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PROMOTING EMPLOYMENT BY TAMING  
THE UNIONS**

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# **SUBSIDIZING EXTRA JOBS: PROMOTING EMPLOYMENT BY TAMING THE UNIONS**

**Andreas Knabe\* and Ronnie Schöb\*\***

**October 2007**

## *Abstract*

We study the subsidization of extra jobs in a general equilibrium framework. While the previous literature focuses on *symmetric* marginal employment subsidies where firms are rewarded when they increase employment but punished when they reduce their workforce, we consider an asymmetric scheme that only rewards employment expansion. This changes the incidence substantially. In the asymmetric case without punishment, it becomes less costly for firms to lay off a substantial fraction of their workforce when trade unions raise wages. This tames the unions, which causes wage moderation and raises aggregate employment and welfare. For moderate subsidy rates, all unions prefer to restrain their wage claims. At sufficiently high subsidy rates, labor market conditions improve so much that some unions enforce higher wages and let their firms shrink. This displacement of firms might have a negative impact on employment and welfare.

*Keywords:* unemployment, marginal employment subsidies, general equilibrium  
*JEL classification:* J38, J68, H25

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## 1. Introduction

When wages are above market-clearing levels and cause unemployment, wage subsidies could alleviate unemployment by narrowing the gap between the labor cost borne by firms and the minimum compensation demanded by workers. Comprehensive wage subsidy schemes, however, are associated with two major drawbacks. First, subsidizing wages creates large windfall gains for all workers already in employment. This makes wage subsidization very expensive. Second, if the subsidy can be redistributed through the wage bargaining process, the effectiveness of wage subsidies in promoting employment is endangered. The literature on tax incidence in economies with imperfect labor markets has demonstrated that under certain institutional arrangements wage subsidies may actually be shifted completely into higher net wages. In this special case, labor costs stay constant and unemployment does not fall (see Layard Nickell and Jackman 1991).<sup>1</sup>

The fiscal cost of wage subsidization can be significantly reduced by restricting the subsidy to extra jobs created in addition to a firm's incumbent workforce. Layard and Nickell (1980) were the first who showed that this type of *marginal* employment subsidies can generate more jobs than general subsidies costing the same amount.<sup>2</sup> Indeed, the theoretical findings are all together very much in favor of a marginal rather than general wage subsidy scheme and several field experiments with such subsidy schemes confirmed the theoretical findings.<sup>3</sup>

Surprisingly, academic research lost interest in marginal employment subsidies in the meantime. The positive summarizing evaluation of marginal employment subsidies in the otherwise so influential book by Layard, Nickell and Jackman (1991) should have stimulated further research. Instead, it appears to have become the closing words on this issue – so far.

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<sup>1</sup> Nevertheless, recent theoretical and empirical work shows that wage subsidies generally have a positive effect on employment (see e.g. Daveri and Tabellini 2000, Nickell and Layard and 1999).

<sup>2</sup> This result was qualitatively confirmed in different theoretical frameworks by Chiarella and Steinherr (1982), Whitley and Wilson (1983), Oswald (1984), Hart (1989), and Layard, Nickell and Jackman (1991).

<sup>3</sup> In the 1970s, many countries experimented with marginal employment subsidies. Examples are the New Jobs Tax Credit in the United States (see Perloff and Wachter 1979, Bishop and Haveman 1979), the French *Prime d'incitation à la création d'emploi* (see Kopits 1978), the Small Firms Employment Subsidy in Great Britain (see Layard 1979), and the *Lohnkostenzuschüsse* in Germany (see Schmidt 1979).

This paper aims at reviving the interest in the analysis of marginal employment subsidies. We develop a general equilibrium model with imperfect labor and output markets (Section 2) for which we can identify a benchmark scenario in which wage taxes and general subsidies do not affect employment at all (Section 3). We then turn to the formal analysis of marginal employment subsidies (Section 4). The theoretical literature has focused on *symmetric* marginal wage subsidies where firms are rewarded when they increase employment but are punished when they reduce their workforce. Real-life marginal wage subsidy programs, however, are *asymmetric*. They subsidize employment expansions but do not punish shrinking firms. This small difference has severe consequences for the incidence of marginal employment subsidies. One might expect that the additional punishment of layoffs under symmetric subsidization may be good for employment, but the opposite is true. The punishment threat of a symmetric marginal employment subsidy makes it more costly for firms to lay off workers when trade unions aggressively raise wages. Trade unions can thus shift a large share of the wage subsidy towards higher net wages. In our benchmark case, this effect is so strong that symmetric marginal subsidies do not affect employment at all. In the asymmetric case without punishment, by contrast, the firm may be more willing to shrink and lay off a substantial fraction of its workforce when wages become too high. This tames the trade unions. Rather than shifting the whole wage subsidy into higher gross wages, trade unions can raise the wage at most to the level at which the firm becomes indifferent whether to hire more workers or to shrink and lay off workers. This wage restraint leads to positive employment effects of asymmetric marginal employment subsidies.

When the subsidy raises aggregate employment, the threat of shrinking becomes less frightening for the trade union. High subsidy rates may induce some trade unions to let their firm shrink while other firms continue to expand. The general equilibrium thus exhibits displacement between incumbent firms. Although this displacement may lower employment, we show that the government can promote employment further if it sets the wage subsidy sufficiently high. However, these additional employment gains come at a huge welfare loss. Employment will be concentrated in very few firms which sell their goods at low prices while the majority of firms shrink and sell their goods at higher prices. This distorts the optimal

consumption pattern: the variety of goods is diminished substantially. Our numerical simulations (Section 5) illustrate that employment and welfare move in the same direction for moderate subsidy levels, but that the trade-off becomes severe for larger subsidy rates.

In how far these results carry over to the long run with free entry crucially depends on the way in which new firms are treated (Section 6). When they are eligible for the subsidy, their whole workforce has to be subsidized. Any incumbent firm could take advantage of this by setting up a new firm to which it relocates all its business activities. A marginal employment subsidy would then become equivalent to a general wage subsidy in the long run. Alternatively, the government could grant the subsidy to incumbent firms only. In this case, the marginal employment subsidy will continue to tame the trade unions even in the long run. As marginal subsidies normally reduce profits, new firms will not enter and our short-run analysis carries over to the long run. Only if the desire for variety is very high, profits rise and new firms will enter. This may lead to lower, though still positive employment effects compared to the short run, but the larger variety of goods will increase welfare even further.

## 2. The model

We apply a general equilibrium model as laid out by Layard, Nickell and Jackman (1991). Rents are created by firms who can set prices above marginal costs in monopolistically competitive goods markets. These rents are distributed between firm owners and workers through collective wage setting by firm-level labor unions. Unemployment arises because it reduces union's wage pressure to the level where rent claims by firms and unions are compatible with each other.<sup>4</sup>

### *Worker households*

The economy consists of many identical worker-consumer-households, the number of which we normalize to one. Each household  $j$  provides one unit of labor and derives its utility from

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<sup>4</sup> Simplified versions of these type of models are presented in e.g. Heijdra and van der Ploeg (2002) and Sørensen and Whitta-Jacobsen (2005).

consuming a variety of  $m$  goods. Following Dixit and Stiglitz (1977) and Blanchard and Giavazzi (2003), we formulate the utility function as

$$V_j = m_0^{1/(1-\sigma)} \left[ \sum_{i=1}^m C_{ij}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}, \quad (1)$$

where  $\sigma$  is elasticity of substitution between the various product varieties and  $C_{ij}$  is the amount of variety  $i$  consumed by household  $j$ .  $m_0$  is the initial number of goods available and serves as a normalization parameter. The household's budget constraint reads

$$\sum_{i=1}^m P_i C_{ij} = \begin{cases} W_j(1-t) + \Pi_j & \text{when employed} \\ B + \Pi_j & \text{when unemployed} \end{cases} \quad (2)$$

where  $W_j$  is the nominal wage rate of household  $j$ ,  $\Pi_j$  is the profit share of household  $j$ ,  $B$  is the nominal unemployment benefit payment,  $t$  is the wage tax, and  $P_i$  is the price of variety  $i$ .

Summing up over all households yields

$$\sum_j \sum_{i=1}^m P_i C_{ij} \equiv PC, \quad (3)$$

with

$$C = \left( \frac{m}{m_0} \right)^{1/(1-\sigma)} \sum_j V_j \quad \text{and} \quad P = \left[ \frac{1}{m} \sum_{i=1}^m P_i^{1-\sigma} \right]^{1/(1-\sigma)} \quad (4)$$

being the quantity and price indices (see Dixit and Stiglitz 1977 for details).<sup>5</sup> Aggregate demand for good  $i$  is then given by:

$$C_i = \frac{C}{m} \left( \frac{P_i}{P} \right)^{-\sigma}. \quad (5)$$

### *Firms*

Each firm produces one good for which it is a monopolist. We assume a constant-returns-to-scale technology with labor being the only input factor, i.e.  $y_i = N_i$ ,  $i \in [1, \dots, m]$  where  $N_i$  is

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<sup>5</sup> In the case with identical prices for all goods, the quantity index reduces to  $C = mC_i$  and the price index to  $P = P_i$ .

the amount of labor employed by firm  $i$ , and  $y_i$  is its output level. The firm can set the good's price  $P_i$  but takes the gross wage  $W_i(1-s)$  as given, where  $s$  is an ad-valorem wage subsidy. Firm  $i$ 's profit is  $\Pi_i = (P_i - W_i(1-s))N_i$ .<sup>6</sup> Using  $C_i = y_i = N_i$ , the profit-maximizing price set by firm  $i$  is

$$P_i = (1 + \mu_f)(1-s)W_i \quad (6)$$

with  $\mu_f = (\sigma - 1)^{-1}$  denoting the (constant) markup the firm sets over marginal cost. Conditions (5) and (6) give us the labor demand functions

$$N_i(s, \dots) = C_i = \frac{C}{m} \left( \frac{P_i}{P} \right)^{-\sigma} = \frac{C}{m} \left( \frac{(1 + \mu_f)(1-s)W_i}{P} \right)^{-\sigma} \quad \forall i. \quad (7)$$

Firms have to pay start-up costs  $F$  when they enter the market. These costs are sunk after the firm has entered.

