The Value of Delegation in a Dynamic Agency

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Abstract

In this paper we analyze the value of delegation in a two-period agency. A central management hires an agent to perform a personal effort in each period. Due to time constraints or lack of ability this effort can not be performed by central management. Besides personal effort firm value is influenced by the decision to launch a project which has to be made at the beginning of period two. The project decision can either be delegated to the agent (decentralization) or it can be made by central management (centralization). Under decentralization the agent observes the project’s contribution before its decision. While this captures the benefit of delegation its cost is that the project decision is unobservable and must be motivated together with personal effort via the same incentive contract. In the centralized regime, in contrast, no incentives for the project decision are necessary, however, the project’s actual contribution will not be observed such that the project decision has to be made based on expectations. We analyze optimal performance measurement for both regimes in a linear contracting setting and analyze the variables that affect the value of delegation. We do this for two different contracting regimes: long-term commitment and long-term renegotiation-proof contracts. Trade-offs under both contracting environments differ substantially. In particular, under renegotiation-proof contracts, decentralization might become optimal even if its direct benefit in terms of acquiring specific knowledge about the project vanishes. The reason is that with delegation of the project decision central management implicitly commits to a higher second period incentive rate as personal effort and the project decision must be controlled via the same incentive contract. This is beneficial if renegotiation-proofness requires central management to set too low second-period incentives compared to long-term commitment. A necessary condition for that is, that intertemporal correlation is negative. Contrary to the classical view this result implies that the incentive problem under centralization may become more severe than under decentralization.
Abstract:
In this paper we analyze the value of delegation in a two-period agency. A central management hires an agent to perform a personal effort in each period. Due to time constraints or lack of ability this effort can not be performed by central management. Besides personal effort firm value is influenced by the decision to launch a project which has to be made at the beginning of period two. The project decision can either be delegated to the agent (decentralization) or it can be made by central management (centralization). Under decentralization the agent observes the project’s contribution before its decision. While this captures the benefit of delegation its cost is that the project decision is unobservable and must be motivated together with personal effort via the same incentive contract. In the centralized regime, in contrast, no incentives for the project decision are necessary, however, the project’s actual contribution will not be observed such that the project decision has to be made based on expectations. We analyze optimal performance measurement for both regimes in a linear contracting setting and analyze the variables that affect the value of delegation. We do this for two different contracting regimes: long-term commitment and long-term renegotiation-proof contracts. Trade-offs under both contracting environments differ substantially. In particular, under renegotiation-proof contracts, decentralization might become optimal even if its direct benefit in terms of acquiring specific knowledge about the project vanishes. The reason is that with delegation of the project decision central management implicitly commits to a higher second period incentive rate as personal effort and the project decision must be controlled via the same incentive contract. This is beneficial if renegotiation-proofness requires central management to set too low second-period incentives compared to long-term commitment. A necessary condition for that is, that intertemporal correlation is negative. Contrary to the classical view this result implies that the incentive problem under centralization may become more severe than under decentralization.
1 Introduction

The benefits and costs of decentralization (relative to centralization) have been largely discussed in the literature. The main comparative advantage of delegating decision rights is seen in the ability to use specific knowledge of local managers for decision making. Its disadvantage is the occurrence of incentive and control problems. Taking a mechanism-design view on the problem, decentralization does never outperform centralization: The revelation principle ensures that every mechanism in which decision authority is delegated to an agent can be replicated by a mechanism in which the subordinate transmits the relevant information to central management which then makes the decision. Importantly, this result does apply only in a world with costless communication and an unconstraint contracting space. In practice, however, costly communication is a major reason for decentralized organizational structures (see Kaplan/Atkinson (1989)). Thus, there is a need to investigate the value of delegation in settings in which the revelation principle is not valid. There are many papers that compare decentralization and centralization under different circumstances (e.g., Baiman/Rajan (1995), Bushman/Indjejikian/Penno (2000), Stein (2002)).

In this paper we take a dynamic perspective on the value of delegation. We consider a two-period agency problem where central management hires an subordinate (agent) to perform an action in each period. These actions which we call personal effort must be delegated due to time constraints or lack of expertise by central management. In addition, at the beginning of the second period a decision about launching a project has to be taken. This decision can either be made by central management or it can be delegated to the agent. If the decision authority over the project is delegated to the agent (decentralization), he privately learns the project’s true contribution to firm value at the end of the first period. If central management decides upon the project (centralization) the project’s contribution will neither be observed by central management nor by the subordinate such that the decision has to be made based on expectations. The primary trade-off between both organizational forms that is emphasized in the literature shows in our paper as follows: Under decentralization there is superior information about the project’s value but its efficient use through the agent must be controlled via an incentive contract (together with personal effort). Under centralization there is no control problem with respect to the project’s decision but relevant information is missing.

We analyze both organizational forms for two different contracting regimes: long-term commitment and long-term renegotiation-proof contracts. Under long-term commitment the classical view on the trade-off between decentralization and centralization is confirmed. While the con-

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2 See Melumad/Reichelstein (1988) who compare under which conditions a decentralized mechanism without communication leads the same outcome as the optimal mechanism based on the revelation principle.
3 In line with Milgrom/Roberts (1992), p. 544, we assume that decentralization does not only mean that the agent is authorized to decide upon the project but also that he does not reside in the central office but on the scene. Under centralization he works in the central office and has no possibility to gather specific information.
trol problem under decentralization is always as severe as under centralization the effect from the project in each setting is less clear. We provide necessary and sufficient conditions such that decentralization dominates centralization and vice versa. We find that high contributions of both, personal effort and the project, yield that decentralization outperforms centralization. While long-term commitment can be viewed as a theoretical benchmark it is implausible from a practical perspective as it assumes that contracting parties can credibly commit to stick to an incentive contract that is ex post inefficient. Therefore we analyze in a second step initial long-term contracts that are robust against renegotiation (renegotiation-proof). We show that the trade-off between both organizational forms with renegotiation-proof contracts differs substantially from the commitment setting. In particular, we show that with a renegotiation-proof contract the induced control problem may become more severe under centralization than under decentralization. This implies that decentralization can dominate centralization even if its direct benefit (in terms of specific knowledge) is not present. The reason for this result is that with the decision for a decentralized system central management implicitly commits to a higher second period incentive rate as personal effort and the project decision must be controlled via the same incentive contract. This is beneficial from an ex ante perspective if the renegotiation-proof contract requires central management to set too low second-period incentives as compared to long-term commitment. A necessary condition for this effect is that output is negatively correlated across both periods.

Our paper contributes to the literature that analyzes the value of delegation in agencies with hidden action and hidden information. In this respect it is related to Bushman/Indjejikian/Penno (2000) who analyze a one shot agency where the agent obtains pre-decision information about the contribution of his unobservable effort under decentralization. Under centralization no such information will be observed, however, effort is assumed to be observable and contractible. In contrast to this setting we assume that under both organizational forms there are always incentive problems present as even under centralization some decisions must be delegated. We analyze how the decision whether to delegate the project decision influences the optimal incentive contract and thus personal effort. Similar to Bushman/Indjejikian/Penno (2000) and Raith (2008) we assume that the agent obtains private specific knowledge without any cost. Our focus is on incentives that provide an optimal decentral use of specific knowledge. In contrast, e.g., Demski/Sappington (1987) analyze the value of delegation when information acquisition is costly.

We further contribute to the literature that analyzes mechanisms to mitigate distortions caused by limited commitment in dynamic agencies. Especially in the accounting literature some practices usually labeled inferior have been proven optimal in such settings. E.g., Indjejikian/Nanda (1999) show that aggregation of measures - that reduces degrees of freedom in incentive contracting - might be optimal under limited commitment and Demski (1998), Christensen/Demski/Frimor (2002), and Demski/Frimor (1999) show optimality of earnings management and performance measure manipulation for similar reasons. The strategic value of delegation has been empha-
sized in the early literature by Schelling (1960).\textsuperscript{4} It has been deeper analyzed in the presence of competition (Vickers (1985), Fershtman/Judd (1987), Göx/Schöndube (2004)). Here delegation serves as a commitment to act more aggressively in competition. In a public economics context Melumad/Mookherjee (1989) show in a three-layer game, that it may be optimal for the government to delegate decisions over tax audit policies to revenue collecting agencies. This benefit occurs as the government is unable to credibly commit to tax audit policies which may create considerable welfare losses. Again, delegation of the decision authority here serves as a commitment substitute. To our best knowledge the usefulness of decentralization as a commitment device in a dynamic intra-firm incentive problem has not been shown yet.

