive level of risk aversion $\gamma > \gamma_p$, upon observing $s = 1$ $(s = -1)$, he trades a quantity $\hat{x}(k, \gamma, 1) < \hat{x}(k, \gamma_p, 1)$ $(-\hat{x}(k, \gamma, -1) > -\hat{x}(k, \gamma_p, -1))$. Hence, the problem for the principal is to provide the agent with incentives to take more risk than he would otherwise do when maximizing his own expected utility. If the agent is overconfident, the problem may be different, the reason being that for any $K > k, \lim_{\gamma \to \gamma_p} \hat{x}(K, \gamma, 1) > \hat{x}(k, \gamma_p, 1)$. As a consequence, if the difference $\gamma - \gamma_p$ is small enough, then $\hat{x}(K, \gamma, 1) > \hat{x}(k, \gamma_p, 1)$ $(\hat{x}(K, \gamma, -1) < \hat{x}(k, \gamma_p, -1))$. In such a case, overconfidence generates excessive trading incentives. The problem for the principal is therefore to reduce these trading incentives.

Denote $x(s)$ the quantity traded by the agent after having received a signal s $(s = -1, 1)$, and $x(\emptyset)$ the quantity traded if information is not acquired. Given the two-point distribution of the return of the risky asset, if the agent exerts effort, his portfolio return R is $R_1 = sx(s)$ with probability $\frac{1+k}{2}$ and $R_0 = -sx(s)$ with probability $\frac{1-k}{2}$. Therefore, the problem of the principal is to choose a contract $g^*(R)$ that associates a compensation to a realized portfolio return R. Formally, the principal maximizes

$$\frac{1}{2} \sum_{s \in \{-1,1\}} U_p\left[-sx^*(s) - g(-sx^*(s)), sx^*(s) - g(sx^*(s)), \frac{1 + k}{2}\right] \tag{1}$$

subject to

$$x^*(s) \in \text{argmax}_x U\left[g^*(-sx(s)), g^*(sx(s)), \frac{1 + K}{2}\right] \tag{2}$$

$$x^*(\emptyset) \in \text{argmax}_x U\left[g^*(-x), g^*(x), \frac{1}{2}\right] \tag{3}$$

$$\frac{1}{2} \sum_{s \in \{-1,1\}} U\left[g^*(-sx^*(s)), g^*(sx^*(s)), \frac{1 + K}{2}\right] - c \geqslant U\left[g^*(x^*(\emptyset)), g^*(-x^*(\emptyset)), \frac{1}{2}\right] \tag{4}$$

$$\frac{1}{2} \sum_{s \in \{-1,1\}} U\left[g(-sx^*(s)), g(sx^*(s)), \frac{1 + K}{2}\right] - c \geqslant \overline{U}(\beta, K) \tag{5}$$

Eq. (2) and (3) are the incentive compatibility constraints on risk if the agent has acquired information and has not acquired information, respectively. Eq. (4) represents the incentive-compatibility constraint on effort. Finally, Eq. (5) represents the participation constraint.

We derive the following results regarding the contract proposed by the principal and the trading quantities.

Proposition 2. *Assume that* $c < \frac{K(K+1)}{\gamma}$.

1. *If* $K \leqslant 3k$,
   - *The following contract is optimal*

$$g^*\left(R|\alpha_0^*, \alpha_1^*, X^*\right) = \begin{cases} \alpha_0^* - 1 & \text{if} \quad R < -X^* \\ \alpha_0^* & \text{if} \quad R \in [-X^*, X^*) \\ \alpha_0^* + \alpha_1^* & \text{if} \quad R \geqslant X^* \end{cases} \tag{6}$$

with

$$\begin{aligned} \alpha_0^* &= \overline{U}(\beta, K) - \frac{(1+K)\left(-1+\sqrt{1+4c\gamma}\right)}{2K\gamma} + \frac{(1-K^2)\left(-1+\sqrt{1+4c\gamma}\right)^2}{4K^2\gamma} \\ \alpha_1^* &= \frac{\left(-1+\sqrt{1+4c\gamma}\right)}{K\gamma} \\ X^* &= \frac{k}{2\gamma_p(1-k^2)} + \frac{-1+\sqrt{1+4c\gamma}}{2K\gamma} \end{aligned} \tag{7}$$

   - *The quantity traded by the agent is then* $x^*(s, X^*) = sX^*$.

2. *If* $K > 3k$, *then let*

$$\bar{c} = \frac{K(K-3k)}{2\gamma(1-K^2)}\left[1 + \frac{K(K-3k)}{\gamma(1-K^2)}\right]$$

(i) *If* $\bar{c} < \frac{K(K+1)}{\gamma}$ *and* $c \in \left[\bar{c}, \frac{K(K+1)}{\gamma}\right]$ *then the contract* $g^*\left(.|\alpha_0^*, \alpha_1^*, X^*\right)$ *is optimal and the quantity traded by the agent is* $x^*(s, X^*)$.

(ii) *In all other cases,*

- *the contract* $g^*\left(R|\alpha_0', \alpha_1', X'\right)$, *with*

$$\begin{aligned} \alpha_0' &= \overline{U}(\beta, K) - \frac{(K-3k)}{4\gamma(1-K^2)}(2+K+3k) \\ \alpha_1' &= \frac{(K-3k)}{\gamma(1-K^2)} \\ X' &= \frac{k}{2\gamma_p(1-k^2)} + \frac{(K-3k)}{2\gamma(1-K^2)} \end{aligned}$$

is optimal,

- *the quantity traded by the agent is* $x^*(s, X') = sX'$.

The proposition establishes that two equilibrium situations may arise depending of the level of overconfidence, and describes in each case the optimal contracts and trading strategies. In cases 1 and 2(i) (case 2(ii)), α_1^* (α_1') represents the incentive component of the compensation contract, which provides incentives to exert effort, and α_0^* (α_0') is chosen so that the participation constraint is satisfied. Finally, the performance thresholds X^* and $-X^*$ (X' and $-X'$) aim at solving the risk-related agency problem.