### *Welfare*

The term  $m_0^{1/(1-\sigma)}$  in the utility function (1) normalizes the maximum potential welfare for the initial number of  $m_0$  to one.<sup>7</sup> Maximum welfare is achieved when households consume all  $m = m_0$  goods in equal amounts. In this case, we have  $C = mC_i = 1$ . Intuitively, the concavity of (1) reflects the desire for variety: a household who is indifferent between two consumption bundles (1,0) and (0,1) always prefers the mixed bundle (0.5, 0.5) (see Dixit and Stiglitz 1977, p. 297). Welfare losses may occur when labor is idle due to (involuntary) unemployment or goods are consumed in different quantities. Both types of welfare losses may occur in our model both in the short run and in the long run. In addition, long-run welfare may rise when additional varieties become available.

### *Wage Determination*

Each firm's workforce is organized in a firm-level labor union that can unilaterally determine the firm-specific wage rate. We apply a union's objective function  $\Omega$  of the Stone-Geary type:

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<sup>6</sup> For expositional convenience, we use the term "profit" for the short-run profit that equals the producer rent, i.e. we do not subtract start-up costs.

<sup>7</sup> This can be seen from maximizing  $C$  subject to the economy's resource constraint of one unit of labor.

$$\Omega(W)_i = [(1-t)W_i - W^o]^\phi N_i. \quad (8)$$

The union benefits from firm-level employment  $N_i$  and the difference between the net wage earned by each worker employed by firm  $i$  and the outside option  $W^o$ . The weight  $\phi \in ]0, \sigma[$  indicates the relative importance of wage gains compared to employment.<sup>8</sup>

The outside option  $W^o$  is determined by the expected net wage available to a worker who loses his current job. A laid-off worker will either find employment in a different firm, where he can expect to earn the economy-wide average wage  $W$ , or he will be unemployed and receive unemployment benefits  $B$ . The probability of finding a job is given by the aggregate employment rate  $y$ . Hence, the workers' outside option is

$$W^o = y(1-t)W + B(1-y). \quad (9)$$

Maximizing the trade unions' objective function (8) with respect to  $W_i$  yields

$$W_i = (1 + \mu_u) \frac{W^o}{(1-t)} \quad \text{with} \quad \mu_u = \frac{\phi}{\sigma - \phi}. \quad (10)$$

To maximize its utility, a union will set a markup  $\mu_u$  on the (gross equivalent) of the expected outside wage. Note that both firm-level unions and individual firms take the prices and wages set by other firm-union-pairs as given.

### *General equilibrium*

In general equilibrium, the rents claimed by monopolistically competitive firms and by labor unions have to be compatible with each other. If unions tried to reach a real wage above (below) the level compatible with firms' price setting, inflationary (deflationary) pressures would arise. Hence, the general equilibrium occurs where price stability – or, for that matter, non-accelerating inflation – is secured.

We apply the symmetry conditions for the labor and goods markets:  $W_i = W$  and  $P_i = P$  for all  $i$ . Then, the firm's pricing rule (6) becomes the aggregate price-setting (PS) equation

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<sup>8</sup> The parameter  $\phi$  is useful for numerically simulating the model, because it secures “reasonable” unemployment levels in general equilibrium. The specification (8) encompasses the common utilitarian trade union model with risk-neutral workers as a special case for  $\phi=1$  (see Farber 1986, 1061).



$$\frac{W}{P} = \frac{1}{(1 + \mu_f)(1 - s)}. \quad (\text{PS})$$

Firms add a constant markup onto any nominal wage the trade unions set: the firms will always adjust their goods' prices such that the equilibrium real wage remains constant.

To determine the wage-setting condition, we consider unemployment benefits that are proportional to the average net wage rate, i.e.  $B = b(1 - t)W$ . The aggregate wage setting (WS) equation then follows from (9), (10), and the symmetry condition,

$$\frac{W}{P} = (1 + \mu_u)[y + b(1 - y)]\frac{W}{P} \Leftrightarrow y = \frac{1 - (1 + \mu_u)b}{(1 + \mu_u)(1 - b)} = 1 - \frac{\mu_u}{(1 + \mu_u)(1 - b)}. \quad (\text{WS})$$

The WS-condition shows that, with a constant net replacement ratio, labor taxes and subsidies do not have any general equilibrium employment effects (cf. Layard, Nickell and Jackman 1991). A lower unemployment rate would raise the unions' outside option and lead to higher wage claims, which would lead to continuous increases in the outside option and wages. Since the reverse holds for higher unemployment rates, there is a unique unemployment level compatible with price stability.

The conditions (PS) and (WS) thus specify both the equilibrium unemployment rate and the real wage. In our benchmark case, equilibrium employment is determined in the labor market while the distribution of income, given by the real wage, is determined in the goods markets. The wage tax, which is implicitly determined by the balanced government budget  $y(t - s)W = (1 - y)B$ , influences neither employment nor the level of gross wages, because it affects the unions' outside option proportionally to net wages.

In what follows, we start with the analysis of the short run where the number of firms is fixed at  $m = m_0$ . The long run with free entry of new firms will be postponed until section 6.

### 3. The irrelevance of general employment subsidies

A general employment subsidy (GS) lowers a firm's labor costs and thereby allows the firm to charge a lower price for its product (see equation (6)). Since all subsidized firms reduce their prices, the resulting deflationary effect increases aggregate demand  $C$  and employment

in all firms. Rising aggregate employment lifts the unions' outside option  $W^o$ , which triggers upward wage pressure. This counteracts the deflationary effect of the wage subsidy until price stability is restored. For the special case of a constant net replacement ratio, the whole subsidy is shifted towards higher gross wages (as can be seen from total differentiation of the PS-condition). According to the WS-condition, equilibrium employment does not change either. Since net-of-subsidy labor costs also remain constant, neither firms' profits nor aggregate workers' income change. Hence, in our general-equilibrium framework GS has neither allocative nor distributive effects as it leaves both employment and net wages unchanged. This result will serve as a benchmark and should thus be summarized as

***Proposition 1:** For a constant net replacement ratio, a general employment subsidy has no effect on either employment or the distribution of income.*

This neutrality result confirms a more general insight from the tax incidence literature that (linear) tax instruments will affect employment only when the tax burden can be shifted away from labor income to other income sources.<sup>9</sup> This is ruled out here. Indeed, our result would also hold if unemployment benefits were tax-exempt, in which case a subsidy is just a swap between higher employment subsidies and higher wage taxes. Our assumption of a constant net replacement ratio is thus more restrictive than necessary but will be very helpful to analyze the potential employment effects of marginal employment subsidies in isolation.

## **4. Marginal employment subsidies**

### **4.1 Symmetric marginal employment subsidies**

In the theoretical literature, a specific type of marginal employment subsidies, which we will call "symmetric marginal employment subsidies" (SMS), has been discussed as an alternative policy instrument to general wages subsidies (see Layard and Nickell 1980, Oswald 1984, Layard, Nickell and Jackman 1991). The idea is to subsidize extra jobs, but at the same time

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<sup>9</sup> Pflüger (1997) discusses several tax reforms in a similar framework and finds employment effects only when the government can actually shift the tax burden to other income sources.

tax employment reductions. Formally, such a SMS is equivalent to a general subsidy combined with a tax on a firm's initial workforce. This becomes apparent from a single firm  $i$ 's profit function with a SMS (see Layard et al. 1991, p. 491):

$$\Pi_i = (P_i - W_i)N_i + sW_i(N_i - N_{i0}) = (P_i - (1-s)W_i)N_i - sW_iN_{i0}, \quad (11)$$

where  $N_{i0}$  is the firm's initial employment level. The first way of writing the profit function shows that the subsidized wage applies to all workers hired in excess of  $N_{i0}$ , while the second expression reveals that SMS is effectively a general subsidy on all workers combined with a lump-sum tax. Such a lump-sum tax does not affect the profit-maximizing behavior of a single firm so that the firms' price setting will be the same as under GS. Likewise, firm-level union wage setting will be unaffected by SMS. Thus, SMS and GS have the same effect on gross wages and no effect on employment.

The only difference lies in the lump-sum tax component which reduces the net fiscal expenditures for the SMS program. In general, the balanced-budget tax rate  $t$  will be:

$$tWy = b(1-t)W(1-y) + sW(y - y^*) \Leftrightarrow t = \frac{b(1-y) + s(y - y^*)}{y + b(1-y)}, \quad (12)$$

with  $y^* = 0$  for GS and  $y^* = y_0 = mN_{i0}$  for SMS. As can be seen from  $dt/dy^* < 0$ , the wage tax  $t$  is lower and the net wage is higher with SMS. Net profits are lowered by the lump-sum tax the firm has to pay. This leads to

***Proposition 2:*** *For a constant net replacement ratio, a symmetric marginal employment subsidy has no effect on employment, but redistributes income from profit to labor income.*

While the result of Proposition 1 stems from our assumption of a constant net replacement ratio, the strong result of Proposition 2 also hinges on the assumption of a monopoly trade union. Indeed, our results are in contrast to Layard, Nickell and Jackman (1991) who find a positive employment effect of SMS in a Nash-bargaining framework. With Nash bargaining, the union is unable to raise the wage up to a level where employment is the same as without the subsidy. At this level, the total rent to be distributed would be the same as without the subsidy, but the union would receive a larger share at the cost of firm's profits. This worsens

the union's bargaining position, because it would have more to lose if the firm threatened to stop negotiating, and wages have to fall. Distributional effects do not matter in the monopoly trade union framework though. The monopoly union sets the wage rate irrespectively of the (non-negative) profit level of the firm. Additional benefits for the firm thus do not enter its arbitrage calculus.<sup>10</sup>

Our restrictive assumptions that lead to Propositions 1 and 2 thus ensure that our benchmark scenario is most adverse to favorable employment effects arising from wage subsidies. If any type of wage subsidy has a positive employment effect under these assumptions, it will have a positive employment effect under assumptions more favorable to wage subsidies, *a fortiori*.