The rest of the paper is organized as follows. The next section presents the model. In section 3 we analyze the value of delegation under long-term commitment. We determine the equilibrium solutions for both organizational forms and provide conditions under which delegation has positive value. In section 4 we investigate the optimal organizational structure under renegotiation-proof contracts. Special attention is devoted to the value of delegation to reduce welfare losses from limited commitment. Section 5 concludes.

2 The model

We analyze a two-period agency model where a central management (CM) hires an agent to provide a personal effort in each period. The agent’s effort in period $t = 1, 2$ is denoted by $e_t$. Personal effort is unobservable and must be delegated to an agent, i.e. central management is not able, e.g., due to time constraints, to provide this effort. The firm’s output in period $t$ (gross of the agent’s compensation) is given by

$$x_1 = ge_1 + \eta_1$$
$$x_2 = ge_2 + \theta b + \eta_2$$

$g > 0$ is the marginal contribution of the agent’s effort. $\eta_1$ and $\eta_2$ are normally distributed random variables with mean zero, variance $\sigma^2$ and correlation coefficient $\pi$. Besides managerial effort output in period 2 is influenced by the payoff of a project. This project is initiated at the beginning of the second period by a decision $b$ and leads to a payoff of $\theta b$. The project decision can either be delegated to the agent (decentralization) or it can be made by central management (centralization). With respect to the centralized regime two alternative scenarios are reasonable. Either central management is as capable as the manager and makes the decision on $b$ at the same cost as the manager would do; the cost of $b$ is $b^2/2$. Or (similar to Bushman/Indjejikian/Penno (2000)) the CM is able to force the manager to choose a certain $b$. Both scenarios lead to the same results in our model. In the presentation of our model, however, we stick to the former interpretation.

\textsuperscript{4}See, e.g., p. 142 f.
The factor \( \theta \) of \( b \) in the project’s payoff is modeled as a binary random variable that takes 0 with probability \( p \) and \( \gamma > 0 \) with \( 1 - p \). In a centralized firm \( \theta \) will not be observed before the decision on \( b \) is made. Hence, the optimal \( b \) depends on \( E(\theta) = (1 - p) \gamma \). In the decentralized firm the agent receives specific knowledge about the project’s payoff at the beginning of the second period. More precisely, we assume that the agent privately observes the contribution \( \theta \) of \( b \) immediately before the deciding on \( b \).

We consider the first period after the decentralized structure is implemented as a learning period. The agent, residing outside the central office, obtains general information about the project. This general information enables him to deduce the project’s true contribution \( \theta \) at the beginning of the second period.

We assume that communication is costly such that it is not possible to transmit \( \theta \) to central management under decentralization. Furthermore, under decentralization the \( b \)-decision is unobservable similar to personal effort. Under both regimes \( \theta \) will be observed ex post, i.e. at the end of period 2. Figure 1 summarizes the timeline of events.

The agent is risk- and effort-averse. His personal cost of effort is given by \( C(e) = \frac{e_1^2}{2} + \frac{e_2^2}{2} \).

To capture risk aversion we assume that the agent’s preferences can be represented by a mean-variance functional\(^5\) of the form

\[
\Gamma = E(y) - \frac{r}{2} Var(y),
\]

with \( y = S - C(e) \left( -\frac{b^2}{2} \right) \) under decentralization, as the manager’s residual wealth. \( S \) denotes the agent’s compensation and \( r \geq 0 \) is his risk aversion coefficient. We restrict attention to a long-term linear contract

\[
S = f + s_1x_1 + s_2x_2 + s_3\theta
\]

with \( f \) as a fixed payment and \( s_1, s_2, s_3 \) as incentive weights for the contracting variables \( x_1, x_2, \) and \( \theta \). The agent’s reservation utility for the two-period duration of the agency is 0.

The central management is risk neutral. It selects contract \( S \) (and \( b \) with centralization) to maximize his expected net output \( U = E(x_1 + x_2 - S) \left( -\frac{b^2}{2} \right) \) taking into account that the

\(^5\) Notice that due to the binary random variable \( \theta \) output in period 2 is no longer normally distributed such that we cannot apply the LEN approach that derives mean variance preferences endogenously. To allow for closed form solutions we therefore assume mean-variance preferences exogenously, like, e.g., Feltham/Wu (2001).
contract must be individually rational and incentive compatible for the agent.

We distinguish two different contracting regimes: Under long-term commitment a contract \( S \) is settled at \( t = 0 \) and cannot be revised or renegotiated subsequently. Under a long-term renegotiation-proof contract, we analyze a situation in which contracting parties cannot commit not to renegotiate the contract after the first period. Thus we focus on initial contracts \( S \) that are robust against renegotiation at the end of the first period.

3 Long-term commitment contracts

3.1 Centralized System

The solution to the centralized system can be decomposed into two independent problems: A managerial effort (risk-incentive) problem and a project-decision problem. The project-decision problem is given by

\[
\max_b E \left( b\theta - b^2/2 \right)
\]

and its solution is \( b^c = E(\theta) = (1-p)\gamma \) with corresponding project payoff

\[
B^c = E(\theta)^2/2 = (1-p)^2 \gamma^2/2.
\]

The remaining problem (the managerial effort problem) is to choose the parameters \( s_1, s_2, s_\theta \), and \( f \) of the compensation contract to induce actions \( e_1 \) and \( e_2 \) in the best interest of the central management. The corresponding optimization problem is given by

\[
\max_{s_1,s_2,s_\theta,f} E \left( x_1 + x_2 - S \right)
\]

subject to

\[
\Gamma(S,e) = E(S - C(e)) - \frac{\gamma}{2} Var(S) \geq 0
\]

\[
e = \arg \max_{e'} \Gamma(S,e') = sg.
\]

The first constraint is the participation constraint for the agent which is binding at the optimum. The second constraint is the incentive compatibility constraint for the agent’s effort. The variance of the agent’s compensation is given by \( Var(S) = Var(s_1x_1 + s_2x_2 + s_\theta \theta) \).

**Lemma 1** The optimal weight for \( \theta \) in the incentive contract is given by \( s^{c}_\theta = -s_2 b^c \). Given \( s^{c}_\theta \) the variance of the agent’s wealth becomes \( Var(S) = Var(s_1x_1 + s_2x_2 + s_\theta \theta) = \sigma^2 \left[ s_1^2 + s_2^2 + 2s_1s_2\pi \right] \).

**Proof.** See the Appendix. \( \blacksquare \)

Lemma 1 shows that given the optimal incentive weight \( s^{c}_\theta \) the risk associated with the uncertain project contribution \( \theta \) is completely removed from the agent’s compensation. As \( \theta \) is observable

\[^6\text{The variable } x_2^{-b} \text{ is used to clarify that the payoff from the managerial effort problem is calculated net of the project’s payoff } B^c.\]
at the end of the second period central management can perfectly insure the agent against the risk of $\theta$. By setting $s_\theta$ equal to $s_2 b'c'$ a perfectly negative correlation between $s_\theta \theta$ and the agent’s share $s_2 b'c'$ in the project’s payoff (in $x_2$) is induced. Of course, if $\theta$ was not observable this insurance possibility does not exist. However, it is reasonable to assume that the conditions that affect the project’s payoff become observable ex post. In addition, the result of Lemma 1 - which carries over to the decentralized regime as we will show later - simplifies the analysis considerably as it removes terms that do not influence first order effects: Compensation risk is still included by the performance measure risks $\eta_1$, $\eta_2$.

