Target-Group and Quality Decisions of Inequity-Averse Entrepreneurs

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Abstract

Limited donations force nonprofit entrepreneurs to ration needy individuals by deciding on who is served at what quality level. We propose a positive model of this allocation for applicants with differing incomes under the assumption of perfect user-fee discrimination. By following recent experimental economic research on social preferences, we assume that entrepreneurs behave inequity averse, i.e. they care about the relative consumption possibilities of others. We find that less inequity-averse entrepreneurs prefer to serve wealthier individuals at high reference quality. In contrast, more inequity-averse entrepreneurs care for the poorest individuals but offer minimum quality. Furthermore, as input costs increase, entrepreneurs with low inequity aversion change the target group, while entrepreneurs with high aversion do not.

Keywords: inequity aversion, nonprofit, quality, rationing, social entrepreneur, user fees

JEL Classification: L31, H41, D45
1. Introduction

The quality with which nonprofit organizations provide needy individuals with goods and services is subject to large variations, even within the same branches of the same region: The provision of shelter ranges from a low-quality emergency stay to a long-term accommodation at market standard; food is supplied on a nonprofit basis by soup kitchens as well as higher quality university cafeterias. The choice of quality level follows a specific pattern related to the income of the target group. In cases where only the ability to pay defines the neediness of individuals, the good or service provided to the poorest is frequently of significantly lower quality than comparable market offers. According to the World Bank (2003), in low- and middle-income countries services for poor people are often of low quality characterized by inadequately skilled workers, lacking resources, facilities in disrepair etc. More specifically, for micro-insurance schemes addressing the poor in developing countries a survey by McCord (2001) shows that these insurances’ coverage of health risks is very limited.\(^2\) Similar findings are reported for food assistance programs, which often supply low-quality food.\(^3\) From these observations one may question why nonprofits do not alternatively use their income from donations to lift the service quality to market level at the cost of a lower quantity of recipients.

Our interest in this paper is to provide a theoretical foundation to explain how nonprofits generally choose the quality/quantity mix of social goods and services.

Existing explanations for the low quality of services to the very poor are limited to the role of governmental provision. For example, Glazer and Niskanen (1997) highlight the importance of a poor majority in a public choice setting while Besley and Coate (1991) study governmental measures for redistributing income from the rich to the poor. However, due to the inability of raising taxes, these approaches cannot be adapted to private nonprofit organizations.

A survey of the corresponding literature reveals three different patterns to implement service quality and quantity into the objective function of private nonprofit decision makers. Newhouse (1970) and Rose-Ackerman (1987) follow the established convention that indifference curves between service quality and quantity have the “usual” convex shape. Along a second line, Dor and Farley (1996) as well as Friesner and Rosenman (2004) argue in favor of

\(^2\) The study pinpoints major exclusions and limitations in the coverage of micro-insurance schemes. Moreover, most of the schemes operate with reimbursement limitations.

\(^3\) Food for Survival (2000) studied 971 New York soup kitchens and food pantries and found that the majority of offered food consists of cheap non-perishable goods (rice, pasta, beans, powdered milk, canned foods etc.) while the supply of fresh food is relatively rare.
service intensity-adjusted output, where quality (characterized by service intensity) and quantity are multiplicably dependent within the nonprofit’s utility function. A third specification is given by Blau and Mocan (2002), who apply a Cobb-Douglas objective function in a childcare setting. However, all approaches lack a profound motivation for the specific interaction of quality and quantity within the decision maker’s utility function. Specifically, the intuition of the assumed dependency between the marginal utility of service quality and the absolute level of provided quantity remains unclear.

The present paper fills this gap by assuming that nonprofits are *inequity averse* in making their decisions. We thereby implement one of the major insights of recent experimental economic research on social preferences obtained from distribution games.⁴ Accordingly, in our theoretical model we assume that the decision maker cares about the relative payoff of others and experiences a disutility if the consumption possibilities of an individual deviate negatively from a social reference level. We show that this characterization provides a clear understanding of how nonprofits benefit from service quality, quantity, and the composition of recipients with regard to their initial consumption endowment. Moreover, we show within our theoretical framework that allocations which correspond to the empirical observations mentioned above can be explained. We find the following patterns: Weakly inequity-averse entrepreneurs choose to serve the least needy individuals at (maximum) social reference quality. In contrast, highly inequity-averse entrepreneurs provide the poorest individuals at minimum quality. Allocations between both extremes occur only for entrepreneurs with moderate aversion.