#### 4.2 Asymmetric marginal employment subsidies

Real-life marginal wage subsidy programs are *asymmetric* in the sense that they subsidize extra jobs but do not punish shrinking firms. Examples are the New Jobs Tax Credit in the United States, the French *Prime d'incitation à la création d'emploi*, the Small Firms Employment Subsidy in Great Britain, and the *Lohnkostenzuschüsse* in Germany, all of which applied asymmetric marginal subsidies (AMS).<sup>11</sup> To account for this asymmetry, firm  $i$ 's profit function has to be rewritten as

$$\Pi_i = (P_i - W_i)N_i + sW_i \max(N_i - N_{i0}, 0). \quad (13)$$

Under AMS, a firm can pursue two different strategies. One strategy is to expand employment above its initial level, receive the subsidy for the extra jobs, and sell large amounts of output at low prices ("expansion"). The other strategy is to keep employment at or below its initial level, forgo the subsidy, and sell lower levels of output at higher prices than its competitors ("shrinking"). Depending on the wage set by the firm-level union, a firm chooses whichever strategy yields the larger profit. In the expansion strategy, the firm has to raise its employment

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<sup>10</sup> This result is an application of a general result, according to which comparative statics results for the monopoly union model and the Nash-bargaining model are different when changes affect the firm's revenue function (see Holmlund 1989).

<sup>11</sup> See Perloff and Wachter (1979) and Bishop and Haveman (1979) for the US, Layard (1979) for Great Britain, Kopits (1978) for France, and Schmidt (1979) for Germany.

beyond  $N_{i0}$ . Its marginal cost is then  $(1-s)W_i$ , and it maximizes its profit by setting  $P_i = (1 + \mu_f)(1-s)W_i$ . If the firm decides to shrink, charging a markup on its marginal cost will yield a price  $P_i = (1 + \mu_f)W_i$ . A firm will prefer expanding to shrinking if, for a given wage  $W_i$ ,

$$\pi_i^+ \equiv \mu_f(1-s)W_i N_i(s) - sW_i \frac{y_0}{m} \geq \mu_f W_i N_i(0) \equiv \pi_i^- . \quad (14)$$

The left-hand side of (14) is a firm's profit from the expansion strategy,  $\pi_i^+$ , which consists of the profit the firm would make if the subsidy  $s$  was paid for all employees  $N_i(s)$  (cf. equation (7)) minus the subsidy-exemption for all incumbent workers. The right-hand side denotes the firm's profit in the shrinking strategy,  $\pi_i^-$ .

Figure 1: The firm's output decision for given demand

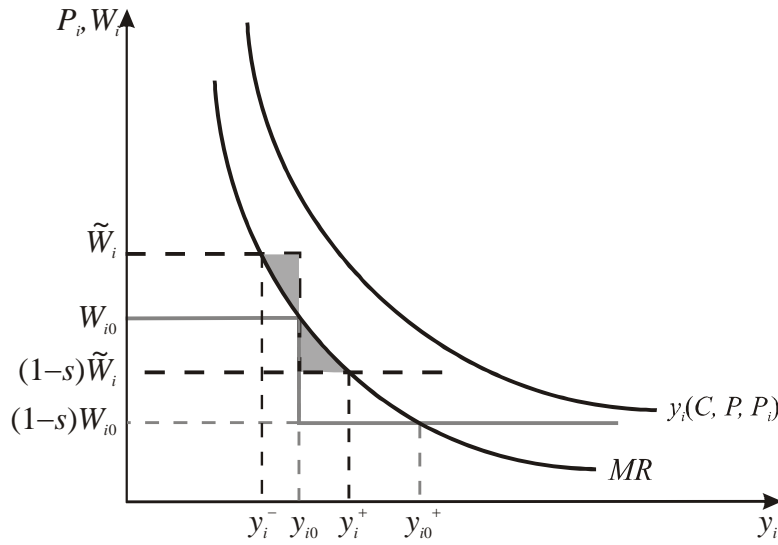


Figure 1 illustrates how the firm's output decision depends on the wage rate set by its union. Without subsidies, the firm faces a wage rate  $W_{i0}$ . It maximizes its profit by selling  $y_{i0}$  units of output. When the marginal subsidy is introduced, and wages do not change, the firm's marginal cost schedule is at  $W_{i0}$  for output levels below  $y_{i0}$ , but at  $(1-s)W_{i0}$  for output levels above this reference level. The firm maximizes its profit by selling the increased output  $y_{i0}^+$  at a lower price. If the firm's union raises its wage to e.g.  $\tilde{W}_i$ , the marginal cost schedule shifts upwards, but retains its downward jump at  $y_{i0}$  to  $(1-s)\tilde{W}_i$  (dashed line). There are two local profit-maxima where marginal revenue  $MR$  equals marginal cost: the firm could either

expand to  $y_i^+$ , or it could shrink to  $y_i^-$ . The firm chooses the output level that yields the higher profit. By switching from  $y_i^-$  to  $y_i^+$ , the firm would make infra-marginal losses on all units between  $y_i^-$  and  $y_{i0}$ , but would make profits on all units between  $y_{i0}$  and  $y_i^+$ . In Figure 1, the infra-marginal losses and profits are represented by the two shaded triangles. We have drawn  $\tilde{W}_i$  as the “indifference wage” at which the two areas have the same size. At  $\tilde{W}_i$ , the firm is indifferent between shrinking and expanding, i.e.  $\pi_i^+(\tilde{W}_i) = \pi_i^-(\tilde{W}_i)$ . For lower wage levels, the firm would strictly prefer to expand. If the firm-level union raises the wage above this threshold, however, the firm would prefer to shrink.

This discontinuity in the firm’s output supply behavior constitutes the main difference between AMS and SMS. It provides firms with a credible threat that they shrink and cut jobs if unions set too high wages. This constrains unions in their ability to shift the subsidy into higher gross wages. If the loss in employment weighs larger than the benefit from higher wages, a union prefers to set the wage just equal to the indifference wage  $\tilde{W}_i$  to extract the maximum rent from a still expanding firm.

To identify potential general equilibria with AMS, we first look at cases where all unions are unconstrained in their wage-setting and set wages according to their first-order condition (10) as the full markup on their (common) outside option. One can show that if all firms decided to shrink, any single firm would have an incentive to deviate and expand. Conversely, if all firms expanded, any single firm would have an incentive to shrink. Hence, we can rule out equilibria in which trade unions set the firm-level wage according to the first-order condition (10). This should be summarized in a

***Lemma 1:*** *There is no Nash equilibrium with AMS in which all unions set their wage as the full markup on their outside option.*

For a formal proof see Appendix 1. In any general equilibrium with AMS, at least some unions have to be constrained in their wage-setting by the indifference wage  $\tilde{W}_i$ . At the indifference wage  $\tilde{W}_i$ , a firm never shrinks in equilibrium because its union would always reduce its wage marginally to induce the firm to expand. This leaves us with only two potential equilibria:

- Case A: all unions set the indifference wage  $\tilde{W}_i$ , and all firms choose to expand,
- Case B: some unions set the indifference wage  $\tilde{W}_i$  so that their firms expand, while other unions choose a higher wage and let their firms shrink.

#### 4.2.1 Case A: all unions prefer expansion, and all firms expand

If all firms try to expand, they will all set their price as a markup over subsidized marginal cost. The PS-condition remains unaffected. Individual unions, however, prefer to deviate from their first-order condition (markup wage-setting) because unconstrained wage-setting would cause their firms to shrink. Unions will set the highest possible wage  $\tilde{W}_i$  that just ensures that firms expand (cf. Figure 1). If all unions behave this way, the new wage-setting equation is given by  $W = \tilde{W}$ , where  $\tilde{W}$  is determined by condition (14), holding with equality,

$$\pi_i^+(\tilde{W}) = \pi_i^-(\tilde{W}). \quad (15)$$

Inserting (PS) into (15), and using  $C = y$  since all firms set the same prices, gives the equilibrium employment rate:

$$y = \frac{sy_0}{\mu_f [(1-s) - (1-s)^\sigma]}. \quad (16)$$

Inspection of (16) shows that  $y > y_0$  and  $dy/ds > 0$  for all  $s \in ]0,1[$  (see Appendix 2). This leads us to

**Proposition 3:** *An asymmetric marginal employment subsidy has a positive employment effect if all unions prefer that their firms expand.*

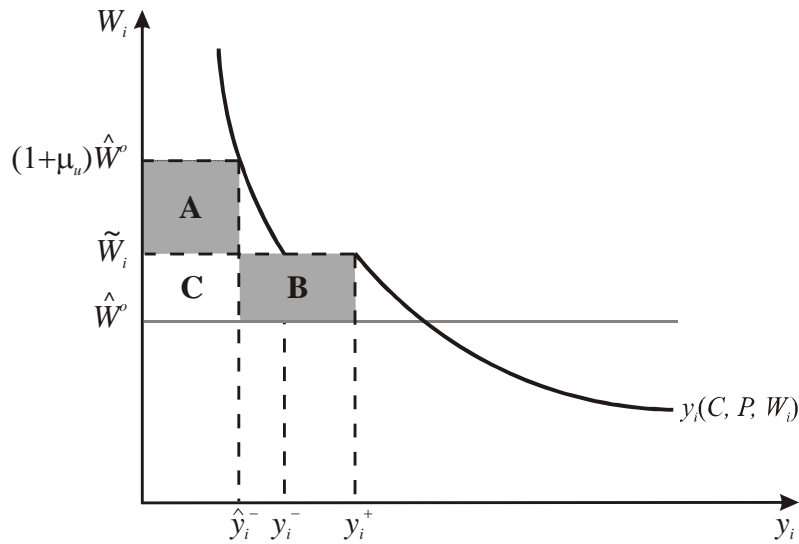
AMS restrict the unions' ability to shift the whole subsidy into higher gross wages because their firms would then prefer to shrink. As long as unions value the potential loss of jobs more than the potential wage gains from shrinking, they will be better off with the indifference wage  $\tilde{W}_i$ . Part of AMS – contrary to SMS – then leads to a reduction in prices, and the resulting deflationary pressure raises output and employment.