>From Lemma 1 it also follows that given $s_\theta$ the agency-problem in (2) formally corresponds to a standard LEN-model with multiple actions and performance measures, which is in depth analyzed by Feltham/Xie (1994). The optimal solution for the incentive weights $s_1$ and $s_2$ is given by

$$s_1^c = s_2^c = \frac{g^2}{g^2 + r\sigma^2 + r\pi\sigma^2}. \quad (3)$$

The incentive rate for each period weights the squared productivity of the agent’s managerial effort $g^2$ in the numerator by a denominator that reflects productivity $g^2$, the risk associated with the period’s compensation $r\sigma^2$, and an intertemporal covariance effect $r\pi\sigma^2$. The latter offers an insurance effect if $\pi < 0$ and for positive correlation it imposes additional risk on the agent’s compensation. For the subsequent investigation it is worthwhile to state the optimal $s_1$ as a function of the second period incentive rate $s_2$:

$$s_1^*(s_2) = \frac{g^2 - r\sigma^2\pi s_2}{g^2 + r\sigma^2}. \quad (4)$$

This "reaction function" is derived from the first order condition for $s_1$ in program (2). It is the same for every contracting environment (full commitment and renegotiation-proofness) and for every organizational structure (centralization and decentralization) we consider. Writing the central management’s payoff from the managerial effort problem as a function of $s_2$ by substituting (4) for $s_1$ into the objective function of (2) leads to

$$M(s_2) = E \left( x_1 + x_2 - b | s_1^* \right) - E (S|s_1^*) = E \left( x_1 + x_2 - b | s_1^* \right) - C(e|s_1^*) - \frac{r}{2} Var(S|s_1^*) \quad (5)$$

$$= \frac{1}{2G} \left[ g^4 + G \left( \gamma (1 - p)^2 \right) + (2g^2G - 2g^2r\sigma^2\pi) s_2 - (g^4 + 2g^2r\sigma^2 + \sigma^4r^2 (1 - \pi^2)) \right],$$

with $G = g^2 + r\sigma^2$. For the centralized regime we analyze in this section the corresponding reaction function for $s_2$, $s_2^*(s_1)$, which is symmetric to (4). As $-r\sigma^2\pi \leq r\sigma^2$, it follows that $s_1 \leq 1$ and $s_2 \leq 1$. If correlation is positive $s_1^*$ is diminishing in $s_2$ and vice versa. This is the additional risk effect mentioned above. It is increasing in $s_2$. For negative correlation increasing reaction functions offer an insurance opportunity. Higher efforts in both periods will be induced due to a decreasing risk premium to be paid to the agent. With zero correlation one obtains the standard one-shot LEN solution for both periods.

Inserting (3) for $s_2$ into (5) leads to the following payoff from the managerial effort problem:

$$M^c = M(s_2^c) = \frac{g^4}{G + r\pi\sigma^2}.$$
and to a total profit of

\[ U^c = M^c + B^c = \frac{g^4}{G} + (1 - p)^2 \gamma^2. \]  

(6)

3.2 Decentralized regime

In the decentralized regime the decision on the optimal volume of the project \( b \) is delegated to the agent. Immediately before his decision the agent observes the true value of \( \theta \). As opposed to the centralized regime, however, the agent does not necessarily choose \( b \) in the best interest of the owners but he chooses the value of \( b \) that maximizes his preference functional \( \Gamma \). For a given incentive weight \( s_2 \) in the second period the agent selects \( b \) to maximize

\[
\max s_2 b\theta - b^2/2
\]

which leads to an optimal \( b \) of

\[
b^d (s_2) = s_2 \theta.
\]

(8)

Inserting \( b^d \) for \( b \) into (7) leads to a project payoff of \( \frac{g^4}{T} s_2 (2 - s_2) \) conditional on \( \theta \) and to an expected payoʃ of

\[
B^d (s_2) = \frac{E [\theta^2]}{2} s_2 (2 - s_2).
\]

(9)

In contrast to the centralized regime, in the decentralized regime it is not possible to decompose the whole problem into a managerial eʃort problem and a project related problem both being independent. \( s_2 \) influences the optimal \( b \) (and \( B^d \)) and the managerial eʃort problem, such that both problems must be considered simultaneously. The central management’s optimization problem to be solved is given by

\[
\max_{s_1, s_2, s_\theta} E (x_1 + x_2 - S) \tag{10}
\]

subject to

\[
\Gamma (S, e, b) = E (S - C (e) - b^2/2) - \frac{T}{2} Var (S - b^2/2) \geq 0
\]

\[
e = \arg \max_{e'} \Gamma (S, e') = sg
\]

\[
b = s_2 \theta.
\]

Problem (10) diʃers in two respects from problem (2) above. First, the agent now decides upon \( b \) which leads to the additional incentive constraint \( b = s_2 \theta \). Second, as \( b \) is random ex ante its personal cost are random, too, and as such the cost influences also the variance of the agent’s wealth.

Similar to the centralized regime we first show that the optimal incentive weight \( s_\theta^d \) eliminates any risk associated with \( \theta \) from the agent’s compensation:

Lemma 2 The optimal weight of \( \theta \) in the incentive contract is given by \( s_\theta^d = -s_2^2 \gamma/2 \). Given \( s_\theta^d \) the variance of the agent’s wealth becomes \( Var \left( S - b^{d^2}/2 \right) = Var (s_1 \eta_1 + s_2 \eta_2) = \sigma^2 \left[ s_1^2 + s_2^2 + 2s_1 s_2 \pi \right] \).
Proof. See the appendix. ■

The following Lemma states the solution for the optimal incentive weights \( s_1 \) and \( s_2 \) and the corresponding surplus of the central management.

**Lemma 3** Optimal incentive weights \( s_1 \) and \( s_2 \) under decentralization are given by

\[
\begin{align*}
    s_1^d &= \frac{G_y (g^2 - r\sigma^2\pi) + r\sigma^2 g^2}{G_y G + r\sigma^2 (g^2 + r\sigma^2 (1 - \pi^2))} \\
    s_2^d &= \frac{G_y G - \pi r\sigma^2 g^2}{G_y G + r\sigma^2 (g^2 + r\sigma^2 (1 - \pi^2))}
\end{align*}
\]

with \( G_y = g^2 + \gamma^2 (1 - p) \). The central management’s equilibrium payoff is

\[ U^d = M^d + B^d \]

with \( B^d = s_2^d (2 - s_2^d) (1 - p) \gamma^2 \) and \( M^d = M (s_2^d) \).

Proof. See the Appendix. ■

### 3.3 Comparison of organizational forms

In this section we clarify the trade-off between both systems and investigate by which variables the trade-off is significantly driven. We also provide comparative static results and conditions for one organizational form being superior to the other. We first analyze differences in the incentive weights \( s_1 \) and \( s_2 \).

**Lemma 4** \( s_2^c \leq s_2^d \). Assume \( r, \sigma > 0 \) and \( p \neq 1 \). Then \( s_1^c \begin{cases} > & \text{iff } \pi = 0 \\ < & \text{iff } \pi = 0 \end{cases} \)

Proof. The proof follows straightforward by factorizing the differences \( (s_t^e - s_t^d), t = 1, 2 \), according to the results of sections 3.1 and 3.2. ■

As in the decentralized regime the second period incentive rate has to control two decisions (\( b \) and \( e_2 \)) it is always (weakly) higher than in the centralized regime. From the reaction function (4) we know that for positive (negative, no) correlation \( \pi \), \( s_1 \) is increasing (decreasing, not varying) in \( s_2 \). Thus, depending on the sign of the correlation coefficient first period incentives under centralization are either higher (\( \pi > 0 \)), lower (\( \pi < 0 \)) or the same compared to decentralization. Given this result we are able to provide a graphical representation of the trade-off between both regimes. Notice, that according to (6) and Lemma 3 CM’s total surplus consists of a surplus \( B \) which refers to the project’s payoff and a surplus \( M \) which is the surplus from the managerial effort problem net of \( B \). Let \( \Delta M \equiv M^c - M^d \) and \( \Delta B \equiv B^c - B^d \). We start analyzing the differences between both regimes with regard to the managerial effort problem.
in figure 2. We plot $M(s_2)$ the maximum surplus from the managerial effort problem as a function of $s_2$ according to the definition in (5). This function has its maximum at $s_2 = s^*_2$, the optimal incentive rate in the centralized regime. As in the decentralized regime the second-period incentive rate is higher than $s^*_2$, the induced managerial risk-incentive problem under decentralization is always (weakly) more severe than under centralization, $\Delta M \geq 0$.

Next we analyze how both organizational forms affect the payoff from the project. Figure 3 plots $\Delta B (s^d_2) = -\gamma^2 (1 - p) (s^d_2 (2 - s^d_2) - (1 - p)) / 2$ as a function of $s^d_2$ for the same parameters as in figure 2. The higher $s^d_2$ the higher the relative benefit from decentralization as with increasing second period incentives the project decision of the agent becomes more congruent with the objectives of CM. In the limit $s^d_2 \to 1$ the "first-best" choice for $b$ is induced and the relative advantage of decentralization with respect to the project’s payoff becomes maximal. As (except for $p = 1$) $\Delta B$ is strictly decreasing in $s^d_2$ and $\Delta B (0) > 0$ and $\Delta B (1) < 0$ the intermediate value theorem guarantees a unique null (critical value) $s^{crit}_2$ such that $\Delta B (s^{crit}_2) = 0$ and $\Delta B < (> ) 0$ if $s^d_2 > ( < ) s^{crit}_2$. For the given parameters of the plot the resulting value is denoted by $s^{d*}_2$.

