Supply chain coordination with information sharing in the presence of trust and trustworthiness: a behavioral model from

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Abstract
The strategic use of private information causes efficiency losses in traditional principal-agent settings. One stream of research states that these efficiency losses cannot be overcome if all agents use their private information strategically. Yet, another stream of research highlights the importance of communication, trust and trustworthiness in supply chain management. The underlying work links the concepts of communication, trust and trustworthiness to a traditional principal-agent setting in a supply chain environment. Surprisingly, it can be shown that communication and trust can actually lead to increasing efficiency losses although there is a substantial level of trustworthiness.
1 Introduction

Previous research on supply chain coordination shows that a supply chain optimal solution (i.e. the solution that maximizes the sum of all net present values of the supply chain members) is typically not achieved as long as the incentives of the independent companies within the supply chain are not aligned (see e.g. Cachon, 2003). The research field of supply chain coordination identifies the supply chain optimal solution (e.g. Kelle and Akbulut, 2005) as well as coordination instruments (e.g. contracts) that align the incentives of the supply chain members (see e.g. Cachon, 2003).

Earlier work on supply chain coordination demonstrates that asymmetric information are a major source of inefficiencies within the supply chain, because the strategic use of private information (e.g. private cost or demand information) can lead to inefficient allocations\footnote{Obviously, private information does not harm supply chain performance, as long as the private information is not decision relevant.}, even though sophisticated menu of contracts (i.e. screening contracts) are utilized (see Corbett and de Groote, 2000, Ha, 2000, Corbett, 2001, Corbett and Tang, 2003, Corbett et al., 2004, Sucky, 2004).

The basic idea behind this menu of contracts stems from the revelation principle (see Myerson, 1986). The specific incentive structure of these contracts ensures that the holder of the private information does reveal his information by his contract choice. However, as the contract choice determines the respective supply chain performance, the information revealed by the contract choice cannot be utilized to design supply chain optimal contracts.

Another stream of research highlights that successful supply chain management and coordination requires communication (e.g. Cachon and Fisher, 2000) and trust (Moore, 1998 and Zaheer et al., 1998). However, in these studies the incentives for communication and trust are not explicitly modeled. The goal of this paper is therefore to investigate, under which circumstances regarding trust and trustworthiness communication is an appropriate coordination instrument when screening contracts are used.

![Figure 1: Supply chain configuration](image)
The model utilized in the underlying study captures a supply chain lotsizing decision under deterministic demand. In a standard principal-agent setting an agent (buyer) holds private information about her respective holding cost. The principal (supplier) tries to induce higher order sizes while compensating the buyer for her holding cost increase with a discount on the purchase price. However, the buyer’s cost increase depends on her holding costs, which are private information. It is assumed that communication takes place via a signal (e.g. verbal or written). Standard-theory assumes that the buyer claims a very high compensation (e.g. quantity discount) for agreeing upon higher order sizes. As the supplier anticipates this behavior, he will simply ignore the buyer’s signals. The buyer, in turn, anticipates that her signal has no impact on her profits. Therefore, even the randomization of signals is an equilibrium strategy. Thus, communication is regarded as cheap-talk and the supply chain members are caught in a “babbling-equilibrium”. In this case, the supplier’s best action is to offer a menu of contracts, which leads to inefficient allocations.

However, there is a vast amount of experimental research that shows that cheap talk can influence the behavior of a decision maker. It is referred to Crawford (1998) for a comprehensive review of these cheap talk experiments. Particularly, the underlying study is motivated by the experimental finding from Inderfurth et al. (2008) as well as Özer et al. (2008) that not all agents use their information entirely strategically. Their findings indicate that there are different types of agents, who can be divided into two subclasses.

One group includes those agents who always communicate their private information truthfully. One explanation for this is, for example, that the honest agent faces intrinsic costs of lying (see Minkler and Miceli, 2004). In the following, these agents are denoted as ‘honest agents’.

On the other hand there are the agents who misrepresent their private information at least sometimes. The behavior of these “deceptive agents” may have different explanations. In the first part of the analysis it is assumed that the deceptive agent does not consider that the principal might take the signals into account while offering the contracts. In this case the ‘deceptive agent’ is assumed to give signals which do not depend on the private information, e.g. she unconditionally randomizes the signal. This is the well-known ‘cheap-talk’ hypothesis. In the second part of the analysis we assume that the deceptive agent follows a strategy which is conditioned on her specific private information.

The remainder of the study is organized as follows: section 2 gives a brief literature review on the concepts trust, trustworthiness and communication. Section 3 gives an introduction to the
screening-model, which is used as a starting point of the underlying study. Section 4 expands the scope of the basic model to the possibility of communication. Furthermore, section 4 shows that the supplier can update his expectations on the signals sent by the buyer. Section 5 and 6 evaluate the impact of this adjustment of beliefs on the buyer’s, supplier’s and overall supply chain’s performance under the unconditional as well as the conditional signaling assumption, respectively. Finally, Section 7 summarizes the results and gives an outlook for further research. Please note that the notation is summarized in appendix 9.2.

2 Literature Review

Honest and deceptive agents:  
One of the main assumptions of this study is the division of agents into two subclasses. This assumption has already been made in a similar principal-agent framework from Severinov and Deneckere (2006). In this study, it is assumed that one subclass of agents is fully rational and opportunistic. They claim, for example, the highest compensation regardless of their true holding cost parameter. On the other hand there is a second subclass of agents who will always communicate their true holding cost, even though they are aware of loosing money by doing this.

Severinov and Deneckere (2006) utilize the fact, that the deceptive agents can be detected, as they will give the same signal, e.g. they will always claim the highest possible compensation. On the other hand, the “honest agents” can be easily identified by a deviation from this behavior. Every agent who does not claim the highest compensation is therefore identified as an honest agent. Severinov and Deneckere (2006) propose a “password”-mechanism, where the password is the signal, which are the deceptive agents supposed to give. Thus, the agent who knows the password (i.e. the deceptive agent) is offered a more favorable contract than the honest agent, who does not know the password. However, previous research from Özer et al. (2008) and Inderfurth et al. (2008) show that even this division in two subgroups may not be sufficient.

Özer et al. (2008) investigate whether information sharing enhances supply chain performance in a supplier-manufacturer supply chain with uncertain end-customer demand. A simple wholesale price contract determines the financial payments in the relationship. In this study, the supplier’s capacity reservation for the manufacturer relies on a demand forecast. However, as the manufacturer is closer to the market, she has more accurate forecast information than the supplier. Under these circumstances, the supply chain optimal solution is
achieved, if the manufacturer reports the demand forecast truthfully. Yet, rational game theory predicts that the manufacturer exaggerates the demand forecast to influence the supplier’s capacity reservation decision. In turn, the supplier treats the manufacturer’s information about the demand forecast as cheap talk.

Interestingly, Özer et al. show in their experiment that the manufacturers inflate the superior forecast information indeed, but they do not exaggerate to the maximum extent. Particularly, the report does linearly depend on the private forecast information. The supplier, in turn, does not treat the report as cheap talk but conditions his capacity decision on the report instead. Özer et al., thus, find partially trust and trustworthiness in their supply chain setting. This leads to a higher supply chain performance than theoretically predicted.

Another experimental investigation that analyses the impact of information sharing on supply chain performance was conducted by Inderfurth et al. (2008). They investigate a stylized joint economic lot sizing model in a supplier-buyer relationship. In this relationship the buyer holds private information about her holding cost. The supplier tries to induce higher order sizes while compensating the buyer for her holding cost increase. In comparison to Özer et al., they investigate a screening contract instead of the wholesale price contract. They show that buyers either tend to exaggerate their holding costs or to report them truthfully. However, they also find that a non negligible portion of reports that understated the respective holding cost. Additionally, only 1 out of 24 buyers was claiming the highest compensation throughout the experiment. Finally, they find an ambiguous effect of communication on the overall supply chain performance. Particularly, the supply chains which manage to build up trust and trustworthiness performed significantly better than the supply chains in which deception and strategic interpretation of reports were prevalent.

Taking these behavioral findings into account, the password-mechanism from Severinov and Deneckere is not applicable as the deceptive agents do not constantly give the same signal (i.e. always exaggerating to the maximum extent). Hence, the underlying study does not assume that the deceptive agents are completely strategic. In fact, the deceptive agents are characterized by signals which are not constantly truthful.

Summarizing the above arguments, the underlying study proposes a behavioral model that evaluates the impact of trust and communication in a standard principal-agent setting where the deceptive agents cannot be easily identified by their signals.
Other studies that incorporate the idea of honest and deceptive agents can be found in Alger and Renault (2005, 2006). However, in contrast to the underlying study there is no direct communication between the principal and the agent.

**Communication:**
In Supply Chain Management, information sharing is regarded as one of the main drivers to improve or even optimize the overall supply chain performance. Chen (2003) gives a comprehensive review on the potential gains from upstream and downstream sharing of information such as demand, cost or capacity information. Furthermore, he describes the screening mechanism, which is utilized in the underlying study, as an enabler to align the incentives for information sharing. Mohr and Spekman (1994) find that there are basically three dimensions of communication behavior that are important to determine the effectiveness of communication in a relationship. One dimension includes all aspects which refer to the quality of the conveyed information such as adequacy, accuracy or timeliness. Another dimension includes the degree of which the communication is “bilateral”, i.e. to what extent both parties within the relationship consider the shared information in joint decision making. Finally, one dimension includes the frequency and extent of information sharing. Mohr and Spekman find that all these dimensions are critical levers to enhance successful partnerships.

In the underlying study it is assumed that all communication fulfills certain quality standards. This is ensured by restricting the buyer’s signal space to only relevant information. Furthermore, we assume a strict sequence of events. This ensures the timeliness of communication. Additionally, it is assumed that the supply chain members interact only once. Hence, the frequency and extent of information sharing is limited by assumption.

**Trust and trustworthiness:**
There is a vast amount of definitions regarding the concept “trust”.\(^2\) However, the way the concept ‘trust’ is considered in the underlying study is rather crude and the determinants of trust are not explicitly considered.