#### 4.2.2 Case B: some firms expand and other firms shrink

Equation (16) shows that if  $s$  becomes sufficiently large, aggregate employment exceeds one. At full employment, however, each firm-level union has an incentive to raise the wages and let its firm shrink because any worker whom the firm lays off could easily find a job at the same wage rate elsewhere but those who remain would be strictly better off. Therefore, Case A cannot be a feasible equilibrium for all values of  $s$ .

As long as there is some unemployment, any individual union has to compare its utility from expansion (at a wage  $\tilde{W}_i$ ) with the utility from shrinking (at a wage  $(1 + \mu_u)W^o > \tilde{W}_i$ ). Figure 2 illustrates the two different strategies.

Figure 2: The union's indifference between shrinking and expanding



As described above, the asymmetry of the AMS schedule causes a jump in a firm's labor demand function at the wage  $\tilde{W}_i$ . When the union sets the wage to  $\tilde{W}_i$ , the firm expands employment to  $y_i^+$ . If the union sets the wage according to (10), the firm reduces the employment level to  $\hat{y}_i^-$ . The union compares the rents gained by its members over their outside option under both strategies. For the low-wage strategy, the rent is given by the areas  $A + C$ . For the high-wage strategy, the rent is given by the two rectangles  $B + C$ . If the value of the outside option is relatively small, i.e.  $W^o < \hat{W}^o$ , the union will prefer the low-wage strategy, and *vice versa* for attractive outside options  $W^o > \hat{W}^o$ . At  $\hat{W}^o$ , the union is



indifferent between both strategies. Figure 2 depicts this critical level of the outside option where A and B are of the same size.

Increasing the subsidy causes expanding firms to become larger and employ more workers. This raises the outside option above  $\hat{W}^o$ , such that some unions would find it beneficial to raise their wages above  $\tilde{W}$  and let their firms shrink. This is the *displacement effect* of AMS. Some firms expand employment, while other firms shrink and lay off a substantial part of their workforce. Since shrinking firms set higher prices, this displacement causes inflationary pressures that counteract the subsidy's deflationary effect. The aggregate employment effect becomes generally ambiguous and depends on whether the inflationary effect from rising prices in shrinking firms is large enough to outweigh the deflationary effect from the subsidy.

In any equilibrium with displacement, some firm-union-pairs expand while some other (of otherwise identical) firm-union-pairs shrink. This requires that all unions are indifferent between the two strategies, which will be the case if the following condition holds:

$$\Omega^+ \equiv \left[ (1-t) \frac{\tilde{W}_i}{P} - \frac{W^o}{P} \right]^\phi y_i^+ = \left[ \mu_u \frac{W^o}{P} \right]^\phi y_i^- \equiv \Omega^-, \quad (17)$$

where

$$y_i^+ = \frac{C}{m} \left( \frac{P_i^+}{P} \right)^{-\sigma} \quad \text{with } P_i^+ = (1 + \mu_f)(1-s)\tilde{W}_i,$$

and

$$y_i^- = \frac{C}{m} \left( \frac{P_i^-}{P} \right)^{-\sigma} \quad \text{with } P_i^- = (1 + \mu_f)(1 + \mu_u) \frac{W^o}{(1-t)}$$

are the employment levels in expanding firms and in shrinking firms, respectively. The left-hand side of equation (17) is a union's utility  $\Omega^+ = \Omega(\tilde{W}_i)$  from setting the indifference wage  $\tilde{W}_i$  as determined by (15) that induces the firm just to expand. The right-hand side is the union's utility  $\Omega^- = \Omega((1 + \mu_u)W^o(1-t)^{-1})$  from setting the full markup according to (10), in which case the firm will shrink.

The  $m$  incumbent firms are divided in  $m^+$  expanding and  $m^- = m - m^+$  shrinking firms. In equilibrium with  $\tilde{W}_i = \tilde{W} \forall i$ , the economy-wide average wage  $\bar{W}$  is then determined by weighting firms' wages with their employment share:

$$\bar{W} = \frac{m^+ y_i^+}{y} \tilde{W} + \frac{m^- y_i^-}{y} \left( \frac{(1 + \mu_u)}{(1 - t)} W^o \right). \quad (18)$$

The aggregate employment level is obtained by

$$y = m^+ y_i^+ + m^- y_i^-. \quad (19)$$

The outside option is then given by

$$W^o = [y + b(1 - y)](1 - t)\bar{W}. \quad (20)$$

The last remaining piece necessary to determine the general equilibrium is the average price level that can be determined by inserting markup prices into equation (4) and dividing by  $P$ :

$$\frac{m^+}{m} \left( \frac{P_i^+}{P} \right)^{1-\sigma} + \frac{m - m^+}{m} \left( \frac{P_i^-}{P} \right)^{1-\sigma} = 1. \quad (21)$$

The six equations (15) and (17)-(21) determine the equilibrium values of  $y$ ,  $C$ ,  $\bar{W}/P$ ,  $m^+$ ,  $\tilde{W}/P$  and  $W^o/P$ . This system of equations does not have a closed-form solution, but it is possible to analyze the employment effects when the subsidy rate is raised to very high levels (see Appendix 3 for analytical details).

**Proposition 4:** *If an asymmetric marginal employment subsidy approaches 100 percent, the economy reaches full employment whereas welfare falls to zero.*

Increasing the subsidy rate induces some firm-union pairs to shrink. These firms set higher prices, which cause inflationary pressures counteracting the subsidy's deflationary effect. If the subsidy rate is raised to sufficiently high levels, however, Proposition 4 shows that the subsidy's deflationary effect dominates its inflationary counter-effect. The power of AMS to impose a wage cap on unions is thus sufficiently strong to allow the economy to run almost at full employment without triggering inflation.

Full employment, however, comes at huge welfare costs. While in Case A, welfare and employment go hand in hand (they are actually equivalent due to the symmetric structure of the model), this is no longer true when firms split up into small, high-price firms and large, low-price firms. At a given level of aggregate employment, this reduces welfare compared to a situation in which all firms behave identically. A higher subsidy rate will induce more firm-union pairs to shrink, while the remaining firm-union pairs become larger. Consumers are left with less variety in their shopping baskets. For the limiting case  $s \rightarrow 1$ , Proposition 4 indicates that the number of expanding firms approaches zero, while the few expanding firms employ almost the complete aggregate workforce ( $m^+ \rightarrow 0$ ,  $m^+ y_i^+ \rightarrow y$ ). Despite larger output levels of the remaining good(s), variety-loving consumers are clearly worse off by this extreme reduction in the available range of (affordable) products. Hence, an AMS sacrifices the consumers' desire for variety for gains in aggregate employment.

Proposition 4 indicates the existence of Case B. The question remains whether Case A always exists so that introducing AMS at a very small rate would always increase both employment and welfare. If there had not been any employment gains compared to the situation without subsidies, the outside option would not have improved and there would not be any incentive to shrink. Therefore, starting at  $s = 0$  and marginally increasing the subsidy rate will always lead to a Case A – equilibrium before switching to Case B (for a proof see Appendix 4) Proposition 5 summarizes.

***Proposition 5:** The marginal introduction of an asymmetric marginal employment subsidy always raises both employment and welfare.*

## **5. Numerical simulation**

Since no closed-form solution of the equilibrium in Case B can be obtained – except for the limiting case of  $s \rightarrow 1$  – we apply a numerical simulation to analyze the effects of AMS for “intermediate” subsidy rates. How strongly welfare is linked to aggregate employment in Case B depends on the consumers' taste for variety, which is represented by the elasticity of substitution  $\sigma$ . To account for its influence, we consider four different scenarios:

- a high desire for variety, which implies a low substitutability ( $\sigma = 1.5$ );
- two intermediate scenarios with  $\sigma = 2$  and  $\sigma = 3$ ;
- a low desire for variety, which is represented by a high substitutability ( $\sigma = 10$ ).

In all scenarios, we set the net replacement rate to  $b = 0.5$  (which is in line with stylized facts for the OECD; see Carone et al. 2004, Table 8). The union's weight on wages  $\phi$  is adjusted to ensure an unemployment rate of ten percent in the initial situation without subsidies.<sup>12</sup>

Figure 3 plots the employment, welfare, and distributional effects of AMS for all four scenarios. There is always a range of moderate subsidy rates for which Case A exists. As was to be expected, higher values of  $\phi$  reduce the maximum attainable employment level in Case A. In the first scenario with  $\phi = 0.075$ , for example, unions value employment much more than wages, such that an AMS with  $s = 0.13$  can increase aggregate employment up to 99.7 percent before unions start to raise wages and let their firms shrink. In the fourth scenario, on the other hand, unions value wages much more ( $\phi = 0.5$ ). The transition from Case A to Case B occurs at a subsidy rate of  $s = 0.02$  and an aggregate employment rate of 95.3 percent. The critical subsidy rate, above which Case B sets in, falls as  $\sigma$  rises. Individual firm-union-pairs react more strongly to a given subsidy rate if they face more elastic product demand. This produces stronger employment effects, and the switch to Case B occurs at lower subsidy rates.

Our simulations suggest that the employment and welfare effects in Case B are not monotonous, aggregate employment can fall immediately after switching to Case B. This non-monotonous employment effect has its roots in the non-monotonous composition of the workforce. At the transition between Cases A and B, the share of workers employed in expanding firms  $m^+ y_i^+ / y$  has to be equal to one. When the first firm decides to shrink, this share has to fall. Workers in shrinking firms earn a higher wage than workers in expanding firms, so average wages increase and wage pressure rises. This inflationary effect counteracts the subsidy's deflationary effect and causes employment to fall. Although the overall effect is ambiguous for some interval in Case B, we know from Proposition 4 that  $m^+ y_i^+ / y$  converges to one for  $s \rightarrow 1$ . Hence, the employment share of expanding firms has to increase

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<sup>12</sup> Empirical estimates of  $\phi$  are typically in the range of 0.2 to 0.4, but can reach values as high as 0.88 for some unions (see Cahuc and Zylberberg 2004, 379, for an overview).