When the agent is rational or moderately overconfident (formally, $K/k \leqslant 3$) or when the level of overconfidence is large but the effort cost is not small enough (i.e., $c > \bar{c}$), the optimal contract is such that both the incentive-compatibility constraint on effort and the participation constraint are binding, as is standard in contract theory in the absence of a limited liability constraint. In this case, we observe that the optimal trading quantity is a decreasing function of the overconfidence level. This result contrasts sharply with those obtained in the case of private investors for which overconfidence increases trading quantities. Here, if an agent is moderately overconfident, then, for a given effort level, overconfidence reduces trading incentives. Hence, overconfident agents take less risk than rational agents.

When the level of overconfidence is large and effort cost is small, i.e., case 2(ii), it is possible for the principal to have simultaneously the incentive component of the contract small and the incentive-compatibility constraint satisfied. In this case, the optimal trading quantity is an increasing function of the overconfidence level, as in the case of individual investors.

The reason for the optimality of the trading strategy $x^*(s,X^*) = sX^*$, when performance thresholds $-X^*$ and X^* are chosen, is the following. Consider cases 1 and 2(i)[4] and assume, without loss of generality, that the agent observes the signal $s = 1$. In such a case, if the agent deviates from the optimal trading strategy and trades a quantity $x' > X^*$, he receives $\alpha_0^* + \alpha_1^*$ if the realized value of the risky asset is $v = 1$, as if trading the quantity X^*. However, if the realized value of the risky asset is $v = -1$, he receives $\alpha_0^* - 1$ if trading x', while he receives α_0^* if trading X^*. It is then straightforward that the deviation $x' > X^*$ is not profitable. Assume now that the agent deviates from the equilibrium trading strategy by trading a quantity $x'' < X^*$. In such a case, he receives α_0^* whatever the realization of the value of the risky asset. This deviation yields a lower but certain compensation with respect to the equilibrium one. If $c \leqslant \frac{K(K+1)}{\gamma}$ then the utility derived from obtaining α_0^* with probability $(1-K)/2$ and $\alpha_0^* + \alpha_1^*$ with probability $(1+K)/2$ is larger that the utility derived from obtaining α_0^* with probability 1.

However, if the trading strategy $x'' < X^*$ is preferred to the trading strategy $x^*(s,X) = sX$ when acquiring information (i.e., $c > \frac{K(K+1)}{\gamma}$), then the agent is better off *not* acquiring information since in such a case, he can obtain the same compensation (i.e., α_0^* with probability 1) and save the effort cost. But, anticipating the strategy of the agent in the continuation game, the principal would not offer the contract $g^*(.|\alpha_0^*, \alpha_1^*, X^*)$. It then implies that $x^*(s,X^*) = sX^*$ is the unique optimal trading strategy in an equilibrium in which the contract $g^*(.|\alpha_0^*, \alpha_1^*, X^*)$ is offered.

Finally, note also that in all cases, the optimal trading quantity is larger than the first-best quantity. This is a direct effect of delegation. As a matter of fact, in cases 1 and 2(i), the optimal trading quantity can be decomposed as follows:

$$x^*(s,X^*) = sX^* = \hat{x}(k,\gamma_p,s) + s\frac{-1+\sqrt{1+4c\gamma}}{2k\gamma} - s\left(1-\frac{k}{K}\right)\frac{-1+\sqrt{1+4c\gamma}}{2k\gamma} \tag{8}$$

The first term represents the first-best quantity. The second term represents the impact of delegation to a rational agent. This term is increasing with the effort cost. The reason is that the principal cares about the portfolio return *net* of compensation costs. The larger the effort cost, the more the agent has to be compensated in case of a good performance. For a given trading quantity, this decreases the risk undertaken by the principal. Therefore, in order to reach the optimal risk level, the principal chooses a larger performance threshold (X^*) which provides the agent with incentives to trade larger quantities. In this environment, the principal is always able to reach the first-best risk level: The variance of the return of the principal is equal to the variance of the return of the first-best investment. Finally, the last term of Eq. (8) represents the impact of overconfidence. The larger the overconfidence level the larger this term in absolute value.

It should also be noted that in case 2(ii), the effect of overconfidence cannot be isolated from that of delegation, since this case never applies when the agent is rational.

Another question we would like to answer is whether a rational principal is better off recruiting rational or overconfident agents. The following corollary provides results on the impact of overconfidence on the utility of the principal.

Corollary 3. *Assume that parameters are such that cases 1 or 2(i) hold, and let* $Y = (1+2\gamma c) - \sqrt{1+4\gamma c}$.

[4] The reasoning for the case 2(ii) is identical.

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- *If* $Y < \frac{\beta^2}{2(1-\beta^2)^2}$ *then* U_p *is an increasing function of K on* $\left[k, \frac{\sqrt{2Y}}{\beta(\beta\sqrt{2Y}+1)}\right]$ *and a decreasing function of K on* $\left[\frac{\sqrt{2Y}}{\beta(\beta\sqrt{2Y}+1)}, 1\right]$.
- *If* $Y > \frac{\beta^2}{2(1-\beta^2)^2}$ *then* U_p *is an increasing function of K.*

When deciding whether to hire a rational or an overconfident agent, the principal has to take two effects into account. First, overconfident agents are more expensive because of their higher outside option. Second, incentives necessary for effort exertion are lower for overconfident agents than for rational agents, due to the overestimation of the probability of success by overconfident agents. If β is small, the effect of overconfidence on the outside option is small. As a consequence, the incentive effect dominates the outside-option effect, hence hiring an overconfident agent provides more utility to the principal. The situation is different when β is large. In such a case, the difference $K - k$ generates a large difference in reservation values between overconfident and rational agents. Thus, the corollary establishes that the incentive effect dominates for moderates level of overconfidence (i.e., $K - k$ small), but the outside-option effect dominates for large levels of overconfidence. As a consequence, when the level of overconfidence is small, the utility of the principal is increasing with the overconfidence level, but when this level gets large, the expected utility of the principal is decreasing in the level of overconfidence.

5. The continuous effort case

In the previous section, it has been assumed that the agent faces a binary effort choice. As mentioned in the introduction, a direct implication is that if information is acquired, then overconfident and rational agents have information of the same precision, though they differ in the perception of its precision. This difference also means that rational and overconfident agents differ in their perception of the productivity of effort (the precision of information acquired per unit cost of effort). As a consequence, if several levels of information were available, it could be the case that overconfident and rational agents choose different effort levels in equilibrium.