The organization and main results of the paper are given as follows. Section 2 introduces a model of the entrepreneur’s allocation problem accounting for applicants with differing incomes and exogenously given donations. Additionally, we allow the entrepreneur to charge perfectly discriminated user fees, which goes in line with common nonprofit practices.⁵ Section 3 analyzes how a variation in donations and input costs impacts the rationing behavior of nonprofits. We show first that an increase in donations leads to an extension of the target group for all entrepreneurs and additionally to an improvement of service quality for highly inequity-averse entrepreneurs. Second, an increase in input costs incites decision makers with less inequity aversion to serve even wealthier individuals at constant (social reference) quali-

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⁴ Seminal work in this field has been done by Fehr and Schmidt (1999) and Bolton and Ockenfels (2000).
⁵ Theoretical aspects of price discrimination by nonprofits are studied by Le Grand (1975) and Steinberg and Weisbrod (2005). A discussion of instruments enabling user-fee discrimination is given by Steinberg and Weisbrod (1998).
ty. More averse entrepreneurs leave the target group unchanged but decrease service quality. We conclude in section 4 with a discussion of these results.

2. The Model

We consider a continuum of individuals \( \mathcal{N} = [n_{\text{min}}, n_{\text{max}}] \subseteq \mathbb{R}^+ \) seeking to satisfy a basic human need. Examples of such needs are food, shelter, clothing, health etc. Each individual \( n \in \mathcal{N} \) is willing to spend a budget \( b(n) \) on purchasing one unit of a need-specific good. We assume that individuals are ordered according to their willingness to pay, such that \( db(n)/dn < 0 \) and \( b(n_{\text{max}}) = 0 \). Different product qualities of the good are available on perfectly competitive markets where firms face zero profits, and the price of the good increases with its quality level. We distinguish individuals only by their budget and, therefore, assume that consumers’ preferences are identical. Moreover, their marginal utility of quality is strictly positive. The latter assumptions reflect the basic-human-need character of the good. Intuitively, for this type of goods consumer preferences are similar and relatively intensive until a minimum quality level is reached. For example, the minimum level for food might be given by a balanced periodical nutrition. Together, our specifications of consumer preferences allow us to treat the terms willingness to pay and payment ability equally and, thus, to differentiate individuals by their income, i.e. poverty level. Accordingly, the individual \( n_{\text{max}} \) is the poorest whereas \( n_{\text{min}} \) represents the wealthiest individual.

Suppose a social entrepreneur is able to perfectly observe individual budgets. This assumption is supported by nonprofit practices, implying that it is quite common to differentiate the financial situation of needy people either through income verification sheets or through appropriate indicators.\(^6\) Moreover, Steinberg and Weisbrod (2005) argue that individuals may be willing to reveal their payment willingness to nonprofit but not to for-profit organizations. The social entrepreneur compares the individual budgets with a subjective social reference level \( b_{sr} \), which might be equal to her own consumption budget or might be deduced from

\(^6\) Steinberg and Weisbrod (1998) provide a general discussion of these indicators. More specifically, FAO (2001) surveys and discusses the application of indicators of several nutrition programs in developing countries (e.g. socio-economic status, education level, age, household size, number of children etc.). Although such practices are supposed to cause so-called targeting costs, we simplify by ignoring them for the following reason: These costs mainly arise due to the identification of suitable income indicators and the screening of individuals. However, since the social entrepreneur must screen all applicants to detect the targeted individuals, targeting costs are independent of the quantity and composition of recipients. Hence, they are fixed costs that simply reduce the amount of donations. A variation in donations is analyzed in section 3.
scientific or regulatory guidelines. This reference level determines the individuals the entrepreneur considers needy. For reasons of simplicity, we assume that all \( n \) individuals own a budget endowment equal or below this level, i.e. \( b_{sr} = b(n_{min}) \). Consequently, the social entrepreneur observes a budgetary inequity of \( q_{eq}(n) := b_{sr} - b(n) \geq 0 \) for the \( n \)th individual, which will be referred to in the following as *ex-ante inequity*.

In order to mitigate the ex-ante inequity the nonprofit entrepreneur offers one unit of a need-specific social good to any preselected individual. This selection is based on two related decisions: Which product quality should be offered and which needy subgroup should be targeted? We make three assumptions about the quality of the social good. First, the good is provided to all recipients at uniform quality, i.e. we do not consider quality discrimination. Second, the marginal costs of producing an additional unit of the social good \( c \in R^*_+ \) are independent of the supplied quantity but positively correlated to the product’s quality level. In the following, we do not distinguish between quality and marginal production costs and denote quality equivalently by \( c \). Third, for reasons of simplicity, it is assumed that the quality of the social good is produced with the same technology as the market good.