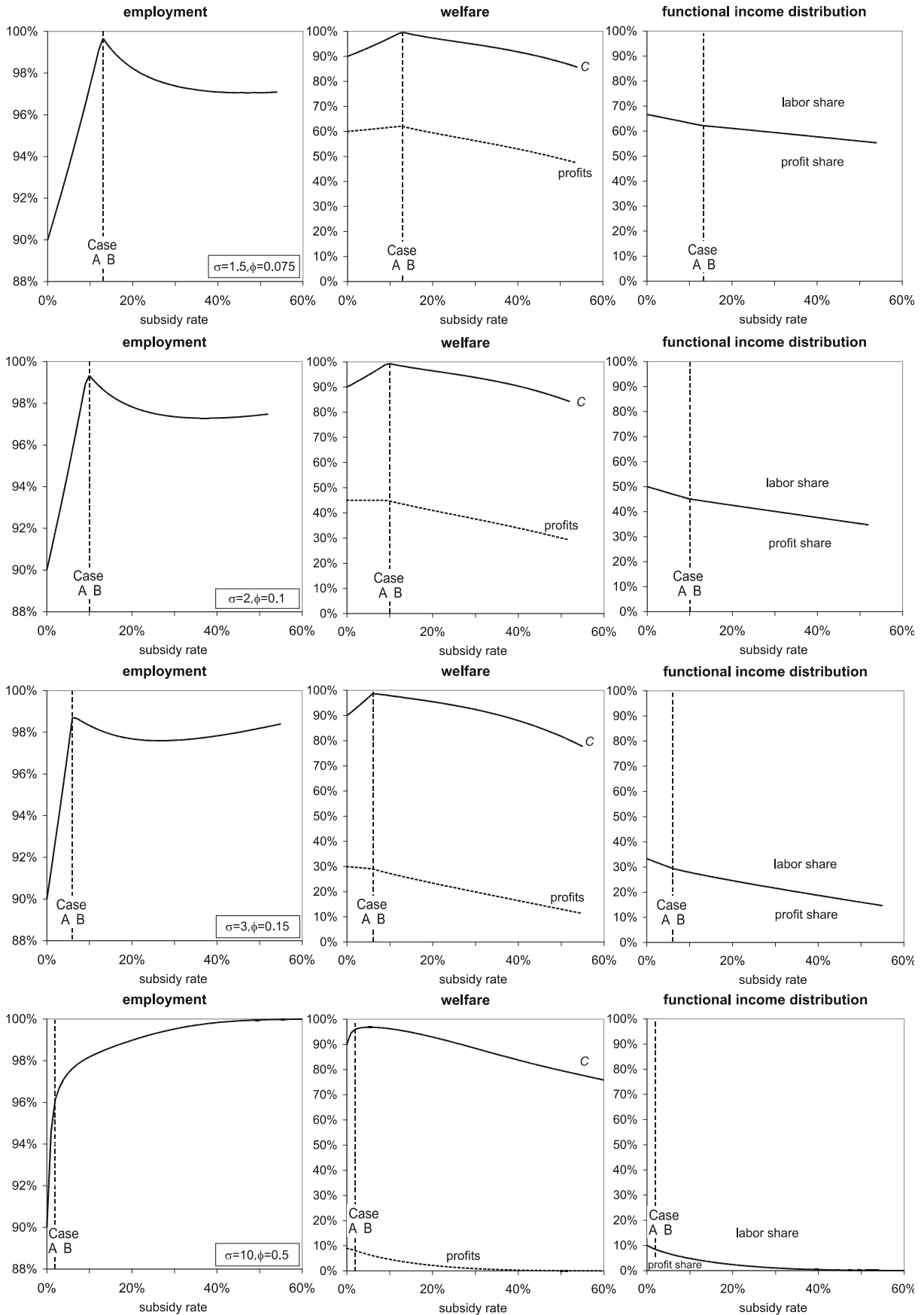
eventually, which lowers the average wage, reduces inflationary wage pressure, and increases employment.

In comparison, the four scenarios show that the negative effect on employment lessens as  $\sigma$  rises. A higher price elasticity of product demand means that shrinking firms become smaller. This lowers the share of workers employed in shrinking firms and thereby reduces their impact on aggregate wage pressure. In the fourth scenario with  $\sigma = 10$ , the upward wage pressure is too weak to overcompensate the subsidy's deflationary effect and employment rises monotonically in  $s$ .

If employment falls, welfare must fall as well. Less employed workers produce less output, and diverging prices between expanding and shrinking firms make it more difficult for consumers to satisfy their taste for variety. This happens in Case B of the first three scenarios. In the fourth scenario, however, aggregate employment rises monotonically in Case B. Moreover, consumers are able to substitute the various goods relatively well, such that variety is less important. Our simulations show that this suffices to raise welfare for a range of subsidy rates even in Case B. While the transition from Case A to Case B takes place at a subsidy rate of 1.2 percent, the welfare index  $C$  is maximized at a higher subsidy rate of 5.3 percent.

AMS has strong distributional effects. Even if unions cannot shift the subsidy fully into higher wages, they are able to lift their members' net wages. Firms have to pay these higher wages for all their employees, but receive the subsidy only for extra workers. This lowers the share of profits in the functional income distribution in all four scenarios. Since total production might increase, however, this does not necessarily mean that absolute profits have to fall. In Case A, profits rise if  $\sigma < 2$ , stay constant if  $\sigma = 2$ , and fall if  $\sigma > 2$ . In Case B, profits are decreasing in  $s$  in all scenarios.

Figure 3: Numerical simulations



## 6. The long run

In the long run, firms may freely enter and exit the market. If, by developing a new variety and selling it on the market, a new firm has good prospects to earn enough profits to cover its start-up costs  $F$ , it enters. If not, it stays out. Incumbent firms stay in the market as long as their short-run profits are positive because their start-up costs are sunk.<sup>13</sup>

The treatment of new firms is crucial for the long-run efficacy of marginal employment subsidies. Since a new firm's reference employment level is zero, all its workers take up extra jobs that, in principle, would be eligible for AMS. This procedure is problematic, however, because any incumbent firm would try to take advantage of it by setting up a new firm to which it would relocate all its business activities and its workforce. It could then enjoy full subsidization even without creating a single new job. If all incumbent firms converted to new firms, all workers in the economy would be subsidized. This would make AMS equivalent to a general subsidy and eliminate all positive employment effects.

Alternatively, the government could exclude new firms that are founded after the reference date from subsidization. New firms would make less profit than incumbent firms, which prevents the conversion of incumbent into new firms.<sup>14</sup> Keeping incumbent firms in the market is essential for a positive long-run employment effect because it is the asymmetry of the subsidy scheme that tames the trade unions. The long-run equilibrium then depends on whether new firms will enter.

Incumbent firms' profits fall in  $s$  if  $\sigma > 2$  (see Appendix 5 for a formal proof). New firms always make less profit than incumbent firms because they are not subsidized. Hence, new firms strictly prefer not to enter the market. For  $\sigma = 2$ , incumbent firms' profits stay constant in  $s$ , such that they earn just enough to cover start-up costs. New firms make less profit and prefer to stay out. Only for  $\sigma < 2$ , where incumbent firm's profits are increasing in  $s$ , is it possible that new firms earn enough to cover their start-up costs. Therefore, there has to be a critical level  $\bar{\sigma} \in ]1,2[$  at which new firms are, in equilibrium, just indifferent between

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<sup>13</sup> For a discussion of sunk cost see e.g. Martin (1993, 304) who also lays out why sunk cost are not just a short-run phenomenon, but remain relevant in the long run.

<sup>14</sup> A second alternative would be to grant higher marginal employment subsidies to incumbent firms until the average employment subsidy becomes the same for new and incumbent firms (see Knabe, Schöb and Weimann 2006 for details).

entering the market and staying out. Only if  $\sigma$  is less than this critical level will new firms enter. Otherwise, the long-run effects of AMS will be exactly the same as in the short run. This result is worth being stated as

**Proposition 6:** *An asymmetric marginal employment subsidy yields the same employment and welfare effects in the long run as in the short run if  $\sigma \geq \bar{\sigma}$  with  $\bar{\sigma} < 2$ , in which case new firms prefer not to enter the market.*

Only if  $\sigma < \bar{\sigma} < 2$ , new firms earn enough profits to cover their start-up costs and enter the market. The resulting equilibrium can then be described by a system of equations similar to that of Case B because new entrants always behave like (unsubsidized) shrinking firms. In fact, equations (15) and (18)-(21) have to be fulfilled unchanged. The total number of firms in the market,  $m$ , is determined by the condition that new firms make just enough profits to cover their start-up cost:

$$\mu_f (1 + \mu_u) \frac{W^o/P}{1-t} y_i^- = \frac{F}{P} = \frac{\mu_f}{1 + \mu_f} \frac{y_0}{m_0}. \quad (22)$$

The left-hand side of (22) is a new firm's profit. The right-hand side is the level of start-up costs that is given by a firm's profit in the initial equilibrium with  $s = 0$ , i.e.  $F = \mu_f W y m_0^{-1}$ .