\(^2\) Castaldo (2007) identified 72 definitions. Four dimensions of trust were mentioned in most of the definitions, namely expectation, willingness, confidence and attitude. Sako and Helper [1998] states that trust “is an expectation held by an agent that its trading partner will behave in a mutually acceptable manner”. In this case, the trustor’s expectation reduces the perceived uncertainty about the trustee’s actions and in turn increases the predictability of these actions. Zand (1972) highlights that an important dimension of trust is the willingness to accept the vulnerability associated with deviations from expected actions. Morgan and Hunt (1994) underline that confidence in the exchange partner’s reliability and integrity is an important aspect of trust. Finally, Ben-Ner and Putterman (2001) argue that trust can be interpreted as an attitude towards taking risky decisions.
Obviously, the principal would like to trust the honest agent and distrust the deceptive agents. The principal, however, does not know whether the signal was sent from an honest or deceptive agent. Trust in this environment is interpreted as the principal’s perceived probability that he is interacting with an honest agent. This interpretation of trust is closest to the definition of Gambetta (1988). Gambetta states that trust is the principal’s subjective probability that an agent acts in a specific way.

Mui and Halberstadt (2002) point out the differences between reputation and trust. Reputation, thus, is a concept that focuses on previous actions whereas trust focuses on future actions. Reputation can affect the level of trust in the relationship. However, they argue that trust can be prevalent even in non-recurring actions. This fact was also highlighted by Eckel and Wilson (2004). They state that the principal’s level of trust is influenced by previous interactions that are similar in nature, even though there have been no interaction with the respective agent. As mentioned before, we only consider a non-recurring interaction between a supplier and buyer. Hence, reputational effects cannot affect the supplier’s beliefs, although it is assumed that the supplier’s have an initial level of trust (which is determined, for example, by experiences from prior interactions with other agents).

Mayer et al. (1995) highlight the difference between trust and trustworthiness. They identify three characteristics that affect the trustworthiness of a trustee. One determinant is the perception that a person is able to perform a specific task, e.g. because the person is very competent in a specific area. Another identified determinant is benevolence, which describes the trustee’s intention to interact without exploitation, even though exploitation is possible. Finally, the trustee’s integrity is a main determinant of trustworthiness. Integrity describes the trustor’s perception that the interactions are based on a set of principles which are acceptable for the trustor. In the underlying study, however, it is simply stated that the honest agent is trustworthy (i.e. she signals her true holding cost realization), whereas the deceptive agent is not.

3 Outline of the model

This paper analyses a dyadic lot-sizing decision between a supplier and a buyer (see figure 1). It is assumed that the buyer’s demand is constant over time and, without loss of generality, standardized to one unit per period. Hence, the period costs equal unit costs. The buyer orders
$q$ units per order cycle and pays a wholesale price of $w$ per every delivered unit. The transfer payment per order between buyer and supplier, thus, results from $w \cdot q$.

The supplier faces several disadvantages from low order sizes, e.g. low capacity utilization and high tied up capital (see Schonberger and Schniederjans, 1984). These disadvantages are captured by the supplier’s fixed cost per period. Let $f$ denote the supplier’s fixed cost that occur for every delivery to the buyer. The fixed costs per unit results from $\frac{f}{q}$. The buyer faces holding cost of $h$ for every unit stored in a period. Thus, the buyer faces holding costs of $\frac{h}{2} \cdot q$ per period that are linearly dependent on the order size $q$. This situation captures a basic conflict of interest in supply chain management, namely that the buyer prefers low order sizes while the supplier prefers high order sizes.

It is assumed that the buyer can choose an alternative supplier who delivers the same items for costs of $R$ per unit. The supplier, thus, cannot dictate an arbitrarily high order size to the buyer as she can simply choose the outside option. However, the supplier can lower his wholesale price $w$ if the buyer agrees upon a higher order size $q$. In this case he can compensate the buyer’s holding cost increase with a lower wholesale price $w$. The supplier, thus, offers a contract which consists of an order size $q$ and a respective wholesale price $w$.

This contract offer $(w, q)$ must ensure that the buyer will not choose the alternative supplier.

Under these assumptions, the supplier yields profits of $P_s = w - \frac{f}{q}$ and the buyer faces cost of $C_b = w + \frac{h}{2} \cdot q$. The supplier with full information (FI), i.e. the supplier who knows the buyer’s costs of the outside option ($R$) and the buyers’ respective holding cost per unit ($h$) will maximize his profits by solving the following linearly constrained convex optimization problem:

**Problem FI:**

$$\max_{q, w} P_s = w - \frac{f}{q}$$

s.t.

$$C_b = w + \frac{h}{2} \cdot q \leq R \quad \text{(participation constraint)}$$

$$q \geq q_{\min}$$
Let $q_{\text{min}}$ denote the supplier’s minimum order size for which he yields profits. It can be shown, that the condition $q > q_{\text{min}}$ does not bind, as long as the costs of the outside option are sufficiently large, i.e. as long as $R \geq \frac{f}{q_{\text{min}}} + \frac{h}{2} q_{\text{min}}$.

The participation constraint in problem FI ensures that the offer $\langle w, q \rangle$ made by the supplier will result in lower or equal costs per unit for the buyer than sourcing from the outside option. It can be easily shown that this participation constraint binds in the optimal solution. Hence, the buyer is indifferent between the offer $\langle w, q \rangle$ and the outside option. The supplier’s optimization problem therefore reduces to $\max_q P_s = R - \frac{h}{2} q - \frac{f}{q}$. The supplier accounts obviously for only the relevant supply chain costs. Hence, the following contract offer is supply chain optimal.

$$\langle w_{\text{SC}}^*, q_{\text{SC}}^*, q_{\text{SC}}^* \rangle = \left( R - \frac{h}{2}, \frac{2 \cdot f}{h} \right)$$

The assumption that the supplier has full information about the buyer’s holding cost is certainly a critical assumption. For this reason it is assumed that the supplier does not know the buyer’s holding costs per period ($h$) with certainty. In turn, the buyer has only an estimation over possible holding cost realizations and the respective probabilities with which these holding costs per period occur. This estimation is formalized with a discrete probability distribution $p_i, i = 1, \ldots, n$ over all possible holding cost realizations $h_i, i = 1, \ldots, n$. This estimation is common knowledge.

Offering a menu of contracts $\langle w_i, q_i \rangle, i = 1, \ldots, n$ is the supplier’s profit maximizing action under asymmetric information (AI). The menu of contracts (i.e. screening contract) results from the following linearly constrained convex optimization problem.

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3 Corbett and de Groote (2000) prove this for the case of a continuous probability distribution.
Problem AI

\[
\max \ E(P_s) = \sum_{i=1}^{n} p_i \left( w_i - \frac{f}{q_i} \right) \\
\text{s.t.} \qquad \begin{align*}
w_i + \frac{h_i}{2} q_i & \leq R, \quad \forall i = 1, \ldots, n \quad \text{(participation constraint)} \\
w_i + \frac{h_i}{2} q_i & \leq w_j + \frac{h_j}{2} q_j, \quad \forall i \neq j ; i, j = 1, \ldots, n \quad \text{(incentive constraint)} \\
q_i & \geq q_{\min}, \quad \forall i = 1, \ldots, n
\end{align*}
\]

Again, it is assumed that \( R \geq \frac{f}{q_{\min}} + \frac{h_i}{2} q_{\min} \) holds. The participation constraint ensures that the buyer will not choose the alternative supplier, regardless of her holding cost per period. The incentive constraint ensures that the buyer facing holding cost \( h_i \) chooses the contract \( \{w_i, q_i\} \) as this contract choice causes lower costs per unit than any other contract \( \{w_j, q_j\}, \forall j \neq i \). Hence, the supplier can infer the buyer’s holding cost from her contract choice. However, this is only an ex-post revelation of information, as this information cannot be used in further interactions. For notational convenience it is assumed that \( p_0 = h_0 = 0 \). The optimal menu of contracts \( \{w_i^*, q_i^*\} \) results from (see Inderfurth and Voigt, 2008):

\[
q_i^* = \sqrt{\frac{2 \cdot f}{h_i + \gamma_i}}, \quad \forall i = 1, \ldots, n \quad \text{where} \quad \gamma_i = \frac{\sum_{t=0}^{i-1} p_t}{p_i} (h_i - h_{i-1})
\]

\[
w_n^* = R - \frac{h_n}{2} q_n^*
\]

\[
w_i^* = \frac{h_i}{2} (q_{i+1}^* - q_i^*) + w_{i+1}^* \quad \forall \ i = 1, \ldots, n - 1
\]

It was already mentioned in the introduction that only situations are considered in which a unique assignment from contract choices to the respective holding cost is possible. Hence, we disregard all situations in which \( q_i^* = q_{i+1}^* \) holds in the optimal menu of contracts. This case is ruled out by the assumption that

\[
h_i + \frac{\sum_{t=0}^{i-1} p_t}{p_i} (h_i - h_{i-1}) < h_j + \frac{\sum_{t=0}^{j-1} p_t}{p_j} (h_j - h_{j-1}) \quad \forall \ i = 1, \ldots, n - 1; j = 2, \ldots, n; j > i
\]
Let \( q_{i,SC}^*, \forall i = 1,\ldots, n \) denote the supply chain optimal order size which refers to the holding costs \( h_i \). Under these conditions, the optimal menu of contracts \( \{w_i^*, q_i^*\}, \forall i = 1,\ldots, n \) has the following properties (see Sappington, 1983).