To investigate this possibility, we consider an environment in which the effort level is continuous. As in the binary-effort case, we assume that, upon exerting effort, the agent receives a signal $s = 1$ or $s = -1$, and $\text{Prob}(v|s = v) = \frac{1+k}{2}$ implying that $\text{Prob}(v|s \neq v) = \frac{1-k}{2}$. In what follows, we will refer to k as a measure of the precision of the signal received by the agent. The effort cost associated to a signal of precision k is a twice continuously differentiable increasing and convex function $C(k)$. Furthermore, we assume that there exists $\bar{k} < 1$ such that $\lim_{k\to\bar{k}} C'(k) = +\infty$.

This last property of the cost function implies that the agent never acquires perfect information. This is necessary for an interior solution. If the agent can acquire perfect information, even at an infinite cost, the principal will set $\alpha_1 = +\infty$ when offering a contract $g(.|\alpha_0, \alpha_1, X)$. This is profitable for the principal since, in absence of trading restrictions, he can set $X > \alpha_0 + \alpha_1$, hence obtaining a positive profit with probability one, despite an infinite effort cost for the agent. In order to discard this case, there are two possibilities. The first one is to make trading increasingly costly. The second one, chosen here, is to take a cost function such that the marginal cost goes to infinity even for a signal precision strictly smaller than 1.

Overconfidence is now modeled as follows. Upon spending an effort cost c^o, the agent believes that the precision of his signal is $\psi C^{-1}(c^o)$ with $\psi \in (1, 1/\bar{k})$. That is, for a given effort cost, the agent overestimates the precision of the signal he receives. Hence, ψ can be interpreted as a measure of overconfidence. Finally, the condition $\psi < 1/\bar{k}$ ensures that the perceived probability of having $v = s$ if a signal $s \in \{-1, 1\}$ is observed is strictly smaller than one.

The maximization problem of the principal is modified from the binary-effort case as follows. The incentive-compatibility constraint (4) has to be replaced by the following condition

$$k^* \in \text{argmax}_k U\left[g^*(sx^*(s)), g^*(-sx^*(s)), \frac{1+\psi k}{2}\right] - C(k)$$

As in the previous section, we focus on equilibrium contracts of the shape of the optimal contract derived in Proposition 2, i.e., $g^*(.|\alpha_0, \alpha_1, X)$. Before proceeding, note that the result about $x^*(s, X) = sX$ being the unique optimal trading strategy in an equilibrium in which the principal propose a contract $g^*(.|\alpha_0, \alpha_1, X)$ is still valid in the continuous effort case, since the proof of the optimality of $x^*(s, X)$ is independent of the number of strictly positive effort levels the agent can exert.

We then start by deriving results about the optimal effort level exerted by the agent, for a given contract proposed by the principal.

Lemma 4. *Assume that the principal offers a contract $g^*(.|a_0, a_1, X)$ and denote $\mathscr{K}$ the set of signal precisions that maximize the expected utility of the agent, i.e.,*

$$\mathscr{K} = \text{argmax}_k \left(U\left(a_0, a_1, \frac{1 + \psi k}{2}\right) - C(k) \right)$$

If there exists $\hat{k} \in \mathscr{K}$ such that $U\left(a_0, a_1, \frac{1+\psi\hat{k}}{2}\right) - C(\hat{k}) \geqslant a_0$, then $k^* = \hat{k}$ is an optimal signal precision for the agent. In all other cases, the agent does not acquire information, i.e., $k^* = 0$.

The lemma derives a condition that the contract parameters must satisfy for the agent to exert effort and trade $x^*(s, X)$ in the continuation game. If the condition is not satisfied, then the agent does not acquire information.

A direct consequence of this lemma is that, for given cost function and risk-aversion parameters, it could be the case that for some overconfidence levels an equilibrium with effort acquisition exists while for other levels of overconfidence, the principal does not offer a contract (since the agent would not exert effort in the continuation game).

In the context of our model, in order to study the impact of overconfidence on trading quantities, it is then necessary to rely on sets of parameters (risk-aversion parameters and effort cost function) for which comparative static can be performed. In other words, sets of parameters for which there exists an interval I_ψ such that for any $\psi \in I_\psi$, an equilibrium in which a contract of the shape of $g^*(.|a_0, a_1, X)$ is offered and the agent exerts effort exists. Therefore, in the rest of the section, we will proceed as follows. In a first step, we will assume that risk aversion coefficients and effort cost functions such that comparative static can be performed exist, and derive the impact of overconfidence on the contract proposed by the agent, the precision of the acquired information, and the quantities traded. Then, in a second step, we will provide an example of a family of cost functions such that the interval I_ψ does exist.

We have the following results regarding the impact of the contract parameters and overconfidence level on the precision of the signal acquired by the agent.

Proposition 5. *Consider intervals I_{a_1}, I_{a_0}, and I_ψ such that for any $a_1 \in I_{a_1}, a_0 \in I_{a_0}$ and $\psi \in I_\psi$ if the principal proposes a contract $g^*(.|a_1, a_0, X)$, the optimal effort level k^* is strictly positive. Then*

1. *k^* is an increasing function of the incentive component of the compensation contract (a_1).*
2. *k^* is an increasing function of the overconfidence level (ψ).*
3. *k^* is independent of a_0.*

The proposition states standard results from contract theory, in the case of our model. The first part of the proposition says that the larger the difference in compensation between a good and a bad performance, the larger the amount of effort exerted by an agent. The second part of the proposition states that the larger the (perceived) marginal productivity of effort, the larger the amount of effort exerted by an agent. Finally, the last part of the proposition says that the compensation in case of bad performances has no effect on the level of effort exerted by the agent.

Given that k^* is increasing in a_1 and ψ, and is independent of a_0, we will denote the optimal effort level as $k^*(a_1, \psi)$ in what follows.

Having determined how the agent responds to a given contract, we can now turn to the problem of the principal (i.e., the determination of the contract parameters a_0, a_1, and X), and study how overconfidence influences these parameters, hence the precision of the information acquired by the agent and the trading volume.