In order to illustrate the setting we have in mind, consider the following application to food-consumption. Here, the good is viewed as a bundle of staple foods of specific quantity and quality. Any change in the composition of the bundle that increases need satisfaction is modeled as an increase in the good’s quality. Hence, an increase in the number and scope of meals through additional food as well as an increase in the quality of a single item enhances the overall quality.

The second decision of the social entrepreneur concerns the composition and size of the target group. As will be argued by the following assumptions, this decision solely requires the choice of the marginally poorest recipient \( \bar{n} \in \mathcal{N} \). First, we define \( n \in [n_{min}, \bar{n}] \) as the marginally wealthiest recipient and we assume that the group of served individuals lies in the closed interval \([n, \bar{n}]\), with the quantity of recipients given by \( \bar{n} - n \). Furthermore, we allow the entrepreneur to perfectly discriminate prices. The differentiation of user fees according to payment ability, which is often observed in practice, is a basic assumption in models of nonprofit

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7 Exemplarily, the UK government (School Food Trust 2007) defined a minimum quality for school food by pinpointing items that have to be offered within a specific period.

8 A different approach is taken by Rose-Ackerman (1987), who argues that the marginal costs of quality for the provision of social goods are zero. Although sharing the opinion that there exist some factors improving quality without additional costs, e.g. changing school teaching from frontal to interactive mode, we account for the majority of dimensions where improvements in quality are costly.
In this regard, Hansmann (1980) as well as Steinberg and Weisbrod (1998) provide numerous examples of nonprofit industries frequently charging sliding-scale fees for different users. In our model, the social entrepreneur charges the $n$th individual a user fee that exactly corresponds to the budget endowment $b(n)$. The individual purchases the social good, if its quality $c$ does not fall short of the user-fee level, i.e. $c \geq b(n)$, or, in other words, if its quality is at least as high as the affordable quality of the market good. Consequently, the entrepreneur’s total user-fee revenues $F$ are given by

$$ (1) \quad F = \int_{\mu}^{\pi} b(n) \, dn. $$

In addition to these revenues, the entrepreneur receives an exogenously given level of donations $D \in (0, D_{\text{max}})$, with

$$ D_{\text{max}} = \int_{n_{\text{min}}}^{n_{\text{max}}} [(n_{\text{max}} - n_{\text{min}}) \cdot b_{sr}] \cdot \int_{n_{\text{min}}}^{n_{\text{max}}} b(n) \, dn $$

as the maximum level at which all individuals are served at social reference quality. In line with the organization’s nonprofit status user-fee revenues and donations have to be spent completely on financing the allocation of the social good to needy individuals, i.e.

$$ (2) \quad F + D = c \cdot (\overline{n} - n). $$

The nonprofit-condition (2) shows that for given levels of donations $D$ and individual budgets $b(n)$, the entrepreneur’s choice of the good’s quality $c$ and the marginally poorest recipient $\overline{n}$ determines the marginally wealthiest recipient $\underline{n} = \underline{n}(c, \overline{n})$ and, likewise, the size of the target group $\underline{n} - \underline{n}(c, \overline{n})$. These dependencies are depicted in figure 1.

Given the individual endowments $b(n)$, the social entrepreneur is confronted with the status-quo budgetary inequity $q_{\text{eq}}(n) = b_{sr} - b(n)$. With donations $D$ at hand, she decides on the quality level $c$ of the social good and determines the specific target group by choice of the poorest recipient $\overline{n}$. Due to the nonprofit-condition, she completely spends donations to cover

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9 Theoretical aspects of price discrimination by nonprofits are studied in Le Grand (1975) and Steinberg and Weisbrod (2005).

10 Recall that firms face zero profits in perfectly competitive markets and use the identical production technology as nonprofit organizations. Consequently, the quality an individual purchases from the market equals the budget which is spent.
the difference between marginal costs $c$ and individual contributions. Starting with the poorest recipient the funds suffice to subsidize $\bar{n} - n(c, \bar{n})$ individuals. Since recipients have to pay a user fee equal to their payment abilities, total user-fee revenues amount to $F(c, \bar{n})$. Subsequent to the allocation of the social good, there remains an inequity with served individuals amounting to $q_{eq}(c) := b_{sr} - c$, which will be referred to as \textit{ex-post inequity} in the following.