1. \( q_{i-1}^* > q_i^* \) and \( w_{i-1}^* < w_i^* \)

2. the participation constraint for the buyer facing holding costs \( h_n \) binds

3. the buyer facing holding costs \( h_i \) (\( i = 1,\ldots, n-1 \)) is indifferent between the contracts \( (w_i^*, q_i^*) \) and \( (w_{i+1}^*, q_{i+1}^*) \), \( \forall i = 1,\ldots, n-1 \)

4. the buyer facing holding cost \( h_i \) (\( i = 1,\ldots, n-1 \)) does have lower cost than her reservation cost \( R \)

5. \( q_i^* < q_{i,SC}^*, \forall i = 2,\ldots, n \) and \( q_1^* = q_{1,SC}^* \)

Condition (1) shows that the wholesale price decreases with increasing order sizes. Hence, the menu of contracts can be interpreted as a quantity discount. Conditions (2) and (3) point out that a buyer is always indifferent between two alternatives. It is assumed that the indifferent buyer chooses the contract which is in the supplier’s best interest. Only this assumption ensures the ex-post revelation of information. Please refer to Inderfurth et al. (2008) for a broader discussion of this assumption and some empirical evidence that this indifference modeling approach can seriously harm the supply chain performance. Condition (4) states that the buyer earns an information rent for her private information. As long as the buyer does not have the highest possible holding cost realization, \( h_n \), she faces lower unit cost than if she would choose the alternative supplier. Hence, the fully rational and strategic buyer has no interest to communicate her holding cost parameter truthfully. Otherwise, the supplier could solve problem FI with the communicated holding cost realization. This would, again, result in the buyer’s reservation cost \( R \). Finally, the probably most important condition (5) states that the resulting menu of contracts is not supply chain optimal, except for the holding cost realization \( h_i \). A closer look at the contract parameters under full and under asymmetric information reveals that \( \gamma_i \) makes the difference compared to the supply chain optimum. As

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4 In the continuous formulation of this problem this situation is typically ruled out by assuming that the probability distribution has a monotone hazard rate. If this assumption does not hold, there is a so called “bunching”, i.e. two or more holding cost realizations refer to only one contract \( \{w_j, q_j\} \). The optimality conditions for this special case can be find in Spence (1980).
\( \gamma_i \geq 0, \forall i = 1, \ldots, n \), it is obvious that the order sizes are too low compared to the supply chain optimum, i.e. there is a downward distortion of order sizes.\(^5\) Standard-theory predicts that this coordination deficit (i.e. the performance deficit that results from a downward distortion of order sizes) cannot be overcome as long as all supply chain members act individually rational and opportunistic. Particularly, this expected coordination deficit, \( CD \), can be expressed as follows:

\[
CD = \sum_{i=1}^{n} p_i \left( \frac{f}{q_i \ast} + \frac{h_i}{2} \cdot q_i \ast - \frac{f}{q_{i,SC} \ast} - \frac{h_i}{2} \cdot q_{i,SC} \ast \right)
\]

The main objective of the underlying analysis, thus, is to investigate whether communication and trust (as defined in section 2) can enhance supply chain performance or leads to a deterioration of the overall supply chain performance.

4 The impact of communication and trust on supply chain performance

In the following it is assumed that the supplier receives a signal from the buyer before he offers the menu of contract. The following sequence of events results: the buyer first learns her holding cost \( \tilde{h} \in [h_1, \ldots, h_n] \). Then, the buyer communicates a holding cost to the supplier. The buyer is restricted to signals \( H \) that are possible holding cost realizations or she can refuse to give a signal, i.e. \( H \in (H_1 = h_1, \ldots, H_i = h_i, \ldots, H_n = h_n, H_{n+1} = " \text{No signal}" ) \). Then, the supplier decides to adjust the a-priori probabilities \( p_i \) to the perceived a-posteriori probability distribution \( \hat{p}_i(h_i | H_i) (\forall i = 1, \ldots, n; k = 1, \ldots, n + 1) \), which is conditioned on the buyer’s signal. Then, the supplier calculates the menu of contracts with respect to the perceived a-posteriori distribution and offers this screening contract to the buyer. Finally, the buyer chooses one contract out of the menu of contracts. The following figure 2 depicts the sequence of events.

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\(^5\) It is referred to Inderfurth and Voigt (2008) for a broader discussion of the distortion caused by asymmetric information.
Next, we analyze the key factors that influence the buyer’s signaling behavior ($t = 1$) as well as the supplier’s adjustment of beliefs ($t = 2$), and the buyer’s contract choice ($t = 3$).

### 4.1 The buyer’s signaling behavior

**Truthful signals**

As mentioned in the introduction, it is assumed that some buyers report their holding costs truthfully, i.e. $H = \tilde{h}$. It is assumed that a fraction $\alpha \in [0,1]$ of the buyers show this behavior. Therefore, the probability that the supplier interacts with an honest buyer is $\alpha$.

**Unconditioned signals**

All buyers that do not report their holding cost truthfully are called ‘deceptive’ buyers. As a fraction $\alpha$ of the buyers are honest, it follows directly that a fraction $(1 - \alpha)$ of the buyers are deceptive. As mentioned in the introduction, we analyze two different types of signaling behavior. On the one hand, the buyer is assumed to simply ignore that the supplier processes the communicated signal. In this case, the buyer gives signals regardless of her holding cost learned at $t = 0$. This behavior is formalized with the unconditioned signaling variables $\phi_i, i = 1, ..., n+1, \phi_i \in [0,1], \sum_{i=1}^{n+1} \phi_i = 1$. The variable $\phi_i$ is therefore the probability of the buyer giving the signal $H_i$. Note that this signal is independent of the buyer’s respective holding cost parameter $\tilde{h}$. Particularly, the signal can either be true, an overstatement or an understatement. Nonetheless, unconditioned signaling also includes the standard hypothesis of the buyer giving the signal $H_n$ constantly, i.e. $\phi_n = 1$. In this case, the buyer always exaggerates to the maximum extent.

**Conditioned signals**

A deceptive buyer uses her signal strategically, if she conditions the signal on the holding cost realization. This behavior is formalized with the conditioned signaling variable...
\[ \phi(h_k), i = 1, \ldots, n + 1, k = 1, \ldots, n, \phi(h_k) \in [0,1]. \] A complete strategy profile requires that 
\[ \sum_{i=1}^{n+1} \phi(h_k) = 1, \forall k = 1, \ldots, n \] holds. As an example, the buyer who always exaggerates her holding cost by one (possible) unit and gives no signal if she faces the highest holding cost realization has the strategy profile \( \phi(h_k) = 1, \forall i = k + 1, k = 1, \ldots, n \) and \( \phi(h_k) = 0, \forall i \neq k + 1, k = 1, \ldots, n \).

4.2 The supplier’s probability adjustment

The supplier is aware of the fact that there are some honest buyers. However, as there are also deceptive buyers, he has to estimate the probability that he is interacting with an honest buyer. This subjective probability is denoted by \( \hat{\alpha} \in [0,1] \). Furthermore, the supplier needs to estimate the unconditioned signaling variables \( \hat{\phi}_i, \forall i = 1, \ldots, n + 1 \) or the conditioned signaling variables \( \hat{\phi}_i(h_k), \forall i = 1, \ldots, n + 1; k = 1, \ldots, n \), respectively. It is assumed that the supplier can observe whether the buyer conditions her signals or not.\(^6\) In the following we will only present the supplier’s adjustment of beliefs in case of a buyer who gives unconditioned signals. The analysis, however, can be easily transferred to the case where the buyer uses conditioned signaling variables instead.

If the supplier assumes that the buyer gives unconditioned signals, he expects the following conjoint probability distribution \( \hat{p}_i(h \cap H_k), \forall i = 1, \ldots, n; k = 1, \ldots, n + 1 \) (see table 1). The actual conjoint probability distribution \( p_i(h \cap H_k), \forall i = 1, \ldots, n; k = 1, \ldots, n + 1 \), in contrast, can be easily obtained by replacing the estimations \( \hat{\alpha}, \hat{\phi}_i \) by their actual counterparts \( \alpha, \phi_i \).

Table 1: Estimated (perceived) conjoint probability distribution \( \hat{p}_i(h \cap H_k) \)

<table>
<thead>
<tr>
<th>0 \leq \hat{\alpha} \leq 1</th>
<th>( H_1 )</th>
<th>( H_i )</th>
<th>( H_{n+1} )</th>
<th>( \sum )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_i )</td>
<td>( \hat{\alpha} p_i + (1 - \hat{\alpha}) p_i \hat{\phi}_i )</td>
<td>( (1 - \hat{\alpha}) p_i \hat{\phi}_i )</td>
<td>( (1 - \hat{\alpha}) p_i \phi_{n+1} )</td>
<td>( p_i )</td>
</tr>
<tr>
<td>( h_i )</td>
<td>( (1 - \hat{\alpha}) p_i \hat{\phi}_i )</td>
<td>( \hat{\alpha} p_i + (1 - \hat{\alpha}) p_i \phi_i )</td>
<td>( (1 - \hat{\alpha}) p_i \phi_{n+1} )</td>
<td>( p_i )</td>
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<td>( h_i )</td>
<td>( (1 - \hat{\alpha}) p_i \phi_i )</td>
<td>( (1 - \hat{\alpha}) p_i \phi_i )</td>
<td>( (1 - \hat{\alpha}) p_i \phi_{n+1} )</td>
<td>( p_i )</td>
</tr>
<tr>
<td>( \sum )</td>
<td>( \hat{\alpha} p_i + (1 - \hat{\alpha}) \phi_i = \phi_i )</td>
<td>( \hat{\alpha} p_i + (1 - \hat{\alpha}) \phi_i = \phi_i )</td>
<td>( (1 - \hat{\alpha}) \phi_{n+1} = \phi_{n+1} )</td>
<td>1</td>
</tr>
</tbody>
</table>

\(^6\) This assumption can be easily relaxed by introducing a variable that denotes the probability that the buyer gives unconditioned signals.
Let \( \hat{p}_i(h_i \mid H_k) \), \( i = 1, ..., n; k = 1, ..., n + 1 \) denote the perceived a-posteriori probability that the buyer giving signal \( H_k \) faces holding cost \( h_i \). These perceived a-posteriori probabilities result from:

\[
\hat{p}_i(h_i \mid H_k) = \frac{\hat{\alpha} p_i + (1 - \hat{\alpha}) p_i \hat{\phi}_i}{\hat{\phi}_i}, \forall i = 1, ..., n
\]

\[
\hat{p}_i(h_i \mid H_k) = \frac{(1 - \hat{\alpha}) p_i \hat{\phi}_i}{\hat{\phi}_k}, \forall i = 1, ..., n; k = 1, ..., n + 1; i \neq k
\]

This distribution is utilized to calculate the menu of contracts, i.e. the supplier solves the problem AI with respect to \( \hat{p}_i(h_i \mid H_k) \), \( i = 1, ..., n; k = 1, ..., n + 1 \). The resulting optimal wholesale prices are denoted by \( w_i^k \) and the respective order sizes are denoted by \( q_i^k \), \( \forall i = 1, ..., n; k = 1, ..., n + 1 \).