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Proposition 6. *Consider a cost function $C(.)$ for which there exist $\bar{\psi}$ and $\bar{\gamma}$ such that if $\psi \in [1, \bar{\psi}]$, and $0 \leqslant \gamma_p < \gamma \leqslant \bar{\gamma}$, then the principal optimally offers a contract $g^*(.|a_0^*, a_1^*, X^{**})$ and the agent chooses a signal of precision $k^*(a_1, \psi) > 0$. Then,*

1. *The performance threshold is*

$$X^{**} = \frac{a_1^*}{2} + \frac{k^*(a_1^*, \psi)}{2\gamma_p\left(1 - k^{*2}(a_1^*, \psi)\right)} \tag{9}$$

2. *There exists $\bar{\bar{\gamma}} \in (0, \bar{\gamma})$ such that if $\gamma < \bar{\bar{\gamma}}$, then the optimal incentive component a_1^* is an increasing function of the overconfidence level.*
3. *As a consequence, if $\gamma < \bar{\bar{\gamma}}$, then the quantity traded by the agent is an increasing function of the overconfidence level.*

The first part of the proposition derives the optimal performance threshold. We observe that it is increasing in k^*. Therefore, for a given incentive parameter a_1, the more overconfident the agent, the higher the precision of the acquired signal, and the larger the equilibrium trading quantity (i.e., $x^*(s, X^{**})$).

The second part of the proposition establishes that when risk-aversion is not too large, then the incentive component of the compensation contract is increasing with the level of overconfidence. The intuition for this result is the following. First, the return taken into account by the principal when designing a compensation contract is the portfolio return *net* of compensation expenses (hereafter, the net return). Given the expression of X^{**}, we derive that the expected utility of the principal as a function of the compensation levels in case of bad performances (a_0^*) and in case of good performances ($a_0^* + a_1^*$) is

$$U_p = -a_0^* - \frac{a_1^*}{2} + \frac{k^{*2}(a_1, \psi)}{4\gamma(1 - k^{*2}(a_1, \psi))}$$

The principal has to take two effects into account when choosing the incentive parameter a_1^*. First, the marginal impact of a_1 on the signal precision k^* is increasing in ψ. This is due to the fact that overconfidence implies that the agent over-estimates his marginal productivity of effort. The consequence is that the more overconfident the agent, the larger the incentives for the principal to set a_1^* high.

Second, as a_1^* increases, the variance of the compensation of the agent increases. It implies that the principal must also increase the compensation in case of a bad outcome (a_0^*) for the participation constraint of the agent to be satisfied. However, the lower the level of risk aversion of the agent, the less a_0^* has to increase as a response to an increase of a_1^*. The proposition then states that a low level of risk aversion ensures that the positive effect of an increase of a_1 on effort dominates the negative effect of an increase of a_1 on the risk borne by the agent. It follows that the larger the overconfidence level of the agent, the larger a_1^*, hence the larger the effort exerted by the agent and the larger the equilibrium trading quantity.

Then, comparing these results with those of the previous section, we deduce that in the case of delegated portfolio management, if overconfidence generates excessive trading and risk-taking, it is only through over-acquisition of information. The main reason for this difference in results between the continuous and binary-effort cases, is that in the latter one, the principal does not have to take into account the marginal impact of the incentive component of the compensation contract (a_1) on the effort level. The principal then focuses on the impact of a_1 on the risk borne by the agent. By choosing a_1 smaller for an overconfident agent than for a rational one, the principal can reduce the risk borne by the overconfident agent, while keeping the incentive-compatibility constraint on effort satisfied. It implies that the compensation of an overconfident agent in the case of a bad performance (a_0) can also be decreased. Now, the variance of the net return being $\frac{(1-k^2)}{4}(2X - a_1)^2$, the consequence of a decrease in a_1 is an increase of the variance of the net return. In order to decrease this risk, the principal sets the threshold X at a lower level for an overconfident agent than for a rational one, implying that overconfident agents trade smaller quantities than rational ones in the binary-effort case.

Finally, we need to show that there exists a cost function $C(.)$ such that the type of equilibria described in Proposition 6 is obtained. We provide an example of such a cost function. Let

$$C(k) = -c\text{Log}(\bar{k} - k)$$

with $\bar{k} < 1$. This cost function is increasing and convex and goes to infinity as k tends to $\bar{k}$, hence ensuring that perfect information is never acquired.

For such a cost function, a closed form solution is obtained for the effort level:

$$k^*(a_1, \psi) = \frac{\gamma\psi\bar{k}a_1 - 1 + \sqrt{(1 + \gamma\psi\bar{k}a_1)^2 - 8c\gamma}}{2\gamma\psi a_1} \tag{10}$$

Then, one checks that it is always the case that $U\left(a_0, a_1, \frac{1+\psi k^*}{2}\right) > U\left(a_0, a_1, \frac{1}{2}\right)$, hence ensuring that if a signal of precision $k^*(a_1, \psi)$ is acquired, then the agent trades $x^*(s, X^{**})$ in the continuation game. Hence, $k^*(a_1, \psi)$ is an optimal signal precision for the agent.

This implies that if the effort cost function of the agent is $C(k) = -c\text{Log}(\bar{k} - k)$, then for any $\psi \geqslant 1$, there exists an equilibrium in which the principal offers a contract $g^*(.|a_0, a_1, X^{**})$, and the agent chooses a strictly positive effort level.

6. Discussion

Overconfidence is often the result of a self-attribution bias which leads agents to falsely attribute a good performance to skills rather than luck, and a bad performance to bad luck rather than skill. Under such circumstances, good performances generate overconfidence (see, for example, Gervais and Odean (2001)). Our results are then consistent with several pieces of empirical evidence.

6.1. Portfolio rebalancing

Two recent studies by Pütz and Ruenzi (2008) and Choi and Lou (2008), measuring trading volume by the turnover ratio, provide evidence that fund managers trade more after a good performance, hence suggesting that fund managers suffer from a self-attribution bias causing overconfidence.[5]

The results of the previous section are consistent with these pieces of evidence since, in our model, the larger the overconfidence level, the larger the trading volume, even when compensation contracts are taken into account. Everything else constant, as overconfidence increases, the precision of the information acquired by the agent increases, hence the trading volume increases (see Eq. (9)). Furthermore, the incentive component of the compensation contract also increases, providing further incentive to acquire information and trade.

6.2. Momentum investment strategies

Daniel et al. (1998, Section III) develop a model in which, if informed agents suffer from a self-attribution bias then they cause a positive autocorrelation in prices in the short run. Hence, after receiving a good (bad) signal, hence buying (selling) and pushing the price upward (downward), informed agents keep buying upon arrival of public information, on average. It follows that agents suffering from a self-attribution bias generating overconfidence follow momentum investment strategies.