With the choice of her allocation the entrepreneur simultaneously shows two types of rationing. First, by choosing the target group she completely rations all individuals $n \notin [n(c, \bar{n}), \bar{n}]$. Second, her determination of a quality level partially rations all recipients since they do not receive the social reference level.

As indicated in the introduction, we characterize the social entrepreneur as an inequity-averse decision maker. Specifically, she draws a negative utility from a deviation of an individual’s consumption possibilities $b(n)$ from the social reference level. By providing needy individuals with the social good she reduces the inequity and, hence, her own disutility. We thereby build on recent experimental economic research which investigates general social preferences by means of simple distribution games, e.g. dictator and ultimatum games, where one individual decides on the distribution of an exogenously given amount of money between herself and other players. In their seminal work Fehr and Schmidt (1999) as well as Bolton and Ockenfels (2000) analyze the results of several experiments and conclude that the inequity-aversion motive is able to explain the observed behavior. Exemplarily, Fehr and Schmidt (1999) thereby use the following definition: “Inequity aversion means that people resist...
inequitable outcomes; i.e., they are willing to give up some material payoff to move in the direction of more equitable outcomes.”

We apply this motive to our model for two reasons. First, the analyzed distribution games are closely related to the decision context of the social entrepreneur in that an exogenously given amount of third-party funds has to be allocated between different individuals. Second, given that the principle of inequity aversion constitutes a building block in understanding the general fairness preferences of individuals, we can expect it to characterize the motivation of social entrepreneurs, in particular these, whose raison d’être lies in the mitigation of existing inequitable allocations. However, we use a broader definition of inequity aversion than Fehr and Schmidt (1999), who model the preferences of the distributor as self-centered inequity aversion, meaning that she cares about her own payoff relative to the payoff of others. In contrast, we do not restrict the reference outcome (in our paper: the social reference level $b_{sr}$) to be the entrepreneurs own budget endowment but, as previously argued, also allow for alternative reference levels, e.g. societal standards.

The inequity-aversion motive is introduced into our model through the parameter $\alpha \in \mathbb{R}_+$. It determines the social entrepreneur’s disutility from inequity by exponentially weighting $q_{ea}(n)$ and $q_{ep}(c)$, respectively. The functional form of her disutility can be written as

$$v(q) = q^\alpha,$$

with $q \in \{q_{ea}(n), q_{ep}(c)\}$.

The parameter $\alpha$ thereby determines the level of the constant elasticity of marginal disutility $\varepsilon = \alpha - 1$ and is likewise a measure for the curvature of value function (3). Additionally, as with the class of Cobb-Douglas utility functions, $\alpha$ characterizes the entrepreneur’s intensity of disutility. Marginal disutility is decreasing with $\alpha \in (0,1)$, constant with $\alpha = 1$, and increasing with $\alpha \in (1,\infty)$. More specifically, an entrepreneur with $\alpha = 0$ does not care about differences in budgetary inequity between individuals and values $q_{ea}(n)$ and $q_{ep}(c)$ identically. In contrast, for any positive $\alpha$ the entrepreneur draws an increased disutility from individ-

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11 Although we do not account for efficiency concerns in our model, the distribution game closest to our model specification is analyzed as treatment R in Engelmann and Strobel (2004). Here, the decision maker is the wealthiest individual and is likewise not able to extract any rents for herself.

12 The elasticity of marginal disutility is defined as $\varepsilon = \left[\frac{dv'(q)}{dq}\right] \cdot \left[\frac{q}{v'(q)}\right]$.

13 With these specifications of marginal disutility we broaden the scope of Fehr and Schmidt (1999), who integrate $\alpha$ multiplicatively into the utility function and, hence, restrict their analysis to lineair inequality aversion, i.e. constant marginal disutility. However, they also observe “a nonnegligible fraction of people who exhibit nonlinear inequality aversion” in dictator experiments (p. 823).
uals being subject to higher inequity. This increase in disutility is the larger the higher the value of $\alpha$ is, and it becomes infinite with $\alpha \to \infty$.$^{14}$ As will be shown later, entrepreneurs with extreme inequity aversion care only for the poorest target group individuals.