Throughout this study it is assumed that the supplier will always offer the ‘full’ menu of contracts, i.e. he always offers \( n \) contracts which satisfy the incentive- and participation constraint in problem AI. This ensures that the supplier offers the contract \( \langle w_i^k, q_i^k \rangle \) even though he adjusts his beliefs to \( \hat{p}_i(h_i \mid H_k) = 0 \). If this assumption does not hold, there might be situations in which the buyer’s participation constraint is not satisfied.

The perceived probabilities determine the supplier’s contract offer \( t = 2 \), whereas the actual probabilities determine the relative frequency of the buyer’s contract choice \( t = 3 \). Please note that the notation is summarized in the appendix 9.2.

**4.3 The buyer’s contract choice and the impact on the supplier profits, the buyers cost and the overall supply chain deficit**

The buyer’s expected contract choice \( t = 3 \) is determined by the actual conjoint probability distribution \( p_i(h_i \cap H_k), \forall i = 1, ..., n; k = 1, ..., n + 1 \). In the following we analyze the deviation from the standard-theory’s predictions (i.e. without communication or trust). The main focus of this analysis, thus, is to investigate whether communication enhances or deteriorates supply chain performance compared to the game theoretic equilibrium without communication.
The expected change of the supplier’s profit, \( E(\Delta P_s) \), results from the profit difference between the screening menu based on the perceived a-posteriori distribution, \( \{ w_i^k, q_i^k \} \), \( \forall i = 1, \ldots, n; k = 1, \ldots, n+1 \), and the screening menu based on the a-priori distribution, \( \{ w_i^*, q_i^* \} \), \( \forall i = 1, \ldots, n \). All of these differences are weighted by the respective actual conjoint probability distribution, i.e. these cost differences are weighted by the probability that a buyer faces holding cost \( h_i \) while signaling \( H_k \). Hence, the expected change of the supplier’s expected profit results from:

\[
E(\Delta P_s) = \sum_{i=1}^{n} \sum_{k=1}^{n+1} p_i (h_i \cap H_k) \Delta P_{s,j}^k \cdot \text{ where } \Delta P_{s,j}^k = w_i^k - \frac{f}{q_i^k} - w_i^* + \frac{f}{q_i^*}
\]

The same calculation can be done for the buyer with respect to her cost function. Then, the expected difference of the honest buyer’s expected costs results from

\[
E(\Delta C_{b,honest}) = \sum_{i=1}^{n} \sum_{k=1}^{n+1} p_i \left( w_i^k + \frac{h_i}{2} q_i^k - w_i^* - \frac{h_i}{2} q_i^* \right)
\]

and the expected difference of the deceptive buyer’s expected costs results from

\[
E(\Delta C_{b,deceptive}) = (1 - \alpha) \sum_{i=1}^{n} \sum_{k=1}^{n+1} p_i \left( w_i^k + \frac{h_i}{2} q_i^k - w_i^* - \frac{h_i}{2} q_i^* \right)
\]

Hence, the total expected difference of the honest as well as the deceptive buyer’s expected costs results from:

\[
E(\Delta C_b) = E(\Delta C_{b,honest}) + E(\Delta C_{b,deceptive}) = \sum_{i=1}^{n} \sum_{k=1}^{n+1} p_i (h_i \cap H_k) \Delta C_{b,i}^k \cdot \text{ where } \Delta C_{b,i}^k = w_i^k + \frac{h_i}{2} q_i^k - w_i^* - \frac{h_i}{2} q_i^*.
\]

Finally, the expected change of the supply chain performance results from:

\[
\Delta CD = E(\Delta C_b) - E(\Delta P_s) = \sum_{i=1}^{n} \sum_{k=1}^{n+1} p_i (h_i \cap H_k) \Delta CD_{i,k}^k \cdot \text{ where } \Delta CD_{i,k}^k = C_{sc}\left( q_i^k \right) - C_{sc}\left( q_i^* \right)
\]

\[
C_{sc}(q_i^k) = \frac{f}{q_i^k} + \frac{h_i}{2} q_i^k
\]

\( \Delta CD_{i,k}^k = C_{sc}\left( q_i^k \right) - C_{sc}\left( q_i^* \right) \) denotes the supply chain cost differences that arise due to the adjustment of the a-priori probabilities. For \( \Delta CD \leq 0 \) the supply chain deficit decreases due to
communication, which is identical to an improvement of the overall supply chain performance. In this case, communication is an appropriate coordination mechanism. Otherwise, it is not.

5 **Impact of communication in case of unconditioned signals**

5.1 **General analysis**

If the buyer believes that the supplier ignores the signal, she is assumed to use unconditioned signals. In this case the following general predictions regarding the supplier’s expected profits, the buyer’s expected costs and the supply chain’s deficit can be derived. All proofs can be find in the appendix 9.1.

**Theorem 1:** The supplier’s expected profits do not decrease due to the adjustment of the a-priori distribution as long as

\[
\hat{\alpha} \leq \min_i \left[ \frac{\phi_i \cdot \alpha}{\phi_i \cdot \alpha + \hat{\phi}_i \cdot \alpha - \phi_i \cdot \alpha} \right], \quad i = 1, \ldots, n
\]

holds.

Theorem 1 shows that the supplier should be cautious to believe too much in the buyer’s signal. This means, that he should rather underestimate the number of buyers who are honest than to overestimate this number. Yet, on the other hand the supplier cannot participate from truthful signals if he chooses \( \hat{\alpha} \) too low because the probability adjustment is not rigorously enough. As an example, the supplier will not adjust his probabilities at all if \( \hat{\alpha} = 0 \) holds although all buyers are honest, i.e. \( \alpha = 1 \). In this case communication has no effect, simply because the supplier does not react to the signals.

Note that the condition in theorem 1 is always satisfied if \( \hat{\alpha} \leq \alpha \) and \( \hat{\phi}_i = \phi_i, i = 1, \ldots, n + 1 \) holds. Thus, if the supplier can perfectly observe the buyer’s unconditioned signals, and if his estimation with respect to the buyer’s trustworthiness is equal or lower than the buyer’s actual trustworthiness, the supplier will always gain from communication. The increase of expected profits for the above parameter combinations can be explained by the finding, that for these parameters the supplier is always closer to the actual a-posteriori distribution than if he would stick to the a-priori distribution, i.e. \( p_i \leq \hat{p}_i(h_i \mid H_i) \leq p_i(h_i \mid H_i) \) and \( p_i(h_i \mid H_k) \leq \hat{p}_i(h_i \mid H_k) \leq p_i, \forall i \neq k \) hold. In fact, if \( \hat{\alpha} = \left( \hat{\phi}_i \cdot \alpha \right) / \left( \hat{\phi}_i \cdot \alpha + \phi_i \cdot \alpha - \phi_i \cdot \alpha \right) \) holds.
for a specific signal $H_i$, then the supplier estimates the actual a-posteriori distribution with respect to the signal $H_i$ accurately whereas the accuracy decreases with decreasing $\hat{\alpha}$.

If the above condition is not satisfied the supplier’s expected profits can increase nonetheless. In this case he overestimates the probability which corresponds to the respective signal, i.e. $p_i \leq p_i(h_i \mid H_i) \leq \hat{p}_i(h_i \mid H_i)$, and underestimates all other probabilities, i.e. $\hat{p}_i(h_i \mid H_k) \leq p_i(h_i \mid H_k) \leq p_i, \forall i \neq k$. In this case, the change in the supplier’s expected profits is dependent on the specific parameter values.

**Theorem 2.1:** The honest buyer’s expected costs increase due to truthful signaling.

The supplier will decrease all order sizes that corresponds to the holding costs that are higher than the signal, i.e. $q_i^k \leq q_i^*, \forall i > k$ given signal $H_k$. The expected cost, thus, will increase due to the ‘indifference’-condition (3) (see chapter 3).

**Theorem 2.2:** The expected costs of the deceptive buyer can either increase or decrease due to communication.

The deceptive buyer can be worse off due to communication, if she reports (accidentally) truthful or if she understates her actual holding costs. The argumentation is equal to theorem 2.1. If the deceptive buyer exaggerates her holding cost constantly to the maximum extent (i.e. $\phi = 1$), however, then she cannot be worse off due to communication. If the deceptive buyer exaggerates not to the maximum extent, though, her expected costs changes are dependent on the specific parameters values.

**Theorem 3.1:** The supply chain optimum is achieved if $\alpha = \hat{\alpha} = 1$ holds. The supply chain performance is worst if $\phi = 1, \alpha = 0$ and $\hat{\alpha} = 1$ holds.

For $\alpha = \hat{\alpha} = 1$ the supply chain faces a decision problem as if under full information. This results in the supply chain optimum.

For $\phi = 1, \alpha = 0$ and $\hat{\alpha} = 1$ it follows, that all buyers are deceptive and constantly claim that they have the lowest possible holding costs. The supplier will, in turn, decrease all order sizes $q_i^k \leq q_i^*, \forall i = 2, ..., n$. As the order size $q_i^k = q_i^*, \forall k = 1, ..., n + 1$ does not change due to an adjustment of the probabilities (see section 3, condition (5)) the downward distortion increases for all order sizes.
Theorem 3.2: As long as \( \alpha \geq \frac{\sum_{i,k \in \mathcal{K}} p_i \phi_i \Delta CD_i^k + \sum_{i} p_i \phi_i \Delta CD_i}{\sum_{i,k \in \mathcal{K}} p_i \phi_i \Delta CD_i^k - \sum_{i} (1 - \phi_i) p_i \Delta CD_i} = \alpha_{cr} (\alpha, \hat{\phi}) \) holds, communication enhances the supply chain performance.

This theorem points out that there are regions of parameter values in which communication improves supply chain performance, but that there are also parameters values for which the supply chain performance deteriorates. Intuitively, communication becomes more attractive from a supply chain perspective the higher the fraction of honest buyers, \( \alpha \). However, as soon as this fraction \( \alpha \) decreases under the critical level \( \alpha_{cr} (\alpha, \hat{\phi}) \), the more likely are situations in which a deceptive buyer unconditionally misrepresents her holding cost realization while the supplier reacts to this signal. From \( \alpha_{cr} (\alpha, \hat{\phi}) \leq 1 \) it follows directly that communication can always be an appropriate coordination mechanism as long as the number of honest buyers is sufficiently large.

Theorem 3.3: Given a certain probability that the supplier interacts with an honest buyer, i.e. \( \alpha \), it is more likely that communication is an appropriate coordination mechanism if the supplier’s trust in the buyer’s signal decrease, i.e. if \( \hat{\alpha} \) decrease.