In a related study on the investment behavior of different categories of investors, Grinblatt and Keloharju (2000) provide evidence that investors following momentum strategies are the most sophisticated ones, foreign institutional investors in their sample.

In our model, institutions acquire more information than individual investors.[6] As a consequence, they realize better performances than individual investors. Hence, if market participants suffer from a

[5] Choi and Lou (2008) use a second measure of trading volume, namely the Active Share, a measure developed by Cremers and Petajisto (2007), which is calculated as one half of the sum of absolute deviations in portfolio weights of an active portfolio from its benchmark index.

[6] This is also true if we assume that individual investors and institutions have access to the same information processing technology (i.e., $\beta = 1$) and have the same level of absolute risk aversion (i.e., $\gamma = \gamma_p$).

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self-attribution bias, portfolio managers investing on behalf of institutions are the participants who suffer from the strongest confidence reinforcement. As a consequence, in the model of Daniel et al. (1998, Section III), they are those who follow momentum strategies.[7] In this respect, our results are consistent with the empirical findings of Grinblatt and Keloharju (2000).

6.3. Evolution of mutual funds' management fees

The results of the previous section show that compensation increases with the level of overconfidence. Therefore if overconfidence is generated by self-attribution bias, our results suggest that mutual fund management fees increase after good past performances, as overconfidence increases, while they do not decrease after bad performances.

This is consistent with empirical evidence provided by Warner and Wu (2005) who study the determinants of changes in mutual fund advisory contracts. They show that good past performances increase the likelihood of an increase in management fees while bad past performances have no impact on the likelihood of a decrease in management fees.

7. Conclusion

The impact of overconfidence on investment strategies and asset prices has received a lot of attention in the last decade. However, until recently, attention has concentrated on individual investors' behavior, while one of the main categories of financial market participants are institutions, which are characterized by the delegation of investment decisions to professional money managers. This delegation of tasks requires the design of a compensation contract which influences managers' investment decisions.

In this article, we have shown that results on overconfidence obtained in the case of private investors (i.e., excessive trading volume and risk-taking incentives) may not hold in the case of institutional trading. The reason is that institutions can propose contracts that mitigate risk-taking incentives generated by overconfidence. More precisely, we show that in equilibria in which rational and overconfident agents acquire information of the same precision, overconfident agents trade *lower* quantities than rational agents.

However, if we consider the precision of the information to be acquired as a continuous variable, we have shown that overconfident agents are proposed compensation contracts with a larger incentive component. As a consequence, acquire more precise information, trade larger quantities, and take more risk than rational agents do.

Based on psychological evidence that overconfidence is generated by a self-attribution bias, our results are then consistent with recent empirical evidence about mutual fund managers' investment strategies, namely an increase in portfolio rebalancing after good performances and the use of momentum strategies. Our results are also consistent with evidence that good past performances have a positive impact on the likelihood of increasing management fees.

Appendix

Proof of Proposition 1. If exerting effort and observing a signal s, the agent maximizes the following function over the trading quantity x

$$U\left(-x, x, \frac{1+K}{2}\right) = -\frac{1-K}{2}xs + \frac{1+K}{2}xs - \gamma(1-K^2)x^2$$

The first-order condition (sufficient here) yields $x = \frac{Ks}{2\gamma(1-K^2)}$.

Given that for all $s, s^2 = 1$, It follows that the utility obtained by the agent is $\frac{K^2}{4\gamma(1-K^2)}$. Hence, if $c \leqslant \frac{K^2}{4\gamma(1-K^2)}$, the agent exerts effort and acquires information. □

[7] Even if individual investors are also overconfident, but to a lower extent than professional money managers, they will trade against institutions, hence follow contrarian investment strategies.

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Proof of Proposition 2. Given that $P(v=1|s=1)=P(v=-1|s=-1)$, and the two-point distribution of $\tilde{v}$, it is straightforward that if x^* is an optimal trading quantity when observing $s=1$, then $-x^*$ is an optimal trading quantity when observing $s=-1$. Therefore, we restrict our attention to the case $s=1$, the proof for the case $s=-1$ being identical.

Assume that the principal wants the agent to trade a quantity x^o upon observing $s=1$. Given the distribution of $\tilde{v}$, the principal knows that if the agent trades a quantity x^o, the possible returns are $R=-x^o$ and $R=+x^o$. Denote $q>1/2$ the probability of observing the return $R=x^o$ when trading a quantity x^o, given the beliefs of the agent.

Then, a contract of the shape

$$g(R|\alpha_0,\alpha_1,x^o)=\begin{cases}\alpha_0-1 & \text{if} \quad R<-x^o\\ \alpha_0 & \text{if} \quad R\in[-x^o,x^o)\\ \alpha_0+\alpha_1 & \text{if} \quad R\geqslant x^o\end{cases} \tag{11}$$

with $\alpha_1\leqslant\frac{1}{\gamma(1-q)}$ ensures that the agent trades a quantity x^o.

The reason for this result is the following. If the agent deviates from the equilibrium trading strategy and trades a quantity $x'<x^o$, he receives α_0 whatever the realization of the value of the risky asset. Conversely, if he trades a quantity x'' larger than x^o, he receives $\alpha_0+\alpha_1$ with probability q, and α_0-1 with probability $1-q$. Now, if trading the quantity x^o, the agent receive α_0 with probability $1-q$ and $\alpha_0+\alpha_1$ with probability q. Hence, trading the quantity $x''>x^o$ provides a lower expected compensation and a larger variance of the compensation than trading x^o. Therefore, trading x^o is always preferred to trading $x''>x^o$. The situation is different when comparing the strategies x^o and $x'<x^o$. If trading x', the agent receives α_0 with probability 1, hence faces no risk. Trading x^o is then preferred to trading $x'<x^o$ if

$$\alpha_0\leqslant\alpha_0+q\alpha_1-\gamma q(1-q)\alpha_1^2 \tag{12}$$

which is equivalent to $\alpha_1\leqslant\frac{1}{\gamma(1-q)}$.

Now, assume that the agent does not acquire information. Then, given the assumptions of the model, the probability of observing the return $R=x^o$ when trading a quantity x^o is equal to 1/2. Before proceeding, note that the right-hand side of inequality (12) is increasing in q since $q>1/2$. It implies that if the agent trades $x'<x^o$ when acquiring information, he also trades a quantity smaller than x^o if not acquiring information.