Equation (22) can be simplified to

$$\left[ (1 + \mu_f)(1 + \mu_u) \frac{W^o/P}{(1-t)} \right]^{1-\sigma} \frac{C}{y_0} \frac{m_0}{m} = 1. \quad (23)$$

Again, all  $m$  firms are divided into  $m^+$  expanding and  $m^-$  shrinking firms. As in the short run, in equilibrium unions are either indifferent between expanding and shrinking or strictly prefer expansion. The number of expanding firms is restricted to  $m^+ \leq m_0$  because only incumbent firms receive a subsidy and can pursue the expansion strategy. Similarly to (17), we can write this condition as

$$\Omega^+ \geq \Omega^- \text{ and } m^+ \leq m_0, \quad (24)$$



where at least one of the two expressions has to hold with equality. Hence, the long-run equilibrium if new firms enter is given by the system of equations (15), (18)-(21), (23), and (24). The long-run employment and welfare effects can then be described as

***Proposition 7:** If  $\sigma < \bar{\sigma}$ , new firms enter. If all incumbent unions are indifferent between the expanding and shrinking strategies (as in the short-run Case B), AMS yield the same employment effect in the long run as in the short run. If all incumbent unions prefer the expanding strategy, long-run employment is less than short-run employment but larger than in the initial equilibrium. AMS always increase welfare in the long run.*

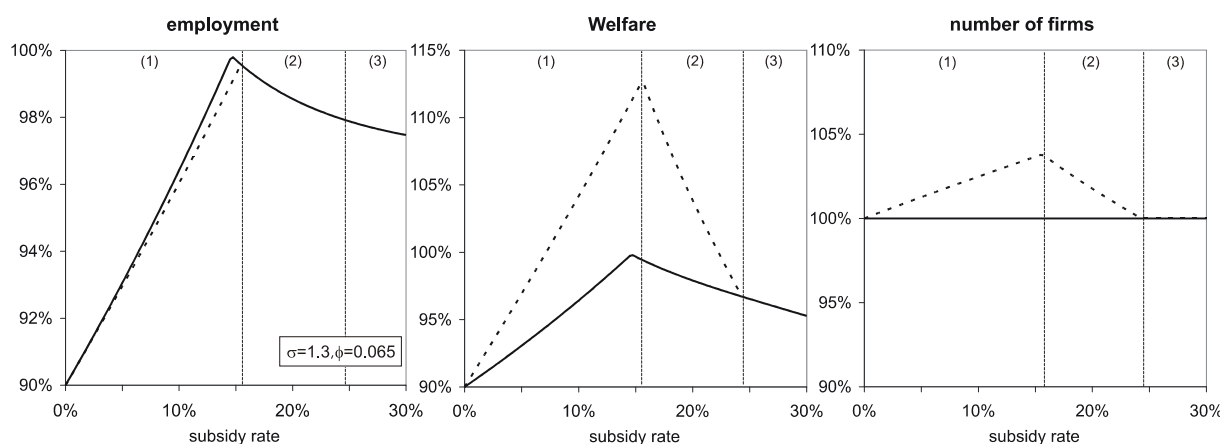
For the proof, see Appendix 6. The first part of the proposition describes the situation where the entry of new firms raises the number of shrinking and expanding firms proportionally. In this case, the relative division of firms is unchanged, and employment is unaffected by the entry of new firms. In its second part, Proposition 7 refers to the case where the entry of new firms increases the share of shrinking firms in the economy. Since shrinking firms set higher prices than expanding firms, this releases inflationary pressure that reduces employment compared to the short run. This inflationary counter-effect, however, is not sufficient to outweigh the positive effects of AMS altogether. Hence, AMS will increase employment compared to the initial equilibrium even in the long-run. Since consumers love variety, an increase in the number of firms always increases welfare for a given level of aggregate production. We have shown that AMS always increase aggregate employment above its initial level, from which follows that AMS always increase welfare also in the long run.

We conduct a numerical simulation to illustrate the difference between the short-run and long-run effects of AMS (Figure 4). For  $\sigma \geq \bar{\sigma}$ , short-run and long-run effects coincide, so we can restrict our attention to the case of  $\sigma < \bar{\sigma}$ . For an initial unemployment rate of 10 percent and  $b = 0.5$ , the critical value  $\bar{\sigma}$  is around 1.58. In Figure 4, we choose a very small value of  $\sigma = 1.3$  to make the difference between the short run (solid line) and the long run (dashed line) visible. The left and middle figures show the results of Proposition 7. In Section 1 (for subsidy rates between 0 and 15.5 percent), new firms enter but unions in incumbent firms prefer to expand. In this case, the share of shrinking firms increases, and the resulting

inflationary pressure reduces employment. Welfare increases more in the long run than in the short run because new firms enter and increase the variety of available goods. At  $s = 0.155$ , welfare increases from 0.9 to 0.995 (increase by 10.5 percent) in the short run, but rises to 1.126 (increase by 25.1 percent) in the long run because of an increase in firms (varieties) by 3.8 percent. In Section 2 (subsidy rates between 15.5 and 24 percent), new firms enter but incumbent unions remain indifferent between expanding and shrinking. As Proposition 7 shows, the entry of new firms does not affect aggregate employment compared to the short run. Welfare, however, increases due to the entry of new firms. In Section 3, new firms do not want to enter, and the long run equilibrium is the same as the short-run equilibrium.

Figure 4 shows that the entry of new firms might harm aggregate employment, although the quantitative effect is rather small. The welfare of variety-loving consumers, however, increases substantially.

Figure 4: The short run (solid line) and the long run (dashed line)



## 7. Conclusion

Asymmetric marginal employment subsidies that support extra jobs without punishing layoffs are superior to both general employment subsidies and symmetric marginal employment subsidies. The driving force behind this result is the fact that the asymmetry of the subsidy scheme makes it less costly for firms to lay off a substantial fraction of their workforce when trade unions raise wages too aggressively. The credible threat of the firm to shrink tames the unions, causes wage moderation and raises aggregate employment and welfare. For moderate

subsidy rates, all unions prefer to restrain their wage claims and let their firms expand. In this case, raising the subsidy rate improves both employment and welfare. At high subsidy rates, labor market conditions improve so much that some unions start to enforce higher wages and let their firms shrink. This displacement of firms might have an ambiguous effect on employment but definitely lowers welfare as it distorts the households' consumption decisions. This shows that asymmetric marginal wage subsidies are an effective means to fight unemployment and to increase welfare. Nevertheless, they should be applied with caution.

We have discussed several features that prevent the exploitation of the subsidy scheme by incumbent firms. One practical problem remains. Even though incumbent firms may not simply convert themselves into new firms, they could outsource their workforce to another incumbent firm. In our model, the insourcing firm would then produce two varieties. Net employment would not change but the 'transferred' workforce would be eligible for AMS. In the extreme case, all incumbent firms merge into one single firm and (almost) the complete workforce would be subsidized. AMS would degenerate to a general subsidy, and aggregate employment and welfare would return to their initial, non-subsidized levels. Such outsourcing activities, however, could be reduced by setting a threshold above which employment expansions in incumbent firms are not subsidized anymore. Such restrictions have already been implemented in real-life marginal subsidy programs. For example, the New Jobs Tax Credit in the United States restricted the maximum subsidy to the smaller of 25 percent of a firm's total wage bill or 100,000 US-\$ per firm and year (Perloff and Wachter, 1979). Another option would be to restrict the subsidy to a certain number relative to the incumbent workforce. For example, one could introduce a ceiling at twice the reference employment level (see Knabe, Schöb and Weimann 2006). Such restrictions can prevent misuse of the subsidy and thereby preserve the asymmetry of the subsidy scheme. One should keep in mind, however, that even if these precautions fail, and firms manage to circumvent the marginal subsidy and get their entire workforce subsidized, the resulting long-run equilibrium would be the same as the one without any subsidy. AMS would then still be a welfare-enhancing policy due to its favorable short-run effects.

Our analysis clearly shows that institutions, and their correct implementation in economic models, matter. The very fact that a small modification in the modeling of marginal subsidy schemes leads to substantially different results emphasizes the importance of institutional details for economic analysis, and in particular for the study of tax incidence. Besides contributing to the literature on the incidence of employment subsidies, this paper therefore also fits into the growing literature that reintroduces institutions in economic theory.

## Appendices

### Appendix 1: proof of Lemma 1

If no firm wants to expand and receive the subsidy, and their unions set their wages accordingly, the resulting equilibrium is the initial equilibrium without any subsidy. The aggregate price setting equation PS is then given by  $W/P = (1 + \mu_f)^{-1}$ , and the wage setting equation is given  $y = y_0$ . Any single firm would strictly prefer shrinking if

$$\mu_f (1-s) W_i \frac{C}{m} \left[ (1 + \mu_f)(1-s) \frac{W_i}{P} \right]^{-\sigma} - s W_i \frac{y_0}{m} < \mu_f W_i \frac{C}{m} \left[ (1 + \mu_f) \frac{W_i}{P} \right]^{-\sigma}. \quad (\text{A.1})$$

By inserting  $W/P = (1 + \mu_f)^{-1}$ , the wage setting equation, and using the symmetry condition  $C = y$  for the quantity index,<sup>15</sup> the condition for a preference to shrink (A.1) simplifies to

$$\mu_f \left[ (1-s)^{1-\sigma} - 1 \right] - s < 0. \quad (\text{A.2})$$

The left-hand-side of (A.2) is zero for  $s = 0$ . For all  $s \in ]0,1]$ , however, it is increasing in  $s$ :

$$\frac{\partial}{\partial s} \left[ \mu_f \left[ (1-s)^{1-\sigma} - 1 \right] - s \right] = (1-s)^{-\sigma} - 1 \geq 0, \quad \forall s \in [0,1], \quad (\text{A.3})$$

i.e. (A.1) cannot hold. If all other firm-union-pairs shrink, any single firm would prefer to expand. There is no Nash equilibrium in which all firms strictly prefer to shrink.