Based on the introduced disutility concept, we now characterize the social entrepreneur’s utility from allocating one unit of the social good to a target group individual by the following functional form:

$$u(c, n) = v(q_{ea}(n)) - v(q_{ep}(c)) = [b_{sr} - b(n)]^\alpha - (b_{sr} - c)^\alpha.$$  

Her utility equals the difference between the weighted ex-ante and ex-post inequity, i.e. the reduction of disutility through provision of the social good. As intuitive result, a non-inequity-averse entrepreneur ($\alpha = 0$) receives no utility from allocating the good independent of the type of recipient. Hence, she does not engage in the social-good provision.

As previously argued, by simultaneously choosing the quality level $c$ of the social good and the poorest recipient $n$, the entrepreneur, due to nonprofit condition (2), indirectly determines the wealthiest recipient $\bar{n}(c, \bar{n})$ and, hence, also the quantity of served individuals, $\bar{n} - n(c, \bar{n})$. Aggregating the utility values of equation (4) for each recipient then yields the following total utility level:

$$U(c, \bar{n}, n(c, \bar{n})) = \int_{n(c, \bar{n})}^{\bar{n}} [b_{sr} - b(n)]^\alpha - (b_{sr} - c)^\alpha \ dn.$$  

For reasons of tractability, the notation of utility function (5) includes the entrepreneur’s decision variables $c$ and $\bar{n}$ as well as their influence on the value of the wealthiest recipient $n(c, \bar{n})$. We thereby allow for a precise characterization of the entrepreneur’s scope of alternatives: Under consideration of nonprofit-condition (2), the entrepreneur can (directly or indirectly) vary two of the variables with the third kept constant. The maximization problem of the entrepreneur is given by

$$\max_{c, \bar{n}} U(c, \bar{n}, n(c, \bar{n}))$$

s.t. $D - \int_{n}^{\bar{n}} [c - b(n)] dn = 0$.  

$^{14}$ Note that the case $\alpha = \infty$ corresponds to maximin-preferences.
In the following, we prove the existence of corner and interior solutions to maximization problem (6).\textsuperscript{16}

**Proposition 1:** Weakly inequity-averse entrepreneurs ($\alpha \in (0,1)$) choose the maximum quality ($c^* = b_{sr}$) and provide only the wealthiest individuals ($n(c^*, \bar{\pi}^*) = n_{min}$). On the other hand, highly inequity-averse entrepreneurs ($\alpha \in (1,\infty)$) serve only the poorest applicants ($\bar{n}^* = n_{max}$) at the lowest feasible quality ($c^* = b(n(c^*, n_{max}))$). Finally, interior optima ($c^* \leq b_{sr}$ and $\bar{n}^* \leq n_{max}$) only exist if $\alpha = 1$.

**Proof:** See Appendix.

If donations are insufficient to serve all needy individuals, the social entrepreneur chooses the mix of quality and recipients that maximizes her utility from reduced inequity under the fulfillment of nonprofit-condition (2). As proposition 1 shows, a first maximum is given for weakly inequity-averse entrepreneurs ($\alpha \in (0,1)$). Their marginal utility of serving the next poorer recipient is always lower than both their marginal utility of an improvement in quality (given a constant wealthiest recipient) and their marginal utility of serving the next wealthier recipient (given a constant quality). Consequently, the entrepreneur maximizes the social-good quality ($c^* = b_{sr}$) and serves only the wealthiest recipients ($n(c^*, \bar{\pi}^*) = n_{min}$). Intuitively, weakly inequity-averse entrepreneurs show the highest marginal disutility of inequity for marginal deviations of individual budgets from the social reference level. As immediate consequence, the first unit of donations (in form of the social good) is used to completely eliminate the inequity of the wealthiest needy individual ($n \rightarrow n_{min}$) which requires the entrepreneur to choose the maximum quality for the good. Until the entire donations are spent, individuals are successively supplied according to the next higher inequity. The characterized corner solution is depicted in figure 2, panel (a).

\textsuperscript{15} Employing equation (1) into nonprofit-condition (2) and rearranging it with respect to $D$ yields $D = \int [c - b(n)] dn$.

\textsuperscript{16} Utility function (5) is similar to the normative poverty measure put forward by Foster et al. (1984). Applying this measure Bourguignon and Fields (1990) analyze optimal governmental subsidies to individuals. Their findings resemble the results of proposition 1.
Second, interior optima \( c^* \leq b_{sr} \) and \( \bar{n}^* \leq n_{max} \) exist for moderately inequity-averse entrepreneurs \((\alpha = 1)\). Their marginal utility of a change in each of the three variables is equally large, which allows for any values that satisfy nonprofit-condition (2). Entrepreneurs in this category show a constant marginal disutility of inequity and, thus, do not care for which applicants and to what level inequity is reduced. An arbitrary interior solution is characterized in figure 2, panel (b). Throughout the rest of the paper the case of \( \alpha = 1 \) will no longer be analyzed. Independent of the subsequently considered parameter variations it can be shown that the marginal utilities of quality, the wealthiest and the poorest recipient remain equally large. Consequently, any allocation satisfying nonprofit-condition (2) is optimal and, therefore, \( \alpha = 1 \) has no further explanatory value.