Given that the expected utility obtained when exerting *no* effort and trading a quantity smaller than x^o is α_0, and the expected utility obtained when exerting effort and trading a quantity smaller than x^o is α_0-c, we derive that either the agent acquires information and trades x^o or does not exert effort and trades a quantity $x'<x^o$. More precisely, if $q\alpha_1-\gamma q(1-q)\alpha_1^2\geqslant c$, the agent acquires information and trades the quantity x^o $(-x^o)$, when observing $s=1$ $(s=-1)$. Otherwise, he does not acquire information and trades a quantity $x'\in(-x^o,x^o)$.

In the rest of the proof, we restrict our attention to contracts of the shape of $g(.|\alpha_0,\alpha_1,X)$. The problem of the principal is then to maximize $U_p\left(-X-\alpha_0,X-\alpha_0-\alpha_1,\frac{1+k}{2}\right)$, subject to the IC constraint on effort and the participation constraint.

Denote λ and μ, the Lagrangian multipliers associated to the participation constraint and the IC constraint, respectively. The principal maximizes

$$U_p\left(X-\alpha_0,X-\alpha_0-\alpha_1,\frac{1+k}{2}\right)+\lambda\left[U(\alpha_0,\alpha_1,\frac{1+K}{2})-c-\overline{U}(\beta,K)\right] \\ +\mu\left[U(\alpha_0,\alpha_1,\frac{1+K}{2})-c-U\left(\alpha_0,\alpha_1,\frac{1}{2}\right)\right] \tag{13}$$

The first-order condition of the maximization problem faced by the principal are

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$$\frac{\partial L}{\partial \alpha_0} = -1 + \lambda = 0 \tag{14}$$

$$\frac{\partial L}{\partial \alpha_1} = -\frac{1+k}{2} - \gamma_p(2x - \alpha_1)(1-k^2) + \lambda\left(\frac{1+K}{2} - \frac{\gamma\alpha_1(1-K^2)}{2}\right) + \mu\frac{K}{2}\left(1 + \gamma\frac{\alpha_1 K}{2}\right) = 0 \tag{15}$$

$$\frac{\partial L}{\partial X} = k - \gamma_p(2X - \alpha_1)(1-k^2) = 0 \tag{16}$$

It is straightforward that $\lambda > 0$ so that the participation constraint is binding. Therefore, two possibilities have to be considered. Either $\mu > 0$ so that the IC constraint of effort is binding, or $\mu = 0$.

First, assume that the IC constraint is binding. In such a case we obtain the solution

$$\alpha_0^* = \overline{U}(\beta, K) - \frac{1+K}{2\gamma K}\left(-1 + \sqrt{1+4\gamma c}\right) + \frac{(1-K^2)}{4\gamma K^2}$$

$$\alpha_1^* = \frac{1}{K\gamma}\left(-1 + \sqrt{1+4\gamma c}\right)$$

and X^* is obtained from the first-order condition (16) and α_1^*. Finally, one checks that, given the expression of $\alpha_1^*, c \leqslant \frac{K(K+1)}{\gamma}$ is equivalent to

$$\alpha_0^* \leqslant \alpha_0^* + \frac{1+K}{2} - \gamma\frac{1-K^2}{4}\alpha_1^{*2} - c$$

This ensures that if the contract $g^*(.|\alpha_0^*, \alpha_1^*, X^*)$ is offered, effort is exerted. Hence the optimal trading strategy of the agent is $x^*(s, X^*) = sX^*$.

Assume now that $\mu = 0$. From Eq. (14), we deduce that $\lambda = 1$. Consequently, Eqs. (15) and (16) imply that

$$\alpha_1' = \frac{K - 3k}{\gamma(1-K^2)}$$

and

$$X' = \frac{\alpha_1}{2} + \frac{k}{2\gamma_p(1-k^2)}$$

It directly follows that this solution is not possible if $K < 3k$. However, if $K > 3k$, it must also be checked that the IC constraint on effort is satisfied. This is equivalent to

$$\frac{K(K-3k)}{2\gamma(1-K^2)}\left[1 + \frac{K(K-3k)}{\gamma(1-K^2)}\right] \geqslant c$$

The left-hand side of this last inequality is equal to $\bar{c}$.

Hence if $\bar{c} < \frac{K(K+1)}{\gamma}$ and $c \in \left[\bar{c}, < \frac{K(K+1)}{\gamma}\right]$ then the contract $g^*(.|\alpha_0^*, \alpha_1^*, X^*)$ is optimal and the quantity traded by the agent is $x^*(s, X^*)$.

Conversely, in all the other cases $g^*(.|\alpha_0', \alpha_1', X')$ is an optimal contract. Furthermore, one checks that it is always the case that

$$\alpha_0' \leqslant \alpha' 0 + \frac{1+K}{2} - \gamma\frac{1-K^2}{4}\alpha_1'^2$$

hence ensuring that if the contract $g^*(.|\alpha_0', \alpha_1', X')$ is offered, then the optimal trading strategy of the agent is $x^*(s, X') = sX'$.

Consequently, if $c > \bar{c}$ this last inequality does not hold and there is no solution such that the IC constraint on effort is not binding. □

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Proof of Corollary 3. From the quantity traded by an informed agent and the contract $g^*(.|\alpha_0^*, \alpha_1^*, X^*)$, we deduce that the expected utility of the principal:

$$U_p = \frac{-Y + 2c\gamma K^2}{2K^2\gamma} + \frac{k^2\gamma(1-\beta^2K^2) - \beta^2K^2(1-k^2)\gamma_p}{4\gamma\gamma_p(1-k^2)(1-\beta^2K^2)}$$

Differentiating U_p with respect to K, we obtain

$$\frac{\partial U_p}{\partial K} = \frac{Y}{2\gamma K^3} - \frac{\beta^2 K}{2\gamma(1-\beta^2K^2)^2}$$

The first-order condition $\frac{\partial U_p}{\partial K} = 0$ has two positive solutions, the smaller one being

$$K^* = \sqrt{\frac{\sqrt{2Y}}{\beta\left(\beta\sqrt{2Y}+1\right)}}$$

Given that $\frac{\partial U_p}{\partial K}$ is positive at $K = 0$, the smaller solution represents a maximum. The rest of the proof directly follows. □