If all firms pursue an expansion strategy, and their unions expect them to do so, the resulting equilibrium is the same as with a SMS: the aggregate price setting equation (PS) and the

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<sup>15</sup> If all firms are identical and set the same price, the quantity index in (4) simplifies to  $C = mC_i = y$ .

aggregate wage setting equation (WS) apply. Any single firm will strictly prefer expansion if condition (17) holds as a strict inequality

$$\mu_f (1-s) W_i \frac{C}{m} \left[ (1+\mu_f)(1-s) \frac{W_i}{P} \right]^{-\sigma} - s W_i \frac{y_0}{m} > \mu_f W_i \frac{C}{m} \left[ (1+\mu_f) \frac{W_i}{P} \right]^{-\sigma}. \quad (\text{A.4})$$

As all other firms expand, we can substitute in the (PS) condition on both sides. By further inserting  $y = 1 - [\mu_u / ((1+\mu_u)(1-b))] = y_0$ , derived from (WS), and using again the symmetry condition  $C = y$  for the quantity index, the condition (A.4) for a single firm to expand simplifies to

$$\mu_f [(1-s) - (1-s)^\sigma] - s > 0. \quad (\text{A.5})$$

At  $s = 0$ , the left-hand side of (A.2) is zero. For all  $s \in ]0,1[$ , condition (A.2) becomes smaller in  $s$  (using  $\mu_f = (\sigma - 1)^{-1}$ ):

$$\frac{\partial}{\partial s} [\mu_f [(1-s) - (1-s)^\sigma] - s] = (1+\mu_f) [(1-s)^{\sigma-1} - 1] \leq 0, \quad \forall s \in [0,1]. \quad (\text{A.6})$$

Condition (A.4) cannot hold. If all other firm-union-pairs expanded, an individual firm would prefer to shrink. There is no Nash equilibrium in which all firms strictly prefer to expand.

## Appendix 2: proof of Proposition 3

With  $s = 0$ , using L'Hôpital's rule, we have for condition (16):

$$\lim_{\substack{s \rightarrow 0 \\ s > 0}} y = \lim_{\substack{s \rightarrow 0 \\ s > 0}} \frac{y_0}{\underbrace{\mu_f}_{=1/(\sigma-1)} [\sigma(1-s)^{\sigma-1} - 1]} = y_0. \quad (\text{A.7})$$

For marginal changes in  $s$ , we find

$$\frac{\partial y}{\partial s} = \frac{y_0 \mu_f [(1-s) - (1-s)^\sigma] - s y_0 \mu_f [\sigma(1-s)^{\sigma-1} - 1]}{\mu_f^2 [(1-s) - (1-s)^\sigma]^2} \quad (\text{A.8})$$

Let the numerator of (A.8) be denoted by  $A$ . Since we have  $A|_{s=0} = 0$  and  $\partial A / \partial s = y_0 \mu_f s \sigma (\sigma - 1) (1-s)^{\sigma-2} > 0 \forall s \in ]0,1[$ , equilibrium employment is increasing in  $s$ .

### Appendix 3: proof of Proposition 4

We discussed in the main text that, for a given level of  $y$ ,  $C$  is maximized if all varieties are consumed at the same level. This is the case if all goods have the same prices. In this case, the quantity index simplifies to  $C = y$ . Since aggregate employment is restricted to  $[0,1]$ , it is clear that we must also have  $C \leq 1$ . Rewriting (15), inserting (7) and (14), yields

$$\frac{\tilde{W}}{P} = \left( \frac{sy_0}{\mu_f(1+\mu_f)^{-\sigma} C [(1-s)^{1-\sigma} - 1]} \right)^{-1/\sigma}, \quad (\text{A.9})$$

from which, for  $C \leq 1$ , we have

$$\lim_{s \rightarrow 1} \left[ (1-s) \frac{\tilde{W}}{P} \right] = \lim_{s \rightarrow 1} \left[ \left( \frac{sy_0}{\mu_f(1+\mu_f)^{-\sigma} C [(1-s) - (1-s)^\sigma]} \right)^{-1/\sigma} \right] = 0 \quad (\text{A.10})$$

Inserting in (17) with the explicit forms of  $P_i^+$  and  $P_i^-$  yields

$$1 = \lim_{s \rightarrow 1} \left[ \frac{m^+}{m} \underbrace{\left( (1+\mu_f)(1-s) \frac{\tilde{W}}{P} \right)^{1-\sigma}}_{\rightarrow \infty} \right] + \lim_{s \rightarrow 1} \left[ \frac{m - m^+}{m} \left( \frac{(1+\mu_f)(1+\mu_u) W^o}{(1-t) P} \right)^{1-\sigma} \right], \quad (\text{A.11})$$

which implies  $\lim_{s \rightarrow 1} m^+ = 0$ . Moreover, the second term on the right-hand-side of (A.11) must not be infinitely large. This requires that the term in round brackets must not be zero, i.e.

$$\lim_{s \rightarrow 1} \frac{W^o/P}{1-t} > 0. \quad (\text{A.12})$$

From (17), we know that

$$\lim_{s \rightarrow 1} \left\{ \left[ \frac{\tilde{W}}{P} - \frac{W^o/P}{(1-t)} \right]^\phi \underbrace{\left( (1+\mu_f)(1-s) \frac{\tilde{W}}{P} \right)^{-\sigma}}_{\rightarrow \infty} \right\} = \lim_{s \rightarrow 1} \left\{ (\mu_u)^\phi (1+\mu_f)^{-\sigma} (1+\mu_u)^{-\sigma} \left[ \frac{W^o/P}{(1-t)} \right]^{\phi-\sigma} \right\}. \quad (\text{A.13})$$

(A.12) then requires the right-hand-side of (A.13) to be finite. The left-hand-side of (A.13) can only be finite if

$$\lim_{s \rightarrow 1} \frac{\tilde{W}}{P} = \lim_{s \rightarrow 1} \frac{W^o/P}{(1-t)}. \quad (\text{A.14})$$

By combining (18) and (20), we obtain

$$\frac{W^o/P}{(1-t)} = \frac{m^+ y^+ \frac{[y+b(1-y)]}{y}}{1-(1+\mu_u)m^- y^- \frac{[y+b(1-y)]}{y}} \frac{\tilde{W}}{P}. \quad (\text{A.15})$$

For  $s \rightarrow 1$ , using (A.14), we obtain after some rearranging

$$1 = \left[ \frac{m^+ y^+}{y} + (1+\mu_u) \left( 1 - \frac{m^+ y^+}{y} \right) \right] [(1-b)y + b]. \quad (\text{A.16})$$

$\lim_{s \rightarrow 1} y = 0$  cannot constitute an equilibrium. By inserting  $y = 0$  into the second bracketed term in (A.16), this condition reduces to  $1 - (1+\mu_u)b = -\mu_u b (m^+ y^+) / y \leq 0$ . Substituting into the (WS) condition shows that this requires  $y_0 \leq 0$ , which is incompatible with a positive employment rate in the initial equilibrium.

Other solutions of (A.16) can be found for  $0 < y < 1, m^+ y^+ < y$ . These solutions cannot describe a general equilibrium for  $s \rightarrow 1$  either. Multiplying both sides of (21) by  $C$  gives

$$\begin{aligned} C &= m^+ \frac{C}{m} \left( (1+\mu_f)(1-s) \frac{\tilde{W}}{P} \right)^{1-\sigma} + (m-m^+) \frac{C}{m} \left( \frac{(1+\mu_f)(1+\mu_u) W^o}{(1-t) P} \right)^{1-\sigma} \\ &= m^+ y^+ \underbrace{\left( (1+\mu_f)(1-s) \frac{\tilde{W}}{P} \right)}_{\rightarrow 0} + \underbrace{m^- y^-}_{> 0} \underbrace{\left( \frac{(1+\mu_f)(1+\mu_u) W^o}{(1-t) P} \right)}_{> 0} \end{aligned} \quad (\text{A.17})$$

Equation (A.17) requires  $\lim_{s \rightarrow 1} C > 0$ ., which would give  $\lim_{s \rightarrow 1} \tilde{W}/P \rightarrow \infty$  (from A.9) and  $\lim_{s \rightarrow 1} (W^o/P)/(1-t) \rightarrow \infty$  (from A.14). From (A.17), it then follows that  $\lim_{s \rightarrow 1} C \rightarrow \infty$ , which, as we stated in the beginning of this proof, is impossible.

The only valid solution of (A.16) left is  $m^+ y^+ = y = 1$ . For the case  $\lim_{s \rightarrow 1} (W^o/P)/(1-t) = \text{const.}$ , we use the second line of (A.17) to obtain

$$\lim_{s \rightarrow 1} C = \lim_{s \rightarrow 1} \left[ m^+ y^+ \underbrace{\left( (1+\mu_f)(1-s) \frac{\tilde{W}}{P} \right)}_{\rightarrow 0} + \underbrace{m^- y^-}_{\rightarrow 0} \underbrace{\left( \frac{(1+\mu_f)(1+\mu_u) W^o}{(1-t) P} \right)}_{> 0} \right] = 0. \quad (\text{A.18})$$

For  $\lim_{s \rightarrow 1} (W^o / P) / (1-t) \rightarrow \infty$ , we make use of the first line of (A.17):

$$\lim_{s \rightarrow 1} C = \lim_{s \rightarrow 1} \left[ \underbrace{m^+ y^+ \left( (1 + \mu_f)(1-s) \frac{\tilde{W}}{P} \right)}_{\rightarrow 0} + \left( \underbrace{\frac{m^- \left( \frac{(1 + \mu_f)(1 + \mu_u) W^o}{P} \right)^{1-\sigma}}{(1-t) P}}_{\rightarrow 0} \right)^{\frac{m}{m-1}} \right] C. \quad (\text{A.19})$$

Equation (A.19) is only fulfilled if  $\lim_{s \rightarrow 1} C = 0$ . This concludes the proof.