This theorem gives an interesting insight into the interaction between trust (\( \hat{\alpha} \)) and trustworthiness (\( \alpha \)) and the impact of this interaction on supply chain performance. Particularly, this theorem shows that more trust does not necessarily increase the supply chain performance. In contrary, the more the supplier trusts the buyer’s signal, the more likely is a deterioration of supply performance. The effectiveness of communication decreases on the other hand, if the supplier’s trust decreases because he simply does not adjust the probabilities rigorously enough. Hence, it is an important challenge of supply chain management to identify an appropriate level of trust.

5.2 Numerical Example

In the following the previous analysis for the case of two possible holding cost realizations, i.e. \( h \in [h_1, h_2] \), is illustrated. Suppose that \( (\hat{\phi}_1, \hat{\phi}_2, \hat{\phi}_3) = (\hat{\phi}_1, \hat{\phi}_2, \hat{\phi}_3) = (1/3, 1/3, 1/3) \), \( (p_1, p_2) = (1/2, 1/2) \), \( (h_1, h_2) = (1; 2) \), \( R = 801 \), \( \hat{\alpha} = 0.5 \), \( \alpha = 0.6 \), \( q_{min} = 1 \) and \( f = 800 \).
Then the following perceived and actual conjoint and a-posteriori probability distribution result (see table 2-5):

<table>
<thead>
<tr>
<th>Table 2: Perceived conjoint distribution $\alpha = 0.5$</th>
<th>Table 3: Actual conjoint distribution for $\alpha = 0.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.5$</td>
<td>$\alpha = 0.6$</td>
</tr>
<tr>
<td>$H_1$</td>
<td>$H_2$</td>
</tr>
<tr>
<td>$h_1$</td>
<td>0.33</td>
</tr>
<tr>
<td>$h_2$</td>
<td>0.08</td>
</tr>
<tr>
<td>$\sum$</td>
<td>0.42</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4: Perceived a-posteriori distribution for $\hat{\alpha} = 0.5$</th>
<th>Table 5: Actual a-posteriori distribution for $\alpha = 0.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\alpha} = 0.5$</td>
<td>$\alpha = 0.6$</td>
</tr>
<tr>
<td>$h_1$</td>
<td>$\hat{p}_1(h_1 \mid H)$</td>
</tr>
<tr>
<td>$h_2$</td>
<td>$\hat{p}_2(h_2 \mid H)$</td>
</tr>
</tbody>
</table>

On the basis of the perceived a-posteriori distribution (see table 4), the supplier will offer the following order sizes and unit prices and given a respective signal (see table 6) and the following changes of the supplier’s profit, the buyer’s cost and the supply chain’s coordination deficit result:: $\Delta CD = 0.11$, $E(\Delta P_b) = 1.01$, $E(\Delta C_b) = 1.12$, $E\left(\Delta C_{b,\text{honest}}\right) = 0.11$ and $E\left(\Delta C_{b,\text{deceptive}}\right) = 1.01$. The supply chain deficit increases if the buyer chooses the even further downwards distorted order size $q_2^1 = 16.33$ while the supply chain deficit decreases if the buyer chooses the upwards adjusted order size $q_2^2 = 26.67$. The expected effect on the supply chain deficit, thus, depends on the frequency with which these order sizes are chosen, i.e. $p_2(h_2 \cap H_1)$ and $p_2(h_2 \cap H_2)$. The total coordination deficit without communication is equal to $CD = 1.17$. The supply chain deficit increases therefore by 9.4%. This, in turn, questions the frequent claim of information sharing within the supply chain. Even though 60% of all buyers actually are trustworthy, and the supplier underestimates this ratio, the
supply chain deficit increases. In fact, the supply chain deficit would only decrease due to communication if \( \alpha > \alpha_{\text{crit}} = 0.67 \) holds (see theorem 3.2).

**Table 6: Order sizes and unit prices in the menu of contracts**

<table>
<thead>
<tr>
<th>Signal</th>
<th>a-priori</th>
<th>supply chain Optimum</th>
<th>a-posteriori</th>
<th>( H_1 )</th>
<th>( H_2 )</th>
<th>( H_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_1^* )</td>
<td>40</td>
<td>( q_{1,sc}^* )</td>
<td>40</td>
<td>( q_1^k )</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>( q_2^* )</td>
<td>23.09</td>
<td>( q_{2,sc}^* )</td>
<td>28.28</td>
<td>( q_2^k )</td>
<td>16.33</td>
<td>26.67</td>
</tr>
<tr>
<td>( w_1^* )</td>
<td>769.45</td>
<td>-</td>
<td>-</td>
<td>( w_1^k )</td>
<td>772.84</td>
<td>767.67</td>
</tr>
<tr>
<td>( w_2^* )</td>
<td>777.91</td>
<td>-</td>
<td>-</td>
<td>( w_2^k )</td>
<td>784.67</td>
<td>774.33</td>
</tr>
</tbody>
</table>

The supplier’s expected profits increase due to communication, which is in line with theorem 1 (as the condition in theorem 1 holds). The buyer, in turn, is worse off, independent of whether she is honest or deceptive. This stresses that unconditioned signals can substantially deteriorate the performance even of the deceptive buyer (see theorem 2.2).

Yet, the previous analysis showed that the effect of communication on supply chain performance is ambiguous. An example in which communication is effective can be easily constructed by setting \( \alpha > \alpha_{\text{crit}} \). In this case, it is more likely that the performance improving order size \( q_2^2 \) instead of the performance deteriorating order size \( q_1^1 \) is chosen (see Table 6).

To test the impact of the several parameters and variables a comparative static analysis is conducted in the next section.

### 5.3 Comparative static analysis

Figure 3 depicts the changes in the coordination deficit, \( \Delta CD \), the supplier’s change in expected profits, \( E(\Delta P_s) \), as well as the buyer’s change in expected costs, \( E(\Delta C_b) \), in dependence of the a-priori probability \( p_1 \), \( p_2 = 1 - p_1 \). Figure 3 is in line with theorem 1. The supplier, thus, cannot be worse off due to the adjustment of beliefs as long as the condition in theorem 1 holds. The coordination deficit decreases in this example for relatively high a-priori probabilities \( p_1 \) (\( p_1 > 0.79 \)). Intuitively, in this case it is more likely that the buyer chooses
the undistorted order size $q_i^1$ instead of the distorted order size $q_i^1$. Furthermore, from

$$q_2^* = \sqrt{\frac{2 \cdot f}{h_2 + \gamma_2}} \quad \text{and} \quad \gamma_2 = \frac{p_1}{p_2} (h_2 - h_1) \quad \text{(see Section 3)}$$

it follows directly that there is a comparably stronger adjustment of the order size $q_2^k, k = 1, 2, 3$ if the a-priori probability $p_2$ is low, because $\frac{\partial \gamma_2}{\partial p_2} = -\frac{1}{p_2^2}$ holds. As coordination potentials are only used through an upward adjustment of $q_2^2$, the coordination deficit decreases for low values $p_2$ or high values $p_1$ respectively (see Section 3, condition (5)). As an example, the supplier adjusts the order sizes from $q_2^* = 12.06$ to $q_1^2 = 7.11$ and $q_2^2 = 18.38$ when $p_1 = 0.9$ holds.

Additionally, figure 3 shows that the supply chain performance does not automatically increase if the supplier’s estimation of the a-posteriori distribution is more accurate (which is always the case if the condition in theorem 1 holds). As the supplier maximizes his own expected profits instead of minimizing the overall expected supply chain costs, the supply chain deficit can increase even if the supplier estimates the buyer’s holding costs more accurately through communication.

![Figure 3: Variation of the a-priori probability $p_1$](image-url)
Figure 4 shows, that the honest buyer cannot decrease her expected costs, regardless of the a-priori distribution (see theorem 2.1.). The deceptive buyer, though, can either increase or decrease her expected costs (see theorem 2.2.).

![Figure 4: Variation of a-priori probabilities](image)

Figure 4: Variation of a-priori probabilities

Figure 5 depicts the robustness of the results in dependence of the buyer’s randomization variables \( \phi_i = \hat{\phi}_i, i = 1, 2, 3 \). The variation of \( \phi_2 \) is considered by setting \( \phi_i = \phi_2 = \frac{\left(1 - \phi_2\right)}{2} \).

![Figure 5: Impact of the unconditioned signals on the buyer’s costs, supplier’s profits and the coordination deficit](image)

Figure 5: Impact of the unconditioned signals on the buyer’s costs, supplier’s profits and the coordination deficit

Figure 5 depicts that an increase of the coordination deficit is observable for a broad range of parameter values. However, one might suggest that it is obvious that the buyer will claim the highest holding costs constantly, i.e. \( \phi_2 = 1 \). Yet, this analysis shows, that even a slight deviation from this
behavior ($\phi_2 < 0.97$), as observed by Inderfurth et al. (2008), can harm the overall supply chain performance.

\[ \phi_1 = \phi_3 = \frac{1-\phi_2}{2} \]

\[ E\left(\Delta C_{b,\text{honest}}\right) \]

\[ E\left(\Delta C_{b,\text{deceptive}}\right) \]

\[ \phi_2 = \hat{\phi}_2 \]

**Figure 6:** Impact of unconditional signals on the honest and deceptive buyer’s expected costs.

Figure 6 again stresses that the rationally deceptive buyer would choose the unconditional signal $\phi_2 = 1$ if she assumes that the supplier updates his beliefs with respect to the signal. An honest buyer, however, benefits from a deviating behavior, as the increase in expected costs is lower for lower values of $\phi_2 = \hat{\phi}_1$. This effect is prevalent, because in this case it is harder for the supplier to distinguish whether signal $H_1$ was given by a honest or by a dishonest buyer.