Proof of Lemma 4. If a contract $g^*(.|a_0, a_1, X)$ is offered by the principal, the agent chooses an signal precision so as to maximize $U\left(a_0, a_1, \frac{1+\psi k}{2}\right) - C(k)$. Hence, the effort level chosen by the agent belongs to

$$\mathscr{K} = \text{argmax}_k\left(U(a_0, a_1, \frac{1+\psi k}{2}) - C(k)\right)$$

Let $\hat{k}$ be an element of $\mathscr{K}$. The agent trades the quantity $x^*(s, X)$ in the continuation game if and only if

$$U\left(a_0, a_1, \frac{1+\psi\hat{k}}{2}\right) \geqslant U\left(a_0, a_1, \frac{1}{2}\right)$$

If this inequality is not satisfied, then by the same argument as that developed in the proof of Proposition 2, we deduce that the agent trades a quantity smaller (larger) than $x^*(1, X)$ ($x^*(-1, X)$) when observing a signal $s = 1$ ($s = -1$). Then, again, from the proof of Proposition 2, we know that in such a case the agent is better off not acquiring information. □

Proof of Proposition 5. Given a contract $g^*(.|a_0, a_1, X)$, the expected utility of the agent is

$$U = a_0 + \frac{1+\psi k}{2}a_1 - \gamma\frac{1-\psi^2k^2}{4}a_1^2 - C(k)$$

The first-order condition of the maximization problem of the agent is then

$$\frac{\partial U}{\partial k} = \frac{\psi}{2}a_1 + \frac{\gamma\psi^2 k}{2}a_1^2 - C'(k) = 0$$

Denote $F(k, a_1, \psi)$ the left-hand side of this last equation as a function of k, a_1, and ψ. Then the optimal effort level k^* satisfies $F(k^*, a_1, \psi) = 0$ with $\frac{\partial F}{\partial k}|_{k=k^*} < 0$ (i.e., the second-order condition of the maximization problem is satisfied).

From the implicit function theorem, we deduce that

$$\frac{dk^*}{da_1} = -\frac{\partial F/\partial a_1}{\partial F/\partial k}$$

Given that $\partial F/\partial dk < 0$, we deduce that $dk^*/da_1 > 0$ is equivalent to $\partial F/\partial a_1 > 0$. Simple differentiation yields

$$\frac{\partial F}{\partial a_1} = \frac{\psi}{2} + \gamma a_1 \psi^2 k$$

we have the desired result. Proceeding similarly, given that

$$\frac{\partial F}{\partial \psi} = \frac{a_1}{2} + \gamma a_1^2 \psi k$$

We obtain that $dk^*/d\psi > 0$.

Finally, given that the expression of $\frac{\partial U}{\partial k}$ is independent of a_0, it is straightforward that k^* is independent of a_0. □

Proof of Proposition 6. Assume that the principal proposes a contract $g^*(.|a_0, a_1, X)$. We know from Lemma 4 and Proposition 5 that the precision of the information acquired by the agent is $k^*(a_1, \psi)$. Furthermore, if $k^*(a_1, \psi) > 0$, the optimal trading strategy of the agent is $x^*(s, X)$.

The problem of the principal is then to choose the parameters a_0, a_1, and X so as to maximize

$$U_p = -a_0 - X + \frac{1 + k^*(a_1, \psi)}{2}(2X - a_1) - \gamma_p \frac{1 - k^{*2}(a_1, \psi)}{4}(2X - a_1)^2$$

Now, denote $k^o(\beta)$ the precision of the information acquired by the agent if trading for his own account. His reservation utility is then $\overline{U}(\beta, k^o(\beta))$ and if his participation constraint is binding, we have

$$\overline{U}(\beta, k^o(\beta)) = a_0 + \frac{1 + \psi k^*(a_1, \psi)}{2} a_1 - \gamma \frac{1 - \psi^2 k^{*2}(a_1, \psi)}{4} a_1^2 - C(k)$$

It implies that a_0 must satisfy

$$a_0 = \overline{U}(\beta, k^o(\beta)) - \frac{1 + \psi k^*(a_1, \psi)}{2} a_1 + \gamma \frac{1 - \psi^2 k^{*2}(a_1, \psi)}{4} a_1^2 - C(k) \quad (17)$$

Substituting a_0 by the expression derived in Eq. (17) and rearranging, the expected utility of the principal can be rewritten as

$$U_p = -\overline{U}(\beta, k^o(\beta)) + \frac{k^*(a_1, \psi)}{2}(\psi - 1)a_1 + Xk^*(a_1, \psi) - \gamma \frac{1 - \psi^2 k^{*2}(a_1, \psi)}{4} a_1^2 - C(k^*(a_1, \psi)) - \gamma_p \frac{1 - k^{*2}(a_1, \psi)}{4}(2X - a_1)^2$$

The principal then chooses a_1 and X so as to maximize U_p. The first-order conditions of this maximization problem are

$$\frac{\partial U_p}{\partial X} = k^*(a_1, \psi) - \gamma_p [1 - k^{*2}(a_1, \psi)](2X - a_1) = 0 \quad (18)$$

and

$$\frac{\partial U_p}{\partial a_1} = \frac{\psi - 1}{2}\left(k^*(a_1, \psi) + a_1 \frac{\partial k^*}{\partial a_1}\right) + X\frac{\partial k^*}{\partial a_1} - \frac{\gamma}{2}\left(a_1(1 - \psi^2 k^{*2}(a_1, \psi)) - a_1^2 \psi^2 k^*(a_1, \psi)\frac{\partial k^*}{\partial a_1}\right) + \frac{\gamma_p}{2}(2X - a_1)\left((2X - a_1)k^*(a_1, \psi)\frac{\partial k^*}{\partial a_1} + (1 - k^{*2}(a_1, \psi))\right) - C'(k^*(a_1, \psi))\frac{\partial k^*}{\partial a_1} = 0 \quad (19)$$

From Eq. (18), we deduce that

$$X^{**} = \frac{a_1}{2} + \frac{k^*(a_1, \psi)}{2\gamma_p [1 - k^{*2}(a_1, \psi)]} \quad (20)$$