#### Appendix 4: proof of Proposition 5

A Case A-equilibrium exists if all unions prefer to expand after a marginal introduction of the subsidy. This requires (cf. condition (17)):

$$\left[ \frac{\tilde{W}}{P} - \frac{W^o}{(1-t)P} \right]^\phi \left( (1 + \mu_f)(1-s) \frac{\tilde{W}}{P} \right)^{-\sigma} > \left[ \mu_u \frac{W^o}{(1-t)P} \right]^\phi \left( (1 + \mu_f)(1 + \mu_u) \frac{W^o}{(1-t)P} \right)^{-\sigma}. \quad (\text{A.20})$$

By inserting the equilibrium outcomes  $\frac{1}{(1-t)} \frac{W^o}{P} = [y + b(1-y)] \frac{\tilde{W}}{P}$  and  $\frac{\tilde{W}}{P} = \frac{1}{(1 + \mu_f)(1-s)}$ ,

(A.20) simplifies to

$$\underbrace{\left[ (1-b)(1-y) \right]^\phi - \left[ (1-b)y + b \right]^{\phi-\sigma} (\mu_u)^\phi (1-s)^\sigma (1 + \mu_u)^{-\sigma}}_{\equiv A} > 0. \quad (\text{A.21})$$

For  $s = 0$ , inserting (WS) into (A.21) gives  $A|_{s=0} = 0$ . Introducing AMS at the margin at  $s = 0$  then yields

$$\frac{\partial A}{\partial s} \Big|_{s=0} = \sigma + \frac{dy}{ds} \Big|_{s=0} (1-b) \underbrace{\left[ (\sigma - \phi)(1 + \mu_u) - \phi \left( \frac{1 + \mu_u}{\mu_u} \right) \right]}_{=0} > 0, \quad (\text{A.21})$$

which implies that (A.20) is always fulfilled, i.e. in the equilibrium arising from a marginal introduction of AMS, unions will always prefer expansion.

#### Appendix 5: proof of Proposition 6

We start with Case A. We first show that new, unsubsidized firms always make less profit than incumbent firms. The profit of a new firm that does not receive any subsidy is given by (using  $W_i/P = (1 + \mu_u)(1-t)^{-1}(W^o/P)$  and equation (9)):



$$\frac{\Pi^{new}}{P} = \mu_f \frac{W_i}{P} \frac{C}{m} \left[ (1 + \mu_f) \frac{W_i}{P} \right]^{-\sigma} = \mu_f \frac{C}{m} (1 + \mu_f)^{-\sigma} (1 + \mu_u)^{1-\sigma} [(1-b)y + b]^{1-\sigma} \left( \frac{\tilde{W}}{P} \right)^{1-\sigma}. \quad (\text{A.23})$$

An incumbent firm's profit is given by

$$\frac{\Pi^{inc}}{P} = \mu_f (1-s) \frac{\tilde{W}}{P} \frac{C}{m} \left[ (1 + \mu_f)(1-s) \frac{\tilde{W}}{P} \right]^{-\sigma} - \frac{\tilde{W}}{P} \frac{sy_0}{m_0}, \quad (\text{A.24})$$

where  $m_0$  is the initial number of firms. Inserting (A.7) and comparing (A.23) and (A.24) at  $m = m_0$  then gives

$$\frac{\Pi^{inc}}{P} > \frac{\Pi^{new}}{P} \Leftrightarrow (1 + \mu_u)[(1-b)y + b] > 1. \quad (\text{A.25})$$

Since  $(1 + \mu_u)[(1-b)y + b] = 1$  at  $y = y_0$  (where  $y_0$  is defined by the WS condition), (A.25) is fulfilled for all  $y > y_0$ : new firms always make less profits than incumbent firms.

Next, we determine how an incumbent firm's profit depends on  $s$ , given that no new firm has yet entered. Inserting (19) into (A.24), and applying  $\left[ \frac{(1-s)^{1-\sigma} - 1}{(1-s) - (1-s)^\sigma} \right]^{1/\sigma} = \frac{1}{1-s}$  to

simplify the resulting expression, gives

$$\frac{\Pi^{inc}}{P} = \left( \frac{s}{(1-s)^{2-\sigma} - (1-s)} \right) (1 + \mu_f)^{-1} \frac{y_0}{m}. \quad (\text{A.26})$$

Differentiating with respect to  $s$  shows that

$$\frac{d(\Pi^{inc}/P)}{ds} = \frac{(1-s)^{2-\sigma} - (1-s) - s[1 - (2-\sigma)(1-s)^{1-\sigma}]}{[(1-s)^{2-\sigma} - (1-s)]^2} (1 + \mu_f)^{-1} \frac{y_0}{m}. \quad (\text{A.27})$$

As the second derivative yields  $d^2(\Pi^{inc}/P)/ds^2 = (1-\sigma)(2-\sigma)(1-s)^{-\sigma}$ , we have

$$\frac{d(\Pi^{inc}/P)}{ds} \begin{cases} > \\ = \\ < \end{cases} 0 \Leftrightarrow (1-s)^{2-\sigma} + (2-\sigma)s(1-s)^{1-\sigma} \begin{cases} > \\ = \\ < \end{cases} 1 \Leftrightarrow \sigma \begin{cases} 1 < \sigma < 2 \\ \sigma = 2 \\ \sigma > 2 \end{cases}. \quad (\text{A.28})$$

Incumbent firms make zero profits if  $s = 0$ , so that they make (weakly) negative profits for  $s > 0$  and  $\sigma \geq 2$ . As new firms make less profits, they will not enter the market.

For case B, we first show that new firms would always make negative profits if  $\sigma \geq 2$ . By inserting (A.7) into (A.24), we can rewrite the profit of an expanding incumbent firm as

$$\frac{\Pi^{inc}}{P} = C^{1/\sigma} \left[ \frac{\mu_f (1 + \mu_f)^{-\sigma} [(1-s)^{1-\sigma} - 1]}{s y_0} \right]^{1/\sigma} \frac{y_0}{m} \left[ \frac{(1-s)^{1-\sigma}}{(1-s)^{1-\sigma} - 1} - \frac{m}{m_0} \right]. \quad (\text{A.29})$$

We know that Case B is characterized by a consumption distortion, so that  $C$  is less than in Case A, *ceteris paribus*. (A.29) shows that  $\partial \Pi^{inc} / \partial C > 0$ , such that profits in Case B will be even less than they would have been if Case A had prevailed at the same  $s$ . Hence new firms never want to enter if  $\sigma \geq 2$ .

### Appendix 6: proof of Proposition 7

We distinguish two cases. In the first case, unions are indifferent between expanding and shrinking. Equation (24) reads  $\Omega^+ = \Omega^-$  and  $m^+ \leq m_0$ . In this case, the system of equations is solved exactly like in the short run. Hence, equations (15) and (17)-(21) solve the equilibrium values of  $y$ ,  $C$ ,  $\bar{W}/P$ ,  $m^+/m$ ,  $\tilde{W}/P$  and  $W^o/P$ . Equation (23) determines  $m$ , but has no influence on the other equilibrium outcomes. Thus,  $y$  and  $C$  retain their short-run values. If  $C$  is constant, but  $m$  increases, it follows from (4) that welfare ( $\sum_i V_i$ ) has to increase.

In the second case, all incumbent unions prefer expansion, i.e.  $\Omega^+ > \Omega^-$  and  $m^+ = m_0$ . Rewriting (19) with the explicit forms of  $y_i^+$  and  $y_i^-$  gives

$$y = \frac{m_0}{m} C \left( \frac{P_i^+}{P} \right)^{-\sigma} + \frac{m - m_0}{m} C \left( \frac{P_i^-}{P} \right)^{-\sigma}. \quad (\text{A.30})$$

For given short-run equilibrium values of  $y$  and  $C$ , an increase in  $m$  decreases the RHS of (A.30) because  $P_i^+ < P_i^-$ .  $C$  is restricted by  $C \leq y$ , such that an increase in  $C$  cannot equilibrate (A.30). Since  $dP_i^- / dy > 0$  (via  $y$ 's impact on the outside option),  $y$  has to fall to restore the equality. Hence, aggregate employment will be less in the short-run than in the long-run.

To compare the long-run effect with the initial equilibrium, we combine (18) and (20), and insert  $m^+ = m_0$ ,  $m^- = m - m_0$ , and the explicit forms of  $y_i^+$  and  $y_i^-$ , to obtain

$$\begin{aligned} \frac{W^o/P}{(1-t)} - [y + b(1-y)] \frac{C}{y} \left( 1 - \frac{m_0}{m} \right) (1 + \mu_f)^{-\sigma} (1 + \mu_u)^{1-\sigma} \left( \frac{W^o/P}{(1-t)} \right)^{1-\sigma} \\ = [y + b(1-y)] \frac{m_0}{m} \frac{C}{y} (1 + \mu_f)^{-\sigma} (1-s)^{-\sigma} \left( \frac{\tilde{W}}{P} \right)^{1-\sigma}. \end{aligned} \quad (\text{A.31})$$

We know from (A.14) and (A.10) that  $\partial(\tilde{W}/P)/\partial s > 0$ . Also,

$$(1-s)^{-\sigma} \left( \frac{\tilde{W}}{P} \right)^{-\sigma} = \frac{y_0}{\mu_f (1 + \mu_f)^{-\sigma} C} \frac{s}{(1-s) - (1-s)^\sigma}, \quad (\text{A.32})$$

with

$$\frac{\partial}{\partial s} \frac{s}{(1-s) - (1-s)^\sigma} = \frac{(1-s) - (1-s)^\sigma + s[1 - \sigma(1-s)^{\sigma-1}]}{[(1-s) - (1-s)^\sigma]^2}. \quad (\text{A.33})$$

Denoting the numerator of (A.33) by  $A$ , we obtain  $A|_{s=0} = 0$  and  $\partial A / \partial s = s\sigma(\sigma-1)(1-s)^{\sigma-2} > 0 \forall s \in ]0,1[$ . Hence, (A.32) is increasing in  $s$ . It follows that, for any equilibrium values of  $C$  and  $y$ , increasing  $s$  increases the right-hand side of (A.32), which requires an increase in  $(W^o/P)/(1-t)$  to balance the equation. From (23), it follows that  $C$  has to increase. The entry of new firms raises  $m$ , which strengthens the necessary increase in  $C$ . At  $s = 0$ , we have  $C = y = y_0$ , so that increasing  $s$  raises welfare and employment above their initial levels.

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