In the following, we investigate the impact of the supplier’s trust as well as the buyer’s trustworthiness. For this purpose, the level of the supplier’s trust is fixed at $\hat{\alpha} = 0.5$ (as in the numerical example) while the buyer’s trustworthiness, $\alpha$, varies. Figure 7 shows that the expected profits of the supplier can decrease, if the overestimation of the buyer’s trustworthiness is relatively high (see theorem 1). In the numerical example the suppliers expected profits would decrease for $\alpha < 0.27$. In turn, the coordination deficit decreases for $\alpha_{\text{crit}} > 0.67$. Figure 8 depicts the critical levels of trustworthiness, $\alpha_{\text{crit}}$, in dependence of the supplier’s trust, $\alpha$, for which the supply chain performance does not change compared to the benchmark without communication. Furthermore, the arrows depicts in which regions the supply chain deficit would increase and decrease. These regions directly follow from theorem 3.3.

\[ \text{Surprisingly, if it is assumed that the supplier can perfectly observe the buyers trustworthiness, i.e. } \alpha = \hat{\alpha}, \text{ then the coordination deficit would only decrease for } \alpha = \hat{\alpha} > 0.98. \]
Figure 7: Variation $\alpha$

Figure 8 shows that communication becomes less attractive from a coordinational point of view, the higher the supplier’s trust in the buyers signals, because $\alpha_{crit}$ is monotonically increasing with increasing $\alpha$. This counterintuitive result highlights that the unilateral claim for more trust in supplier-buyer relationships might not always be justifiable. However, note that Figure 8 does not depict the size of the changes in the coordination deficit. Obviously, if the supplier totally mistrust the buyers signals, i.e. $\hat{\alpha} = 0$, then communication would have no impact on supply chain coordination and $\alpha_{crit} = 0$ would hold. In this case, though, neither the supplier nor the supply chain could benefit from trustworthy signals. The same analysis could be conducted for changes of $\phi_i$. However, it can be shown that a variation of $\alpha$ and $\phi_i$ have a similar impact on the supply chain performance. Hence, this analysis is omitted.

Figure 8: Critical level of trustworthiness ($\alpha_{crit}$) in dependence of trust ($\hat{\alpha}$)
6 Impact of communication in case of conditioned signals

The previous analysis concentrated on unconditioned signals, i.e. the buyer who chooses her signal independently from her actual holding cost realization. Now, it is assumed that the buyer conditions her signals on the actual holding cost realization. Therefore, conditioned signaling variables, \( \phi (h_k) \) \( \forall i,k \), are defined. These variables denote the probability that the deceptive buyer facing holding cost \( h_k \) gives the signal \( H_i \). The deceptive buyer might, for example, exaggerate her holding cost to the maximum extent, i.e. \( \phi_i(h_k) = 1 \forall i = 1,\ldots,n \). Alternatively, she might always exaggerate her actual holding cost realization by one (possible) unit, i.e. \( \phi_i(h_k) = 1 \forall i = k + 1 \).

Please note, that the buyer might convey information if she uses strategic signaling variables and the supplier correctly anticipates this behavior. If the buyer always exaggerates by one unit, and the supplier anticipates this correctly, the supplier could infer the holding cost from the signal. In this case, the supplier has actually full information, which in turn leaves no information rent to the deceptive buyer.

Again, it is assumed that the supplier estimates the buyer’s conditioned signaling variables. This estimation is denoted by \( \hat{\phi}_i(h_k) \).

The conditioned signaling variables are obviously a generalization of the assumption that the buyer gives unconditioned signals. Particularly, if \( \phi_i = \phi_i(h_k) \) and \( \hat{\phi}_i = \hat{\phi}_i(h_k), \forall k = 1,\ldots,n \) holds, the same results apply. Hence, it is not surprising that the analysis becomes more complex. Especially the previous theorems cannot be easily transferred. For this reason a numerical example combined with a comparative static analysis demonstrates the differences that emerge in contrast to the previous section, i.e. in contrast to unconditioned signals.

6.1 Numerical example and comparative static analysis

As a starting point, it is assumed that the supplier can perfectly observe the buyer’s conditioned signaling variables. As an example, it is assumed that

\[
\begin{align*}
\phi_1(h_1) &= \hat{\phi}_1(h_1) = 0, \\
\phi_2(h_1) &= \hat{\phi}_2(h_1) = 1, \\
\phi_3(h_1) &= \hat{\phi}_3(h_1) = 0, \\
\phi_1(h_2) &= \hat{\phi}_1(h_2) = 1/3, \\
\phi_2(h_2) &= \hat{\phi}_2(h_2) = 1/3 \\
\phi_3(h_2) &= \hat{\phi}_3(h_2) = 1/3.
\end{align*}
\]

All remaining parameter values from Section 5 apply. The buyer, thus, follows the strategy to always exaggerate her holding cost to the maximum extent (i.e. by one unit) if she faces holding costs \( h_1 \). Yet, if she faces the holding
costs $h_2$, she gives all signals with the same frequency of 33%. Note that the buyer’s costs do never change if she faces the highest possible holding cost realization (see Section 3, condition 2). Truthful reporting as well as an over- and understatement, thus, has no impact on her performance. In contrast to the previous section, unconditional signals given the highest possible holding cost realization $h_n$ can indeed reflect the buyer’s signaling behavior even though she anticipates the supplier’s adjustment of beliefs. The following Figure 9 depicts the expected changes of the supplier’s profits, $E(\Delta P_s)$, the buyer’s costs disaggregated by honest and deceptive buyers, $E(\Delta C_{b,honest})$, $E(\Delta C_{b,deceptive})$, and the overall supply chain deficit, $\Delta CD$, in dependence of the buyer’s trustworthiness, $\alpha$.

![Figure 9: Variation of $\alpha$ under conditional signaling](image)

Again, the supply chain deficit is vulnerable if communication takes place, i.e. if the supplier adjusts his probabilities with respect to the buyer’s signal. The expected coordination deficit does only decrease if the buyers trustworthiness is relatively high, i.e. if $\alpha > 0.95$ holds. The disaggregated view on the honest and deceptive buyers point out that the deceptive buyers cannot be worse off if they choose a suitable signaling strategy. The honest agents, however, are always worse off due to reporting truthfully. This example shows, that even a buyer’s rational signaling strategy, i.e. always exaggerating to maximum extent if she faces a holding...

---

8 In the previous section it was assumed that the buyer gives unconditional signals as she simple ignores that the supplier updates his beliefs with respect to the signal.
cost realization that is lower than $h_n$, while giving unconditional signals if she faces the holding cost realization $h_n$, can significantly harm the overall supply chain performance. To highlight the impact of the buyer’s unconditional signaling while facing the holding cost realization $h_n$, the buyers signaling strategy given $h_n$ is varied. Figure 10 captures the changes of the respective expected cost and profit changes for a change of $\phi_2(h_2)$. The variation of $\phi_2(h_2)$ is considered by setting $\phi_1(h_2) = 0$ and $\phi_1(h_2) = 1 - \phi_2(h_2)$. Furthermore, it is assumed that the supplier expects that the buyer always exaggerate to the maximum extent, i.e. $\hat{\phi}_2(h_2) = \hat{\phi}_2(h_2) = 1$. All other values from the previous example apply.

This analysis points out that the supply chain performance is heavily dependent on the supplier’s perception of the buyer’s signaling strategy and the signaling strategy itself. As an example, for $\phi_2(h_2) = 0.2$ the supply chain deficit increases by $\Delta CD = 148$, i.e. the coordination deficit is about 126 times higher than without communication. In contrary, the buyer’s expected costs do not change (see Section 3, condition 2). The supplier, thus, should carefully estimate the buyer’s unconditioned signaling variables. However, if this is not possible he should be cautious to adjust the a-priori probabilities at all.

Figure 10: Variation of the conditional signaling strategy
7 Managerial Insights & Conclusion

There is a growing body of work that analyses the inefficiencies of supply chain interactions under asymmetric information in traditional principal-agent settings. Traditionally, it is assumed that the principal has an a-priori probability distribution over the agent’s private information. However, there is usually little said on how the principal obtains this distribution. Moreover, it is stressed that the assessment of the a-priori distribution is not influenced by communication, as information sharing is simply treated as cheap talk.

This study shows that the introduction of communication in the presence of trust and trustworthiness can substantially affect the predictions of principal-agent models under asymmetric information. The behavioral model introduced in this study therefore provides a general framework which allows to investigate the impact of different information processing and information sharing behavior in a general principal-agent framework.

In a typical supply chain lotsizing setup, it is shown that buyers have an incentive to misrepresent their order size related costs. However, previous empirical work in the field of supply chain coordination shows that not all buyers exploit their superior information. In fact, there is a substantial level of honesty in supply chain interactions.

The underlying work shows that the effect of information processing on the supplier’s performance is ambiguous. Particularly, if the supplier manages to assess the buyer’s information sharing behavior accurately while underestimating the probability of receiving credible signals, the supplier can indeed improve his performance. However, if the supplier does not anticipate the buyer’s information sharing behavior accurately while being overconfident in receiving credible information, a decrease in the supplier’s performance level is likely.

Surprisingly, the results show that the overall supply chain performance can seriously deteriorate, even though the supplier can utilize the shared information to assess a more accurate estimation of the buyer’s private information. This fact stresses the basic conflict in supply chain management, i.e. the supplier’s optimization attempts do not automatically lead to supply chain optimal solutions. Therefore, communication can not be regarded as an appropriate coordination instrument without considering the specific supply chain environment. Managers should carefully evaluate whether their respective supply chain is more likely to gain or to loose from information sharing. Hence, the ever increasing unilateral claim for trust in supplier-buyer relationships does not seem to be appropriate.
The behavioral model introduced in the underlying work is subject to some limitations. First, it is assumed that communication, trust and trustworthiness are exogenously determined. Yet, another approach might treat these variables endogenously by incorporating reputational effects in recurring interactions.

A further direction for research might be the extensive testing of this behavioral model according to Inderfurth et al. (2008). This research might give valuable empirical data which allows for an estimation of the behavioral parameters of the model.