Replacing X by its optimal value X^{**} in Eq. (19), we obtain that the first-order condition $\frac{\partial U_p}{\partial a_1} = 0$ can be rewritten as

$$\frac{\psi-1}{2}\left(k^*(a_1,\psi)+a_1\frac{\partial k^*}{\partial a_1}\right)+\left(\frac{a_1}{2}+\frac{k^*(a_1,\psi)}{2\gamma_p[1-k^{*2}(a_1,\psi)]}\right)\frac{\partial k^*}{\partial a_1}$$
$$-\frac{\gamma}{2}\left(a_1(1-\psi^2k^{*2}(a_1\psi))-a_1^2\psi^2k^*(a_1,\psi)\frac{\partial k^*}{\partial a_1}\right)$$
$$+\frac{k^*(a_1,\psi)}{2[1-k^{*2}(a_1,\psi)]}\left(\frac{k^{*2}(a_1,\psi)}{\gamma_p(1-k^{*2}(a_1,\psi))}\frac{\partial k^*}{\partial a_1}+(1-k^{*2}(a_1,\psi))\right)-C'(k^*(a_1,\psi))\frac{\partial k^*}{\partial a_1}=0 \tag{21}$$

Denote $G(a_1,\psi)$ the left-hand side of this last equation as a function of a_1 and ψ. Then, from the implicit function theorem, we obtain that

$$\frac{da_1^*}{d\psi}=-\frac{\partial G/\partial\psi}{\partial G/\partial a_1}$$

The second-order condition of the principals maximization problem implies that $\frac{\partial G}{\partial da_1}<0$ at $a_1=a_1^*$ and $X=X^{**}$. As a consequence, $da_1^*/d\psi>0$ is equivalent to $\partial G/\partial\psi>0$.

Computing $\frac{dX^{**}}{d\psi}$, we obtain

$$\frac{dX^{**}}{d\psi}=\frac{1+k^{*2}}{\gamma_p(1-k^{*2})^2}\frac{\partial k^*}{\partial\psi}>0$$

It implies that $\frac{dX^{**}}{d\psi}$ tends to infinity as γ goes to zero. Then, given the expression of $G(a_1,\psi)$, it is straightforward that there exists $\bar{\bar{\gamma}}>0$ such that if $\gamma<\gamma_p<\bar{\bar{\gamma}}$ then $\partial G/\partial\psi>0$. Hence, for such risk-aversion parameters, $\frac{da_1^*}{d\psi}>0$. As a consequence, the quantity traded by the agent is an increasing function of the overconfidence level. □

References

Admati, A., Pfleiderer, P., 1996. Does it all add up? Benchmark and the compensation of active portfolio managers. J. Bus. 70, 323–350.
Barber, B., Odean, T., 2002. Online investors: do the slow die first? Rev. Finan. Stud. 15, 455–487.
Benos, A., 1998. Aggressiveness and survival of overconfident traders. J. Finan. Markets 1, 353–383.
Bhattacharya, S., Pfleiderer, P., 1985. Delegated portfolio management. J. Econ. Theory 36, 1–25.
Caballé, J., Sákovics, J., 2003. Speculating against an overconfident market. J. Finan. Markets 6, 199–225.
Choi, D., Lou, D., 2008. A test of self-serving attribution bias, evidence from mutual funds. Yale School of Management (unpublished manuscript).
Cohen, R., 1999. Asset allocation decisions of institutions and individual. Harvard Business School (unpublished manuscript).
Cohen, R., Gompers, P., Vuolteenaho, T., 2002. Who underreact to cash flow news? Evidence from trading between individuals and institutions. J. Finan. Econ. 66, 409–462.
Cohen, S., Starks, L., 1988. Estimation risk and incentive contracts for portfolio managers. Manage. Sci. 34, 1067–1079.
Cremers, M., Petajisto, A., 2007. How active is your fund manager? A new measure that predicts performance. Yale University (unpublished manuscript).
Daniel, K., Hirshleifer, D., Subrahmanyam, A., 1998. A theory of overconfidence, self-attribution, and security market under- and over-reaction. J. Finance 53, 1839–1885.
Daniel, K., Hirshleifer, D., Subrahmanyam, A., 2001. Overconfidence, arbitrage, and equilibrium asset pricing. J. Finance 53, 921–965.
Diamond, P., 1998. Managerial incentives: on the near linearity of optimal compensation. J. Polit. Economy 106, 931–957.
Ekholm, A., Pasternack, D., 2008. Overconfidence and investor size. Europ. Finan. Manage. 14, 82–98.
Gervais, S., Odean, T., 2001. Learning to be overconfident. Rev. Finan. Stud. 14, 1–27.
Gervais, S., Heaton, J., Odean, T., 2006. Overconfidence, investment policy and manager welfare. Haas School of Business, UC Berkeley (unpublished manuscript).
Gort, C., 2007. Overconfidence and active management: an empirical study across Swiss pension plans. University of Zurich (unpublished manuscript).
Grinblatt, M., Keloharju, M., 2000. The investment behavior and performances of various investor types: a study of Finland's unique data set. J. Finan. Econ. 55, 43–67.
Hackbarth, D., 2008. Managerial traits and capital structure decisions. J. Finan. Quant. Anal. 43, 843–882.
Kyle, A., Wang, F., 1997. Speculation duopoly with agreement to disagree: can overconfidence survive the market test? J. Finance 52, 2073–2090.
Lewellen, J., 2008. Institutional investors and the limits of arbitrage. Dartmouth College (unpublished manuscript).
Odean, T., 1998. Volume, volatility, price and profit when all traders are above average. J. Finance 53, 1887–1934.
Palomino, F., Prat, A., 2003. Risk taking and optimal contract for money managers. RAND J. Econ. 34, 113–137.

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Pütz, A., Ruenzi, S., 2008. Overconfidence among professional investors: evidence from mutual fund managers. University of Cologne (unpublished manuscript).
Stoughton, N., 1993. Moral hazard and the portfolio management problem. J. Finance 48, 2009–2028.
Wang, F., 1998. Strategic trading, asymmetric information and heterogeneous prior beliefs. J. Finan. Markets 1, 321–352.
Wang, F., 2001. Overconfidence, investor sentiment, and evolution. J. Finan. Intermediation 10, 138–170.
Warner, J., Wu, J., 2005. Changes in mutual fund advisory contracts. University of Rochester (unpublished manuscript).

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