Hence, for the parameters of the plots centralization dominates decentralization with respect to the incentive problem but decentralization dominates centralization with respect to the project’s payoff. While the former holds true for all parameters the latter depends.

In the following proposition we provide sufficient conditions for the dominance of centralization and decentralization, respectively. Of course, non-validity of the conditions are necessary for the
dominance relation being the other way around.

**Proposition 1** *(Sufficient conditions and comparative statics)*

a) If \( p \to 0 \), \( U^c \geq U^d \)  
Assume \( 0 < p < 1 \) in what follows.

b) For \( r = \sigma = 0 \), \( U^c < U^d \).

c) \( \partial (U^c - U^d) / \partial \sigma (r) > 0 \). If \( \sigma (r) \) is sufficiently high (low) \( U^c > U^d \) \( (U^c < U^d) \)

d) If the contribution of managerial action \((g)\) or contribution of the special task in the good case \((\gamma)\) are sufficiently high, \( U^c < U^d \).

**Proof.** See the Appendix.

\( p = 0 \) means that the contribution of the project is \( \gamma \) with certainty. Hence, there is a strict advantage of centralization with respect to both the managerial effort problem and the delegation problem except for the special case of \( \pi = -1 \) that we discuss subsequently to proposition 2.

For strictly positive values of \( p \), if the manager is risk neutral or there is no noise in the output the optimal incentive rates are \( s_1 = s_2 = 1 \) in both regimes. This implies that \( \Delta M = 0 \) and as optimal incentives for \( b \) are motivated \( \Delta B < 0 \). Part c) of the proposition shows that if the manager’s risk aversion or the variance of the noise in the performance measures increases, centralization becomes more beneficial relative to decentralization, and vice versa. An increase in \( r \) or \( \sigma \) has two effects: First, it reduces \( s_2^d \) such that lower incentives for \( b \) are provided and \( B^d \) declines. As \( B^c \) does not depend on \( r \) or \( \sigma \) the loss from delegation \( \Delta B \) becomes larger if \( r \) or
\[ \sigma \text{ increase. Second, higher } r \text{ or } \sigma \text{ reduce } s_2^d \text{ and the impact of the managerial effort problem on the overall surplus becomes smaller. Notice, however, that for low } r \text{ or } \sigma \text{ the difference between } s_2^c \text{ and } s_2^d \text{ is reduced if these parameters increase. Therefore, for low values for } r \text{ and } \sigma \text{ } M \text{ may decrease if risk aversion or variance are increasing. For higher values of both variables, the marginal effect on } s_2^c - s_2^d \text{ and } M \text{ is increasing. However, the overall marginal effect of } r \text{ and } \sigma \text{ on } U^c - U^d \text{ is always positive. Notice that } r \text{ and } \sigma \text{ are always multiplicatively connected in the model. For every set of parameters there exists a critical value for } r \text{ such that } U^c = U^d \text{ and } U^c > (\prec) U^d \text{ if } r \sigma > (\prec) \hat{r} \sigma. \]

The effect of a sufficient highly value for \( \gamma \) is as follows. If \( \gamma \) becomes very large \( s_2^d \) approaches to 1. This means that the induced \( b \) in the decentralized regime is close to the first best one. In addition with \( \gamma \) very high the project’s (expected) contribution rises and therefore the advantage to decide on \( b \) based on actual instead of the expected contribution rises. Overall this implies that \( \Delta B \) becomes very negative if \( \gamma \) becomes very high and in the limit \( \Delta B \to -\infty \). On the other hand, the higher \( \gamma \) the higher the deviation \( s_2^d \) from the optimum \( s_2^c \) in the managerial effort problem. This effect, however, does not approach infinity. If \( \gamma \) becomes large, \( s_2^d \) as mentioned above approaches unity. That means that the maximum effect from an increase in \( \gamma \) with respect to the managerial effort problem is \( M^c(s_2^c) - M^d(1) \). That means, if \( \gamma \) becomes sufficiently high the effect on \( \Delta B \) dominates the effect on \( \Delta M \) and \( U^c < U^d \).

The effect on an increasing effort productivity \( g \) is related to the argumentation before. Ceteris paribus the higher \( g \) the closer \( s_2^d \) at unity and the closer the agent’s project decision at first-best. Thus, if \( g \) becomes sufficiently high decentralization becomes dominant with respect to the project’s payoff. But not only \( s_2^d \) gets closer to 1 if \( g \) increases but also the optimal second period incentive weight under centralization, \( s_2^c \). This implies that the relative advantage of centralization with respect to the managerial effort problem decreases in \( g \) such that the overall marginal effect of an increasing \( g \) favors decentralization.

**Proposition 2** Influence of correlation

\[ a) \frac{\partial}{\partial \pi} (U^c - U^d) > 0 \text{ if } \pi < 0. \]
\[ b) \text{If } \pi \to -1 \text{ } U^c < U^d. \]

With negative correlation, \( \pi < 0 \), centralization becomes more profitable relative to decentralization if the intertemporal insurance opportunities decrease (\( \pi \) increases). This effect is due to an increase of the centralized organization’s relative payoff from the project \( \Delta B(s_2^d) \). As the insurance effect is diminishing \( s_2^d \) is decreasing in \( \pi \) if \( \pi \) is negative and \( \Delta B(s_2^d) \) becomes higher. The effect of an increasing (negative) \( \pi \) on the relative payoff from the managerial effort problem \( \Delta M(s_2^c, s_2^d) \) is not unique. Its sign depends on whether \( s_2^c - s_2^d \) is increasing or decreasing in \( \pi \) which in turn depends on the values of the other parameters of the model. Overall, however, the effect on \( \Delta B(s_2^d) \) is always dominant. A similar effect for positive correlation cannot be shown. The reason is that with positively correlated outputs there is no insurance opportunity and both reaction functions \( s_1^d(s_2) \) and \( s_2^d(s_1) \) are falling. If the reduction of \( s_1 \) due to an increase of \( \pi \)
is very strong the optimal $s_2^d$ may become increasing in $\pi$.

If outputs are perfectly negative correlated, risk is completely eliminated from the manager’s compensation package and the optimal solution is to sell the firm to the agent: $s_1 = s_2 = 1$, independently of the regime. As a consequence the managerial effort problem will be solved first-best under both regimes. Furthermore, optimal incentives will be motivated for the project under decentralization; therefore $U^c < U^d$.

4 Long-term renegotiation-proof-contracts

4.1 Renegotiation and renegotiation proofness

It is hard to believe that contracting parties can credibly commit not to renegotiate a long-term contract if there are ex post gains from renegotiation. In our setting such ex post benefits may result from an improved trade-off between risk and incentives after some uncertainty has been resolved. In what follows we therefore allow the central management to offer the agent a new contract for the second period at the end of the first period, i.e., after output $x_1$ has been observed. It has been shown in the literature\(^7\) that the equilibrium solution of the contracting game with renegotiation coincides with a long-term renegotiation-proof contract. Renegotiation-proofness means that the initial contract has to be sequentially optimal; i.e., at the beginning of the second period the central management must not have an incentive to offer a new contract to the agent. For linear contracting models with two-periods Christensen/Feltham/Sabac (2003) have shown that an initial long-term contract is renegotiation-proof if incentive weights for second period measures are chosen sequentially optimal. In our model there are two contractible measures in the second period, $x_2$ and $\theta$, with incentive weights $s_2$ and $s_\theta$, respectively. For both regimes, centralization and decentralization, we now have to consider the constraints $s_2 = s_2^R$ and $s_\theta = s_\theta^R$ on the initial long-term contract, where $R$ denotes renegotiation-proofness. Similar to the full commitment setting the risk associated with $\theta$ is completely removed from the agent’s compensation in the optimal renegotiation-proof contract as is stated in following lemma:

\textbf{Lemma 5} Lemmas 1 and 2 also apply with renegotiation-proof contracts.