8 References

**Alger, I.; Renault, R. (2007):** Screening ethics when honest agents keep their word,
in: Economic Theory 30, 291-311

**Alger, I.; Renault, R. (2006):** Screening ethics when honest agents care about fairness

**Anderson, J.; Narus, J. (1984):** A model of the distributor’s perspective of distributor-manufacturer working relationships
in: Journal of Marketing 48(Fall), 62-74

**Ben-Ner, A.; Putterman, L. (2001):** Trusting and trustworthiness
in: Boston University Law Review 81(3), 523-551

**Cachon, G. (2003):** Supply chain coordination with contracts

**Cachon, G.P.; Fisher, M. (2000):** Supply Chain Inventory Management and the Value of Shared Information
in: Management Science 46 (8), 1032-1048


**Chen, F. (2003):** Information sharing and supply chain coordination

**Corbett, C.J. (2001):** Stochastic inventory systems in a supply chain with asymmetric information:
Cycle stocks, safety stocks, and consignment stock
in: Management Science 49 (4), 487-500
Corbett, C.J.; de Groote, X. (2000): A supplier’s optimal quantity discount policy under asymmetric information
in: Management Science 46 (3), 444-450


in: Management Science 50 (4), 550-559

Crawford, V. (1998): A survey of experiments on communication via cheap talk
in: Journal of Economic Theory 78(3), 286-298

in: Journal of Economic Behavior & Organization 55, 447-465


Ha, A.Y. (2000): Supplier-buyer contracting: Asymmetric cost information and cutoff level policy for buyer participation
in: Naval Research Logistics 48(1), 41-64

in: FEMM Working Paper No. 16/2008,
http://www.uni-magdeburg.de/bwl6/download/2008_02_FEMM.pdf

in: FEMM Working Paper No. 01/2008,
http://www.uni-magdeburg.de/bwl6/download/2008_01_FEMM.pdf

Kelle, P.; Akbulut, A. (2005): The role of ERP tools in supply chain information, cooperation, and cost optimization

in: The Academy of Management Review 20(3), 709-734
Minkler, L.P. und Miceli, T.J. (2004): Lying, Integrity, and Cooperation
in: Review of Social Economy 62(1), 27-50

in: Strategic Management Journal 15(2), 135-152


in: Journal of Marketing 58(3), 20-38

in: Proceedings of the 35th Hawaii International Conference on System Sciences

Myerson, R.B. (1986): Multistage games with communication
in: Econometrica 54(2), 323-358

Working Paper

in: Journal of Economic Behavior & Organization 34, 387-417

in: Journal of Economic Theory 29, S.1-21

Schonberger, R.J.; Schniederjans, M.J. (1984): Reinventing inventory control
in: Interfaces 14(3), 76-83

Severinov, S.; Deneckere, R. (2006): Screening when some agents are non-strategic: Does a monopoly need to exclude?

in: European Journal of Operational Research 171, 516-535


in: Administrative Science Quarterly 17, 229-239
9 Appendix

9.1 Proofs of theorems

**Theorem 1**: The suppliers expected profits do not decrease due to the adjustment of the a-priori distribution as long as \( \hat{\alpha} \leq \min_i \left[ \frac{\hat{\phi}_i \cdot \alpha}{\hat{\phi}_i \cdot \alpha + \phi_i - \phi_i \alpha} \right], i = 1, \ldots, n \) holds.

**Proof:**

In the following it is shown that the supplier estimates the actual a-posteriori distribution \( p_i(h_i \mid H_i) \) more accurately if the condition in theorem 1 holds.

a.) Adjustment of the probability that corresponds to the signal: \( \hat{p}_i(h_i \mid H_i), \forall i = 1, \ldots, n \)

First, it is shown that the perceived a-posteriori probability is always lower than the actual a-posteriori probability if \( \hat{\alpha} \leq \frac{\hat{\phi} \alpha}{\hat{\phi}_i \alpha + \phi_i - \phi_i \alpha} \) holds.

\[
\hat{p}_i(h_i \mid H_i) = \frac{\hat{\alpha} p_i + (1 - \hat{\alpha}) p_i \hat{\phi}_i}{\hat{\alpha} p_i + (1 - \hat{\alpha}) \hat{\phi}_i} \leq \frac{\alpha p_i + (1 - \alpha) p_i \phi_i}{\alpha p_i + (1 - \alpha) \phi_i} = p_i(h_i \mid H_i)
\]

\[
\Rightarrow \frac{\alpha p_i + (1 - \alpha) p_i \phi_i}{\alpha p_i + (1 - \alpha) \phi_i} \geq 0
\]

\[
\Rightarrow \hat{\alpha} \leq \frac{\hat{\phi}_i \alpha}{\hat{\phi}_i \alpha + \phi_i - \phi_i \alpha}
\]

As the suppliers estimation needs to be more accurately for every signal \( H_i, i = 1, \ldots, n \) (note that there is no adjustment for \( H_{n+1} \), it follows:

\[
\hat{\alpha} \leq \min_i \left[ \frac{\hat{\phi}_i \cdot \alpha}{\hat{\phi}_i \cdot \alpha + \phi_i - \phi_i \alpha} \right], i = 1, \ldots, n.
\]
Second, it needs to be shown that $\hat{p}_i(h_i \mid H_i) \geq p_i$ holds:

$$p_i \leq \frac{\hat{\alpha} p_i + (1 - \hat{\alpha}) \hat{p}_i}{\hat{\alpha} p_i + (1 - \hat{\alpha}) \hat{p}_i}$$

$$\hat{\alpha} p_i + (1 - \hat{\alpha}) \hat{p}_i \leq \hat{\alpha} + (1 - \hat{\alpha}) \hat{p}_i$$

$$\rightarrow p_i \leq 1$$

Hence, it follows that

$$p_i \leq \hat{p}_i(h_i \mid H_i) \leq p_i(h_i \mid H_i), \forall i = 1, \ldots, n.$$ 

b. Adjustment of probabilities that do not correspond to the signal, i.e.

$\hat{p}_k(h_k \mid H_i), \forall k = 1, \ldots, n; i = 1, \ldots, n + 1; i \neq k$

$$\hat{p}_k(h_k \mid H_i) = \frac{(1 - \hat{\alpha}) p_i \hat{\phi}_i}{\hat{\alpha} p_i + (1 - \hat{\alpha}) \hat{\phi}_i} \geq \frac{(1 - \alpha) p_i \phi_i}{\alpha p_i + (1 - \alpha) \phi_i} = p_k(h_k \mid H_i)$$

$$\rightarrow \frac{p_k p_i (-\hat{\phi}_i + \hat{\alpha} \phi_i + \alpha \hat{\phi}_i - \alpha \hat{\phi}_i)}{(\alpha p_i + (1 - \alpha) \phi_i)(\hat{\alpha} p_i + (1 - \hat{\alpha}) \hat{\phi}_i)} \leq 0$$

$$\rightarrow \hat{\alpha} \leq \frac{\hat{\phi}_i \alpha}{\phi_i \alpha + \phi_i - \phi_i \alpha}$$

Hence, it follows:

$$\hat{\alpha} \leq \min \left[ \frac{\hat{\phi}_i \cdot \alpha}{\hat{\phi}_i \cdot \alpha + \phi_i - \phi_i \alpha} \right], i = 1, \ldots, n$$

Furthermore, it needs to be shown that $\hat{p}_k(h_k \mid H_i) \leq p_k$ holds:

$$p_k \geq \frac{(1 - \hat{\alpha}) p_i \hat{\phi}_i}{\hat{\alpha} p_i + (1 - \hat{\alpha}) \hat{\phi}_i}$$

$$\hat{\alpha} p_i + (1 - \hat{\alpha}) \hat{\phi}_i \geq (1 - \hat{\alpha}) \hat{\phi}_i$$

$$\rightarrow p_i \geq 0$$
Hence it follows that

\[
p_i (h_k \mid H_i) \leq \hat{p}_i (h_k \mid H_i) \leq p_k, \forall i = 1,\ldots,n + 1; k = 1,\ldots,n; i \neq k
\]

The same argumentation follows for \( \alpha \geq \min_i \left[ \frac{\phi_i \cdot \alpha}{\phi_i \cdot \alpha + \phi_i - \phi_i \alpha} \right], i = 1,\ldots,n : \)

\[
p_i \leq p_i (h_k \mid H_i) \leq \hat{p}_i (h_k \mid H_i) \forall i = 1,\ldots,n
\]

\[
\hat{p}_i (h_k \mid H_i) \leq p_k (h_k \mid H_i) \leq p_k, \forall i = 1,\ldots,n + 1; k = 1,\ldots,n; i \neq k
\]

In this case, however, the change of the supplier’s expected costs depends on the specific cost structure.

**Theorem 2.1:** The honest buyer’s expected profits decrease due to truthful signaling.

The informational rent denotes the costs savings that occur for the buyer in comparison to the outside option, i.e. the alternative supplier. Let \( IR_i \) denote informational rent of the buyer who faces holding costs of \( h_i \). From condition 2 (see section 3) it follows:

\[
w_n + \frac{h_n}{2} q_n = R
\]

\[
\rightarrow IR_n = R - w_n - \frac{h_n}{2} q_n = 0
\]

\[
w_{n+1} + \frac{h_{n+1}}{2} q_{n+1} = w_n + \frac{h_n}{2} q_n
\]

\[
\rightarrow IR_{n+1} = R - w_{n+1} - \frac{h_{n+1}}{2} q_{n+1} = \ldots = IR_n + \frac{q_n}{2} (h_n - h_{n+1}) > 0, \text{ as } h_{n+1} < h_n
\]

\[
\vdots
\]

\[
IR_i = \ldots = R - w_{i+1} - \frac{h_i}{2} q_{i+1} = IR_{i+1} + \frac{q_{i+1}}{2} (h_{i+1} - h_i)
\]

Hence, if all \( q_{i+1} \), \( q_{n-1} \), and \( q_n \) decrease given a signal \( H_k \), then the informational rent \( IR_i \) decreases as well.

The impact on the order sizes \( q_m \), \( m = i + 1,\ldots,n \) is analyzed by computing the change of \( \gamma_m ^k \) given the changes of \( \hat{p}_t (h_t \mid H_k) \), \( t = i + 1,\ldots,n \). From \( \frac{\partial \gamma_m ^k}{\partial \hat{p}_t (h_t \mid H_k)} \leq 0, \forall t = i + 1,\ldots,n \) it follows directly
that \( q_m \leq q_m^* \) as long as \( p_i \geq \hat{p}_i(h_i | H_k) \). The condition \( p_i \geq \hat{p}_i(h_i | H_k) \) holds for all \( H \leq \hat{h} \) (see theorem 1), i.e. as long as the buyer reports truthfully or understates her holding costs.

**Theorem 2.2:** The expected costs of the deceptive buyer can either increase or decrease due to communication.

The deceptive buyer may either (a) report accidentally truthful, i.e. \( H = \hat{h} \), or (b) misreport her holding cost, i.e. \( H \neq \hat{h} \).