\textbf{Proof.} As $\theta$ and $x_1$ are independent, the optimal $s_\theta$ for a given $s_2$ is identical to the long-term commitment setting. \(\blacksquare\)

Lemma 5 does not imply that the optimal $s_\theta$ under a renegotiation-proof contract is the same as under full commitment, but it is the same conditional on $s_2$ such that the same risk sharing effect is induced.

\(^7\)See, e.g., Fudenberg/Tirole (1990).
Prior research has analyzed the conditions under which renegotiation is harmless and leads to the same results than a full commitment contract.\footnote{See Fudenberg/Holmström/Milgrom (1990) and for linear models Indjejikian/Nanda (1999) and Christensen/Feltham/Sabac (2003).} For the linear model considered here, $\pi = 0$ is a sufficient condition that renegotiation does not destroy the incentives of the ex ante efficient long-term contract. For $\pi = 0$ there is no intertemporal risk-sharing effect such that the optimal renegotiation-proof-contract coincides with the optimal long-term contract. For the following analysis we therefore assume $\pi \neq 0$.

### 4.2 Equilibrium results

Formally the optimal renegotiation-proof contract under both organizational forms is determined by solving the respective full commitment problems from section 3 under the additional renegotiation-proofness constraint $s_2 = s_2^R$. As the optimal $b$ under centralization does not depend on the contracting regime it is the same as under full commitment, $b^c$. In the decentralized regime the manager’s choice of $b$ conditional on $\theta$ and $s_2$ is similar to full commitment, except we have to replace $s_2$ by the renegotiation-proof incentive rate $s_2^R$, $b^d (s_2^R) = s_2^R \theta$. Furthermore, the optimal $s_1$ conditional on $s_2$ is similar to the full commitment setting (4), $s_1 (s_2) = \frac{g^2 - r \alpha^2 \pi s_2}{g^2 + r \sigma^2}$ under both regimes.

The sequentially optimal incentive weight $s_2^R$ maximizes the expected second period surplus of the agency, given the observation of first period output $x_1$:

$$s_2^R = \arg \max E \left( x_2 - C (e_2) - b^2/2 | x_1 \right) - \frac{r}{2} Var (s_2 \eta_2 | x_1). \tag{11}$$

Here we have already included the optimal $s_\theta$ as described in Lemma 5 and its effect on the induced variance of the agent’s wealth. We denote the posterior variance of $\eta_2$ given $x_1$ as $\sigma_2^P$. It is given by $\sigma_2^P = \sigma^2 (1 - \pi^2)$. Solving optimization program (11) for both organizational forms leads to the following result:

**Lemma 6** Second period sequentially optimal incentive weights in the centralized regime (C) and the decentralized regime (D) are given by

$$s_2^{Rc} = \frac{g^2}{g^2 + r \sigma_2^P}; \quad s_2^{Rd} = \frac{g^2 + \gamma^2 (1 - p)}{g^2 + \gamma^2 (1 - p) + r \sigma_2^P}.$$

**Proof.** See the appendix. ■

With renegotiation-proof-contracts, as under full commitment, the second period incentive rate under decentralization is higher than under centralization as the manager’s $b$- choice must be motivated, too. As before let $M (s_2)$ be the central management’s payoff from the managerial effort problem as function of $s_2$. $M (s_2)$ is the same for both regimes and given by (5). In addition recall that the payoffs resulting from the project, $B^c$ and $B^d (s_2^{Rd})$, respectively, are
given by (1) and (9) with $s^d_2 = s^{Rd}_2$. Thus, central management’s equilibrium surplus with renegotiation-proof contracts can be written as

$$U^c = M(s^{Rc}_2) + B^c$$
$$U^d = M(s^{Rd}_2) + B^d(s^{Rd}_2).$$

4.3 Comparison of organizational forms

In this section we compare both organizational forms in terms of the trade-off between managerial effort allocation and induced project decision $b$. As we will see the results may differ substantially from the full commitment setting. First, we compare the ex ante efficient second-period incentive weight with its renegotiation-proof counterpart under centralization$^9$:

$$s^c_2 - s^{Rc}_2 = -\frac{\pi (1 + \pi) g^2 r \sigma^2}{(G - r \sigma^2 \pi) (G - r \sigma^2 \pi)} \begin{cases} < & \text{iff } \pi > 0 \\ > & \text{iff } \pi < 0 \end{cases}$$

From (12) we see that for negatively correlated outputs (except for $\pi = -1$) renegotiation-proofness leads to too low powered second period incentives, while for $\pi > 0$ second period incentives are too high powered compared to the ex ante efficient solution $s^c_2$. The reason is that the posterior variance $\sigma^P_2 = \sigma^2 (1 - \pi^2)$ of the agent’s wealth decreases with increasing quadratic correlation while the prior variance is increasing in $\pi$.$^{10}$ As already argued above, for uncorrelated outputs there is no difference between full commitment and renegotiation-proof contracts.

Notice that depending on the parameters of the model $s^{Rd}_2$ can take any value between zero and one; but it is always greater than $s^{Rc}_2$. This leads to the following result:

**Proposition 3** With renegotiation-proof contracts the solution to the managerial effort problem under decentralization may be preferred, $M(s^{Rc}_2) < M(s^{Rd}_2)$.

This result is in sharp contrast to the solutions to the managerial effort problems under long-term commitment. With long-term commitment under centralization $s^c_2$ was chosen to maximize $M(s_2)$ while under decentralization $s^d_2$ trades-off $M(s_2)$ and $B^d(s_2)$ such that $M(s^c_2) > M(s^d_2)$. With renegotiation-proof-contracts the renegotiation-proofness constraint $s_2 = s^P_2$ generally prevents the central management to maximize $M(s_2)$ under centralization. If $M(s^{Rc}_2)$ is greater or less than $M(s^{Rd}_2)$ depends on how strong $s^{Rc}_2$ deviates from the maximum $s^c_2$ relative to $s^{Rd}_2$ and it depends on whether $s^{Rc}_2$ sets too high or too low powered incentives as compared to $s^c_2$. To clarify this point consider figure 4.

$^9$A special case is $\pi = -1$ which leads to $s^c_2 = s^{Rc}_2 = 1$; see also the paragraph following Proposition 2.

$^{10}$See also Christensen/Feltham/Sabac (2005).
Figure 4 plots $M(s^2_2)$ and second period incentive weights $s^{Rc}_2$, $s^{Rd}_2$ and $s^c_2$ for $[p = 1/2, r = 1, \sigma = 5, \gamma = 2, g = 2, \pi = -3/4]$. $s^c_2$ maximizes $M(s^2_2)$. Under centralization renegotiation-proofness forces the central management to implement a second period incentive rate $s^{Rc}_2$ that is too low compared to $s^c_2$. As under decentralization the renegotiation-proof value of $s^2_2$ is higher than under centralization there is a potential gain of delegating the project with respect to the managerial effort problem. In figure 4 this case indeed occurs: $s^{Rd}_2$ is closer to the maximum than $s^{Rc}_2$. However, this positive effect of delegating $b$ on $M$ can only occur if $s^{Rc}_2$ is lower than optimal, $s^{Rc}_2 < s^c_2$. If $s^{Rc}_2$ is higher than optimal, $s^{Rd}_2$ is always even higher which necessarily leads to a lower $M$. Even if $s^{Rc}_2 < s^c_2$, delegation is only beneficial if $s^{Rd}_2$ is not too far beyond the optimal level. The next Proposition formalizes the latter conditions:

**Proposition 4**

a) If $M(s^{Rd}_2) > M(s^{Rc}_2)$ then $\pi < 0$.

b) Assume $\pi < 0$, then $M(s^{Rd}_2) > M(s^{Rc}_2)$ if $\gamma^2 (\pi + 1) (G - r\sigma^2 \pi) (p - 1) > 2g^2 \pi (G - r\sigma^2 \pi^2)$.

**Proof.** See the Appendix. ■

Negatively correlated outputs are necessary for $M(s^{Rd}_2) > M(s^{Rc}_2)$. $\pi < 0$ ensures that under centralization the second period renegotiation-proof incentive rate is lower than ex ante optimal. Part b) of the Proposition is a sufficient condition. To demonstrate this condition consider the point $(s^{Rc}_2; M(s^{Rc}_2))$ in figure 4 and draw a horizontal line to the right until the curve $M(s^2_2)$ is touched. All values for $s^{Rd}_2$ that are on this line lead to $M$ values that exceed $M(s^{Rd}_2)$. Condition b) of Proposition 4 restricts $s^{Rd}_2$ to take values on this line. Proposition 3 and 4 have an important conclusion which we state as a separate corollary.
Corollary 1 Under the conditions of Proposition 4 the incentive and control problem under centralization is more severe than under decentralization. Hence, decentralization may become optimal even if there is not a direct benefit of it in terms of specific knowledge.

Suppose the agent does not receive any specific knowledge about the project’s payoff (either because \( \theta \) is not random, or because it is not observed by the agent). Then, classical view would argue that delegation cannot be optimal. Indeed, if we look at the direct payoffs from the project if \( \theta \) is not random, we obtain \( \Delta B = \theta^2 s_d^2 (2 - s_d^2) \geq 0 \). In addition, under long-term commitment \( \Delta M \geq 0 \) as the coordination problem with respect to managerial effort is more severe. With renegotiation-proof-contracts, that are even more realistic than full commitment contracts, under the conditions of Proposition 4 there is a direct benefit of decentralization on the incentive problem with respect to managerial effort compared to centralization. Formally, this benefit occurs because decentralization relaxes the renegotiation-proofness constraint \( s_2 = s_{RD}^2 \) relative to centralization. Intuitively, by delegating the decision on \( b \) to the agent the central management implicitly itself to a higher sequentially optimal incentive rate \( s_{RD}^2 \). This commitment is beneficial if the central management has an incentive to set low incentives at the beginning of period two.

Having focussed on the managerial effort problems yet, we now want to analyze the influence of renegotiation-proofness on the outcome from decision \( b \) in what follows. Recall that \( \Delta B(s_{RD}^2) = \frac{s^2(1-p)(1-s_{RD}^2)}{2} \). It is easy to check that \( \partial \Delta B(s_{RD}^2) / \partial s_{RD}^2 < 0 \): The higher the renegotiation-proof second-period incentive weight the higher the relative payoff from decentralization from the project. An increase of \( s_{RD}^2 \), however, my result from different changes of parameters. The following proposition clarifies comparative static effects on \( \Delta B \) and provides a necessary and sufficient condition for decentralization/centralization being optimal with respect to the project decision.

**Proposition 5**

(a) Given \( p < 1 \), \( \Delta B(s_{RD}^2) = \begin{cases} 0 & \text{if and only if } p = \left(1 - s_{RD}^2\right)^2. \\ > & \text{if } p > \left(1 - s_{RD}^2\right)^2. \end{cases} \)

\[
\frac{\partial \Delta B(s_{RD}^2)}{\partial g} < 0, \quad \frac{\partial \Delta B(s_{RD}^2)}{\partial \alpha(r)} > 0, \quad \frac{\partial \Delta B(s_{RD}^2)}{\partial \gamma} < \left(>\right) 0, \quad \text{iff } \gamma > (\leq, <) 0.
\]

**Proof.** See the Appendix. ■

As \( \left(1 - s_{RD}^2\right)^2 \) increases in \( p \), part a) of the proposition implicitly defines a critical value \( p^{\text{crit}} \) such that decentralization becomes dominant if \( p > p^{\text{crit}} \). However, this also implies that the comparative static influence of \( p \) on \( \Delta B \) has two countervailing effects: A direct effect \( \frac{\partial \Delta B(s_{RD}^2)}{\partial p} \big|_{s_{RD}=s_{RD}^2} \) that is positive or negative depending on the sign of \( p - \left(1 - s_{RD}^2\right)^2 \) and an indirect effect via \( s_{RD}^2 \) that is positive. Hence, the total effect depends on the specific parameter setting. Similarly, the marginal effect of \( \gamma \) is not unique: the sign of the direct effect again depends on \( p - \left(1 - s_{RD}^2\right)^2 \) whereas the indirect is negative. In contrast, the comparative statics with respect to \( g \) and \( \sigma(r) \) are unique. To see this note that both \( g \) and \( \sigma \) affect \( \Delta B(s_{RD}^2) \) only
via $s_2^{Rd}$. While $s_2^{Rd}$ is increasing in $g$ and $\pi$ (if positive) it is decreasing in $\sigma$, $r$ and $\pi$ (if negative). The lower $s_2^{Rd}$ the greater the distortion in the manager’s project choice under decentralization such that centralization becomes comparatively advantageous.

We conclude this section by a numerical example using the following parameters: $[p = 0, \pi = -0.95, g = 13, \gamma = 8, r = 5, \sigma = 10]$. $p = 0$ captures the extreme case that decentralization has no direct benefit with respect to the project, as its contribution is certain, $\theta = \gamma$.

<table>
<thead>
<tr>
<th>$s_2^{C}$</th>
<th>$s_2^{Re}$</th>
<th>$s_2^{Rd}$</th>
<th>$\Delta B = B^c - B^d (s_2^{Rd})$</th>
<th>$\Delta M = M (s_2^{Re}) - M (s_2^{Rd})$</th>
<th>$\Delta = \Delta B + \Delta M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.871</td>
<td>0.776</td>
<td>0.827</td>
<td>0.958</td>
<td>-1.174</td>
<td>-0.216</td>
</tr>
</tbody>
</table>

Table 1: Numerical example

Table 1 shows that for the given parameters $s_2^{Re} < s_2^{C}$ which is necessary for $M (s_2^{Rd}) > M (s_2^{Re})$. As $s_2^{Rd} < s_2^{C}$ it follows directly $M (s_2^{Rd}) > M (s_2^{Re})$. By definition, if there is no uncertainty with respect to the project’s contribution, the direct payoff under centralization must exceed its counterpart under decentralization, such that $\Delta B > 0$. However, the effect on the managerial effort problem dominates and thus $\Delta < 0$: decentralization turns out to be optimal.

5 Conclusion

Conventional wisdom suggests that decentralizing decision rights creates costly incentive and coordination problems for firms relative to centralized decision making. On the other hand, the benefits of decentralization include improved decision making due to the use of specific knowledge. We analyze the cost and benefits of decentralization in a two-period agency model. We consider a firm consisting of a central management and a subordinate (agent). Central management is not able to make all operational and strategic decisions. Therefore, authority for some decisions must be delegated to the subordinate. We call these actions personal effort. Personal effort has a certain contribution and has to be performed in every period by the agent. To induce proper decisions an incentive contract is needed. In addition to effort there is a project which requires a decision in period two. The contribution from the project is uncertain. In a decentralized regime the decision right for the project is delegated to the agent. Working on the job, the agent’s receives specific knowledge about the project’s contribution. In a centralized regime central management decides upon the project based on prior information.

Our model captures the main trade-off between centralization and decentralization: Under decentralization specific knowledge can be used to decide upon the project, however, the agent’s project decision has to be aligned with the interests of central management via an incentive contract along with personal effort. We analyze this trade-off for two different contracting settings and provide necessary and sufficient conditions for the preferability of the one or other organizational form.
Under long-term commitment the classical view is confirmed. The incentive problem under decentralization is at least as severe as under centralization. With respect to the project’s payoff centralization or decentralization may be preferred. Which organizational architecture is overall preferred depends on the project’s as well as the personal effort’s contribution and the induced risk of the agent’s compensation. High contributions of both, effort and the project, lead decentralization to be the optimal regime.

Long-term commitment may be regarded as a theoretical benchmark but from a practical perspective it has an important shortcoming: It requires the contracting parties to credibly commit to not renegotiate the initial contract, even if there are ex post benefits from renegotiation. Therefore, we consider long-term renegotiation-proof contracts as the more realistic contracting setting. The main result from this analysis is that the "conventional cost" of decentralization (relative to centralization) may turn into a benefit. More precisely, the incentive problem under centralization may become more severe than under decentralization. The reason is that with renegotiation-proof contracts central management is forced to offer an initial contract that must be sequentially optimal at the end of the first period. Depending on the stochastic output structure this constraint may lead the central management to motivate too low or too high personal effort from an ex ante perspective. Under decentralization central management is forced to set a higher second period incentive weight than under centralization as besides personal effort the project decision has to be controlled, too. Hence, if renegotiation-proofness leads central management to offer a too low second period incentive rate, decentralization may relax the firm’s incentive problem. This implies that decentralization may be preferred even if there is no direct benefit of it in terms of specific knowledge. As this result has been derived in the more realistic setting of renegotiation-proof contracts the classical view may be revisited in dynamic environments.
Appendix: Proofs

Proof of Lemma 1

As \( s_\theta \) does not influence the agent’s incentives it will be chosen to minimize the variance of the agent’s compensation given \( s_1 \) and \( s_2 \):

\[
  s_\theta^c = \arg \min_{s_\theta} \text{Var} (S),
\]

with \( \text{Var} (S) = \text{Var} (s_1 x_1 + s_2 x_2 + s_\theta) = \text{Var} (s_1 \eta_1 + s_2 \eta_2 + (s_\theta + s_2 b) \theta) \). This expression is minimized w.r.t. \( s_\theta \) by \( s_\theta^c = -s_2 b \). Given \( s_\theta = s_\theta^c \) it follows \( \text{Var} (S) = \text{Var} (s_1 \eta_1 + s_2 \eta_2) = \sigma^2 (s_1^2 + s_2^2 + 2s_1 s_2 \pi) \).