Case (a): Accidental truthful reporting

\( H = \hat{h} \) may occur if \( \phi_i > 0 \) holds.

From theorem 2.1 it follows directly that the deceptive buyer is worse off if she reports truthfully.

Case (b): Deceptive reporting

Deceptive reporting is formalized by \( H \neq \hat{h} \). The case of the deceptive reporting is divided into three subclasses, (b1) an understatement of holding cost, (b2) an overstatement of holding cost, but not to the maximum extent and (b3) an overstatement to the maximum extent.

Case (b1): \( H < \hat{h} \) (understatement of holding costs)

From theorem 2.1 it follows that the deceptive buyer can only be worse off if she understates her holding cost.

Case (b2): \( \hat{h} < H < h_n \) (overstatement, not to the maximum extent)

In this case, the deceptive buyer can either be better off or worse off. If the reduction of the informational rents \( IR_k, \ldots, IR_n \) given \( H_k \) and \( \hat{h} = h_i \) is compensated by the increase of the informational rents \( IR_i, \ldots, IR_{k-1} \), then she is better off. Otherwise, she is not. However, this depends on the specific cost structure.

Case (b3): \( h_n = H \geq \hat{h} \) (overstatement to the maximum extent)

From theorem 2.1 and condition (1) (see Section 3) it follows directly that the deceptive buyer can only be better off if she constantly signals \( H_n \). In this case, the supplier adjusts the order quantity \( q_n^n \) upwards, i.e. \( q_n^n > q_n^* \). This leads to an increase of all informational rents \( IR_i, i = 1, \ldots, n-1 \).
Theorem 3.2: As long as \( \alpha \geq \sum_{i, k \in k} \sum_{k} p_i \phi_i \Delta CD_i^k + \sum_{i} p_i \phi_i \Delta CD_i^j - \sum_{i} (1 - \phi) \Delta CD_i^j = \alpha_{\text{crit}} \) holds, communication enhances the supply chain performance.

\[
\Delta CD = (1 - \alpha) \sum_{i, k \in k} \sum_{k} p_i \phi_i \Delta CD_i^k + \alpha \sum_{i} p_i \Delta CD_i^j \\
+ (1 - \alpha) \sum_{i} p_i \phi_i \Delta CD_i^j \leq 0
\]

\[
\sum_{i, k \in k} \sum_{k} p_i \phi_i \Delta CD_i^k + \sum_{i} p_i \phi_i \Delta CD_i^j \\
\leq \alpha \sum_{i} \sum_{k \in k} \sum_{k} p_i \phi_i \Delta CD_i^k - \sum_{i} (1 - \phi) \sum_{i} p_i \Delta CD_i^j = \alpha_{\text{crit}}
\]

As \( \Delta CD_i^j \leq 0 \) it can be easily shown that \( \alpha_{\text{crit}} \leq 1 \) holds.

Theorem 3.3: Given a certain probability that the supplier interacts with an honest buyer, i.e. \( \alpha \), it is more likely that communication is an appropriate coordination mechanism if the supplier’s trust in the buyer’s signal decrease, i.e. if \( \hat{\alpha} \) decrease.

If \( \alpha = \alpha_{\text{crit}} \) holds, it follows that \( \Delta CD = 0 \) (see theorem 3.2). Additionally, \( \frac{\partial \Delta CD}{\partial \alpha} \leq 0. \)

\[
\Delta CD = (1 - \alpha) \sum_{i, k \in k} \sum_{k} p_i \phi_i \Delta CD_i^k + \alpha \sum_{i} p_i \Delta CD_i^j \\
+ (1 - \alpha) \sum_{i} p_i \phi_i \Delta CD_i^j
\]

\[
\frac{\partial \Delta CD}{\partial \alpha} = -\sum_{i, k \in k} \sum_{k} p_i \phi_i \Delta CD_i^k + \sum_{i} p_i \Delta CD_i^j \\
- \sum_{i} p_i \phi_i \Delta CD_i^j \leq 0
\]

\[
\sum_{i} p_i (1 - \phi) \Delta CD_i^j \leq \sum_{i, k \in k} \sum_{k} p_i \phi_i \Delta CD_i^k
\]
As $\Delta CD_i \leq 0 \ \forall i = 1,\ldots,n$ and $CD^+ \geq 0, \forall i, k = 1,\ldots,n; i \neq k$ (see theorem 2.1) it follows directly that $\frac{\partial \Delta CD}{\partial \alpha} \leq 0$ is true. Hence, it follows that

$\alpha < \alpha_{crit} \rightarrow \Delta CD < 0$ and $\alpha > \alpha_{crit} \rightarrow \Delta CD > 0$, respectively. From this fact theorem 3.3 follows directly.

### 9.2 Notation

- $C_b$ : cost buyer [monetary units per period]
- $C_{sc}$ : supply chain cost in dependence of the order size $q^e_i$ [monetary units per period]
- $E(P_s)$ : expected profits supplier [monetary units per period]
- $E(C_b)$ : expected cost buyer [monetary units per period]
- $E(\Delta P_s)$ : expected change in profits of the supplier due to communication [monetary units per period]
- $E(\Delta C_b)$ : expected change in cost of the buyer due to communication [monetary units per period]
- $E(\Delta C_{b,honest})$ : expected change in cost of the honest buyer due to communication [monetary units per period]
- $E(\Delta C_{b,deceptive})$ : expected change in cost of the deceptive buyer due to communication [monetary units per period]
- $f$ : fixed cost per delivery [monetary units]
- $H$ : Set of signals
- $H_i$ : signal from buyer to supplier, where $H_i = h_i, \forall i = 1,\ldots,n$ and „no signal“ for $i = n + 1$
- $h$ : holding cost of the buyer under full information [monetary units per unit and period]
- $h_i$ : possible holding cost realization of the buyer under asymmetric information [monetary units per unit and period]
\( \tilde{h} \): actual holding cost of the buyer in a period [monetary units per unit and period]

\( IR_i \): informational rent of the buyer facing holding cost \( h_i \), \( i = 1, \ldots, n \) [monetary units per unit and period]

\( P_s \): profit supplier [monetary units per period]

\( p_i \): a-priori probability that the buyer faces holding cost \( h_i \), \( i = 1, \ldots, n \)

\( p_i(h_i \mid H_k) \): actual a-posteriori probability, \( i = 1, \ldots, n \); \( k = 1, \ldots, n+1 \)

\( \hat{p}_i(h_i \mid H_k) \): perceived a-posteriori probability \( H_k \); \( i = 1, \ldots, n \); \( k = 1, \ldots, n+1 \)

\( p_i(h_i \cap H_k) \): actual conjoint probability distribution; \( i = 1, \ldots, n \); \( k = 1, \ldots, n+1 \)

\( q \): order size [quantity unit]

\( q_i \): order size that corresponds to the holding cost realization \( h_i \), \( i = 1, \ldots, n \) [quantity unit]

\( q_i^{*} \): optimal order size (w.r.t. problem AI) that corresponds to the holding cost realization \( h_i \), \( i = 1, \ldots, n \) [quantity unit]

\( q_{i,sc}^{*} \): supply chain optimal order size that corresponds to the holding cost realization \( h_i \), \( i = 1, \ldots, n \) [quantity unit]

\( q_i^{k} \): optimal order size (w.r.t. problem AI) that corresponds to the holding cost realization \( h_i \), \( i = 1, \ldots, n \) and is calculated w.r.t. \( \hat{p}_i(h_i \mid H_k) \), \( i = 1, \ldots, n \); \( k = 1, \ldots, n+1 \) [quantity unit]

\( q_{min} \): minimum order size under which the supplier yields (positive) profits [quantity unit]

\( R \): cost of the outside option [monetary units]

\( w \): wholesale price under full information [monetary units]

\( w_i \): wholesale price that corresponds to holding cost realization \( h_i \), \( i = 1, \ldots, n \) [monetary units]

\( w_i^{*} \): optimal wholesale price (w.r.t. problem AI) that corresponds to holding cost realization \( h_i \), \( i = 1, \ldots, n \) [monetary units]

\( w_{sc}^{*} \): supply chain optimal wholesale price under full information [monetary units]
\( w_i^k \): optimal wholesale price (w.r.t. problem AI) that corresponds to the holding cost realization \( h_i \), \( i=1,\ldots,n \) and is calculated w.r.t. \( \hat{p}_i(h_i \mid H_k) \), \( i=1,\ldots,n; k=1,\ldots,n+1 \) [monetary units]

\( \hat{\alpha} \): estimated (perceived) probability of interacting with an honest buyer

\( \alpha \): actual probability of interacting with an honest buyer

\( \alpha_{crit} \): critical portion of honest agents that are required for communication to be a coordination instrument

\( \Delta CD \): expected change of the coordination deficit due to communication [monetary units]

\( \Delta CD_i^k \): expected change of the coordination deficit due to communication given holding cost realization \( h_i \) and signal \( H_k \), \( i=1,\ldots,n; k=1,\ldots,n+1 \) [monetary units]

\( \Delta P_{i,j}^k \): expected change of the buyer’s expected profits due to communication given holding cost realization \( h_i \) and signal \( H_k \), \( i=1,\ldots,n; k=1,\ldots,n+1 \) [monetary units]

\( \Delta C_{b,i}^k \): expected change of the buyer’s expected costs due to communication given holding cost realization \( h_i \) and signal \( H_k \), \( i=1,\ldots,n; k=1,\ldots,n+1 \) [monetary units]

\( \gamma_i^k \): expression that captures the distortion due to asymmetric information, \( i=1,\ldots,n; k=1,\ldots,n+1 \)

\( \hat{\phi}_i \): expected (perceived) randomization probability, \( i=1,\ldots,n+1 \)

\( \phi_i \): actual randomization probability, \( i=1,\ldots,n+1 \)

\( \hat{\phi}_i(h_k) \): expected (perceived) strategic signaling variable, \( i=1,\ldots,n+1; k=1,\ldots,n \)

\( \phi_i(h_k) \): actual strategic signaling variable, \( i=1,\ldots,n+1; k=1,\ldots,n \)

\( \tilde{\phi}_i \): probability of signal \( H_i \) (\( \tilde{\phi}_i = \alpha p_i + (1-\alpha)\phi_i \); \( i=1,\ldots,n+1 \)