Proof of Lemma 2

Similar to the proof before, \( s_\theta \) will be chosen as

\[
  s_\theta^d = \arg \min_{s_\theta} \text{Var} \left(S - \frac{b^d}{2}\right).
\]

With \( b^d = s_2 \theta \) we obtain

\[
  \text{Var} \left(S - \frac{b^d}{2}\right) = \text{Var} \left(s_1 x_1 + s_2 x_2 + s_\theta - \frac{s_2^2 \theta^2}{2}\right)
\]

\[
= s_1^2 \text{Var} (\eta_1) + s_2^2 \text{Var} (\eta_2) + 2 s_1 s_2 \text{Cov} (\eta_1, \eta_2) + s_\theta^2 \text{Var} (\theta) + \left(\frac{s_2^2}{2}\right)^2 \text{Var} (\theta^2) + 2 s_\theta s_2^2 \text{Cov} (\theta, \theta^2)
\]

>From the first-order condition for a minimum

\[
\frac{\partial \text{Var} \left(S - \frac{b^d}{2}\right)}{\partial s_\theta} = 2 s_\theta \text{Var} (\theta) + s_2^2 \text{Cov} (\theta, \theta^2) = 0
\]

we obtain \( s_\theta^d = -\frac{s_2^2}{2} \frac{\text{Cov} (\theta, \theta^2)}{\text{Var} (\theta)} = -s_2^2 \gamma / 2 \). Inserting \( s_\theta = s_\theta^d \) into \( \text{Var} \left(S - \frac{b^d}{2}\right) \) leads to

\[
\text{Var} (S) = \text{Var} (s_1 \eta_1 + s_2 \eta_2) = \sigma^2 (s_1^2 + s_2^2 + 2s_1 s_2 \pi).
\]

Proof of Lemma 3

Inserting in program (10) the binding participation constraint \( \Gamma (S, e, b) = 0 \) and the incentive constraints and \( s_\theta^d = -s_2^2 \gamma / 2 \) into the central management’s objective function leads to

\[
\max_{s_1, s_2} U = g^2 \left(s_1 + s_2 - \frac{1}{2} (s_1^2 + s_2^2)\right) + (1 - p) \gamma^2 s_2 (2 - s_2) / 2 - \frac{r}{2} \sigma^2 (s_1^2 + s_2^2 + 2s_1 s_2 \pi). \quad (13)
\]

The first-order conditions for \( s_1 \) and \( s_2 \) are given by

\[
\frac{\partial U}{\partial s_1} = g^2 (1 - s_1) - r \sigma^2 (s_1 + s_2 \pi) = 0
\]

\[
\frac{\partial U}{\partial s_2} = g^2 (1 - s_2) + (1 - p) \gamma^2 (1 - s_2) - r \sigma^2 (s_2 + s_1 \pi) = 0.
\]
Solving for $s_1$ and $s_2$ we obtain

$$
\begin{align*}
  s_1^d &= \frac{G_y (g^2 - r \sigma^2 \pi) + r \sigma^2 g^2}{G_y G + r \sigma^2 (g^2 + r \sigma^2 (1 - \pi^2))} \\
  s_2^d &= \frac{G (g^2 + r \sigma^2) - \pi r \sigma^2 g^2}{G_y G + r \sigma^2 (g^2 + r \sigma^2 (1 - \pi^2))}.
\end{align*}
$$

As the Hessian

$$
\begin{pmatrix}
  -g^2 - r \sigma^2 & -\pi r \sigma^2 \\
  -\pi r \sigma^2 & -g^2 - r \sigma^2 + (p-1) \gamma^2
\end{pmatrix}
$$

is negative definite, the sufficient condition is fulfilled, too. The central management’s overall payoff $U^d = U (s_1^d, s_2^d) = M^d (s_1^d, s_2^d) + B^d (s_2^d)$ is obtained by substituting $(s_1, s_2)$ by $(s_1^d, s_2^d)$ in (13). We know that $B^d = \frac{E[\theta^2]}{2} s_2^d (2 - s_2^d)$, $E[\theta^2] = (1-p) \gamma^2$. Hence, $M^d = U^d - B^d = g^2 (s_1^d + s_2^d - \frac{1}{2} (s_1^{d2} + s_2^{d2})) - \frac{\gamma}{2} \sigma^2 (s_1^{d2} + s_2^{d2} + 2s_1s_2\pi)$.

**Proof of Proposition 1**

According to (6) and Lemma 3 $U^c - U^d$ is given by

$$
U^c - U^d = \frac{g^2 (g^2 + \gamma^2 (\pi - 1) + g^2)}{(\pi - 1) + \gamma^2 (g^2 + \gamma^2 (1 - \pi) + g^2)} \geq 0.
$$

a) For $p = 0 U^c - U^d$ reduces to

$$
\frac{r \sigma^2 (1 + \pi) \gamma^2 (g^2 + \gamma^2 (1 - \pi) + g^2)}{2 (g^2 (g^2 + \gamma^2 (1 - \pi) + g^2)} \geq 0.
$$

b) For $r = 0$ and/or $\sigma = 0$: $U^c - U^d = (p-1) p \gamma^2 < 0$.

c) 

$$
\frac{\partial(U^c - U^d)}{\partial \sigma} = \frac{r^2 \sigma^3 (p-1) \gamma^2 (1 + \pi) (q_1 \sigma^6 + q_2 \sigma^4 + q_3 \sigma^2 + q_4)}{(\pi - 1) + \gamma^2 (g^2 + \gamma^2 (1 - \pi) + g^2)} \geq 0.
$$

with

\[
\begin{align*}
  q_1 &= -r^3 (\pi - 1) (1 + \pi) (g^2 (\pi - 1) + \gamma^2 (p-1)) < 0 \\
  q_2 &= -2g^2 (\pi - 1) r^2 (-g^2 (\pi + 3) + \gamma^2 (2 + \pi) (p-1)) < 0 \\
  q_3 &= -g^4 r (\gamma^2 (1-p) (5 - 2\pi) + g^2 (6 - 2\pi)) < 0. \\
  q_4 &= g^6 \gamma^2 (p-1) - 2g^8 < 0.
\end{align*}
\]

Hence, $\frac{\partial(U^c - U^d)}{\partial \sigma} > 0$. Furthermore, $\lim_{\sigma \to \infty} \left((U^c - U^d) = \gamma^2 (1-p)^2 / 2\right)$. Therefore, there exists $\sigma' > 0$ such that $U^c = U^d$ and $U^c > (\ell) U^d$ if $\sigma < (\ell) \sigma'$. As $r$ and $\sigma$ are multiplicatively connected the same results apply to $r$.

d) 

$$
\frac{\partial(U^c - U^d)}{\partial g} = \frac{g (p-1) \gamma^2 r^2 \sigma^2 (1 + \pi)^2 [2g^6 + l_1 g^4 + l_2 g^2 + l_3]}{(g^4 + g^2 (g^2 (1-p) + 2r \sigma^2) + g^2 + 2r \sigma^2 (1-p) + r^2 \sigma^4 (1-\pi^2) )^2 (g^2 + r \sigma^2 (1 + \pi))}. \text{ with } l_1 = \gamma (1-p) + r \sigma^2 (6 - 2 \pi), l_2 = 2(\pi - 1) r \sigma^2 (\gamma^2 (p-1) - r \sigma^2 (3 + \pi)) \text{ and}
$$
Proof of Lemma 6

\[ l_3 = r^2 \sigma^4 \left( 2 \sigma^2 (1 + \pi^3 - \pi - \pi^2) + \gamma^2 (1 - p) (1 - 2 \pi - \pi^2) \right). \]

As the factor of \( g^6 \) is positive, \( \frac{\partial (U^c - U^d)}{\partial y} \) becomes zero if \( g = g^{\text{crit}} \) and remains negative for all \( g > g^{\text{crit}} \), where \( g^{\text{crit}} \) depends on \( l_2 \) and \( l_3 \) which depend on \( r, \sigma, p, \pi, \) and \( \gamma \). As \( \lim_{g \to -\infty} (U^c - U^d) = \frac{1}{2} \gamma^2 p (p - 1) < 0 \) from the intermediate value theorem it follows that there exists another critical \( g' > 0 \) for \( g \) such that \( U^c (g') - U^d (g') = 0 \) and \( U^c - U^d < 0 \) for \( g > g' \).

\[
\frac{\partial (U^c - U^d)}{\partial \gamma} = \frac{\gamma (p-1) [1-p] \gamma r^2 (2 \gamma^2 + r \sigma^2 + \gamma^2 (1-p) + r^2 \sigma^2)}{\left( \gamma^2 + r \sigma^2 \right)^2},
\]

with \( m_1 = (1 - p) 2 p \left( \gamma^2 + r \sigma^2 \right) \left( \gamma^2 + r \sigma^2 (1 - \pi) \right) \) and \( m_2 = \left( \gamma^2 + r \sigma^2 (1 - \pi) \right)^2 \left( \gamma^2 (p - 1) (1 + \pi)^2 + pg^4 + 2g^2 r \sigma^2 (1 + \pi) \right). \)

Furthermore \( \lim_{\gamma \to -\infty} (U^c - U^d) = -\infty \). Applying the same arguments as for \( g \) shows that there exists \( \gamma' > 0 \) such that \( U^c (\gamma') - U^d (\gamma') = 0 \) and \( U^c - U^d < 0 \) for \( \gamma > \gamma' \).

**Proof of Proposition 2**

a) For \( \pi = -1 \) we obtain \( U^c - U^d = \gamma^2 p (p - 1) / 2 < 0 \).

b) For \( \pi = 0 \) we obtain \( U^c - U^d = \gamma^2 \left[ \frac{G^2 + \gamma^2 + G \gamma^2 + G \gamma^2}{2G (\gamma^2 - G^2)} \right] \) with \( G = g^2 + r \sigma^2 \). For \( p^2 r_2 < \gamma^2 \) it results \( \frac{\gamma^2 - \gamma^2 (G + \gamma^2) g^2 \sigma^2}{2 \gamma^2 (G^2 + \gamma^2)} < 0 \) and \( \frac{\partial (U^c - U^d)}{\partial \pi} < 0 \) which proves the claim.

c) For \( \pi = 0 \) we obtain \( U^c - U^d = \gamma^2 \left[ \frac{G^2 p^2 - (G^2 + G \gamma^2) r^2 \sigma^2}{2 G (\gamma^2 - G^2)} \right] \) with \( G = g^2 + r \sigma^2 \).

**Proof of Lemma 6**

In the centralized regime (11) results in

\[
\begin{align*}
    s_2^{Rc} &= \arg \max_{s_2} E \left( x_2 - C (e_2) - \frac{b^2}{2} | x_1 \right) - \frac{r}{2} s_2^2 \sigma^2 P \\
    &= \arg \max_{s_2} g e_2 + b^2 E (\theta) + E (\eta_2 | x_1) - \frac{e_2^2}{2} - \frac{(b^2)^2}{2} - \frac{r}{2} s_2^2 \sigma^2 P.
\end{align*}
\]

With \( e_2 = s_2 g \) we obtain

\[
\begin{align*}
    s_2^{Rc} &= \arg \max_{s_2} g^2 s_2 + b^2 E (\theta) + \pi (x_1 - E (x_1)) - \frac{g^2 s_2^2}{2} - \frac{(b^2)^2}{2} - \frac{r}{2} s_2^2 \sigma^2 P \\
    &= \frac{g^2}{g^2 + r \sigma^2 P}.
\end{align*}
\]

For the centralized regime (11) is given by

\[
\begin{align*}
    s_2^{Rd} &= \arg \max_{s_2} E \left( x_2 - C (e_2) - \frac{(b^2)^2}{2} | x_1 \right) - \frac{r}{2} s_2^2 \sigma^2 P \\
    &= \arg \max_{s_2} g^2 s_2 + s_2 E (\theta^2) + \pi (x_1 - E (x_1)) - \frac{g^2 s_2^2}{2} - \frac{s_2^2}{2} E (\theta^2) - \frac{r}{2} s_2^2 \sigma^2 P
\end{align*}
\]

with \( b^d = s_2 \theta \) and \( e_2 = s_2 g \) one obtains

\[
\begin{align*}
    s_2^{Rc} &= \arg \max_{s_2} g^2 s_2 + s_2^2 E (\theta^2) + \pi (x_1 - E (x_1)) - \frac{g^2 s_2^2}{2} - \frac{s_2^2}{2} E (\theta^2) - \frac{r}{2} s_2^2 \sigma^2 P \\
    &= \frac{g^2}{g^2 + E (\theta^2) + r \sigma^2 P}.
\end{align*}
\]
Proof of Proposition 4

a) We know that \( M(s_{2}^{Rd}) > M(s_{2}^{Re}) \) implies that \( s_{2}^{Re} < s_{2}^{c} \). From (12) we know that \( s_{2}^{Re} < s_{2}^{c} \) if and only if \( \pi < 0 \).

b) Assume \( \pi < 0 \). Notice that \( M(s_{2}) \) as defined in 5 is symmetric to its maximum \( s_{2}^{c} \). Hence, \( M(s_{2}^{Rd}) > M(s_{2}^{Re}) \) as long as \( s_{2}^{Rd} < 2s_{2}^{c} - s_{2}^{Re} \) or equivalently, if

\[
2s_{2}^{c} - s_{2}^{Re} - s_{2}^{Rd} = \frac{r\sigma^{2}(\pi + 1) \left[ \gamma^{2}(\pi + 1) (G - r\sigma^{2}(\pi + 1) (p - 1) - 2g^{2}\pi (G - r\sigma^{2}\pi^{2})) \right]}{(G + r\sigma^{2}\pi^{2})(G - r\sigma^{2}\pi^{2})(G - r\sigma^{2}\pi^{2} + \gamma^{2}(1 - p))} > 0.
\]

As the denominator is positive and is positive, too, \( 2s_{2}^{c} - s_{2}^{Re} - s_{2}^{Rd} > 0 \) requires \( (G - r\sigma^{2}\pi)(p - 1) > 2g^{2}\pi (G - r\sigma^{2}\pi^{2}) \).

Proof of Proposition 5

a) If \( p > 0 \), \( \Delta B(s_{2}^{Rd}) = \frac{\gamma^{2}(p - \{1 - s_{2}^{Rd}\}^{2})}{2} > (=) < 0 \) if and only if \( \left[ p - (1 - s_{2}^{Rd})^{2} \right] < (=) > 0 \).

b)

\[
\frac{\partial \Delta B(s_{2}^{Rd})}{\partial \gamma} = \frac{2\gamma^{2}(p - 1) g\sigma^{(2r - \gamma^{2}(\pi - 1)^{2})(\pi + 1)^{2}}}{(G + r\sigma^{2}(\pi + 1) (1 - p) - r\sigma^{2}\pi^{2})^{3}} < 0
\]

\[
\frac{\partial \Delta B(s_{2}^{Rd})}{\partial \sigma} = \frac{2\gamma^{2}(p - 1) \sigma^{3}r^{2}(\pi - 1)^{2}(\pi + 1)^{2}(-g^{2} - \gamma^{2}(1 - p))}{(G + r\sigma^{2}(\pi + 1) (1 - p) - r\sigma^{2}\pi^{2})^{3}} > 0,
\]

\[
\frac{\partial \Delta B(s_{2}^{Rd})}{\partial \sigma} = \frac{\partial \Delta B(s_{2}^{Rd})}{\partial \sigma} \frac{\sigma}{2r} > 0
\]

\[
\frac{\partial \Delta B(s_{2}^{Rd})}{\partial \pi} = \frac{2\gamma^{2}(p - 1) \sigma^{3}r^{2} \pi^{2} \pi^{2} \pi^{2}(-g^{2} - \gamma^{2}(1 - p))}{(G + r\sigma^{2}(\pi + 1) (1 - p) - r\sigma^{2}\pi^{2})^{3}} \times (\pi) < (\pi) > 0 \text{ if } \pi < (\pi) > 0.
\]
References