Third, the marginal utility of highly inequity-averse entrepreneurs \((\alpha \in (1, \infty))\) is lower for an improvement in quality than for a provision of both the next poorer and the next wealthier recipient. The resulting allocation is depicted in figure 2, panel (c). Here, only the poorest recipients \( (\bar{n}^* = n_{max}) \) are served at the minimum quality \( (c^* = b(n(c^*,\bar{n}^*))) \). The intuition runs contrary to that of panel (a). Since the marginal disutility from inequity is largest for the highest inequity level, utility is maximized, if donations are transferred to the poorest individuals \( (n_{max} - n(c^*,\bar{n}^*)) \), such that the ex-post inequity is equal across recipients but highest across all needy individuals. This procedure determines the low quality level of the social good.

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17 Interestingly, this is also the optimal allocation under maximin-preferences.
In addition to these findings, figure 1 (panel (c)) indicates that highly inequity-averse entrepreneurs choose to serve the largest quantity of needy individuals \( \bar{n}^* - n(c^*, \bar{n}^*) \). However, this result only holds if the function of budget endowments \( b(n) \) is convex. More specifically, differences in the chosen target-group quantity depend on both the social entrepreneurs’ inequity aversion and the curvature of the \( b(n) \)-function, as we show formally with the following proposition.

**Proposition 2:** Highly inequity-averse entrepreneurs \( (\alpha \in (1, \infty)) \) serve the maximum quantity of individuals \( \bar{n}^* - n(c^*, \bar{n}^*) \), if the \( b(n) \)-curve is convex. In contrast, if \( b(n) \) is concave, then the quantity of recipients is largest for weakly inequity-averse entrepreneurs \( (\alpha \in (0,1)) \). However, both types of entrepreneurs choose the same and likewise maximum quantity of recipients if \( b(n) \) is a linear function.

**Proof:** See Appendix.

Intuitively, the maximum quantity of individuals is served if the required average subsidy margin, i.e. the average difference between constant marginal production costs \( c \) and the perfectly discriminated user fee \( b(n) \), is lowest. There are two requirements to a minimal average subsidy. First, since marginal production costs are assumed to be equal across individuals, and \( b(n) \) is a decreasing function in \( n \), any target group is served with the lowest possible amount of donations, if the wealthiest recipient receives no subsidy. Otherwise, any positive subsidy to this individual would have to be likewise granted to each other recipient, implying increased spending of donations. Second, a minimum average subsidy margin arises among those individuals whose budgets are most uniformly distributed. For those individuals the gap between costs and user fee \( c - b(n) \) is smallest on average.

Following proposition 1, the first requirement is met for all entrepreneurs with \( \alpha \in (0, \infty) \setminus \{1\} \). However, the fulfillment of the second requirement depends on the curvature of the function of budget endowments \( b(n) \). Given that \( b(n) \) is convex, individual budgets vary least among the poorest individuals, such that highly inequity-averse entrepreneurs \( (\alpha \in (1, \infty)) \) serve the maximum quantity of recipients. In contrast, given a concave \( b(n) \)-function, budgets are most uniformly distributed among the wealthiest individuals which are supplied by weakly inequity-averse entrepreneurs \( (\alpha \in (0,1)) \). Consequently, they serve the maximum quantity of recipients. Finally, there exist no such differences in the distribution of
individual budgets, if the $b(n)$-curve is linear, which implies an equal and maximum target-group quantity for all entrepreneurs with $\alpha \in (0, \infty) \setminus \{1\}$.

3. Variations in Donations and Input Costs

As argued in section 2, the determinants of the social entrepreneur’s allocation decision include available third-party funds and production costs. These financial conditions are likely to change during the lifetime of a social business. A donor might withdraw or extend announced funds or might simply terminate a long-term relationship. Input costs might vary due to periodic shortages or shocks on resource markets. In this section, we analyze the impact of those variations on the entrepreneur’s choice of target group and social-good quality.

In principle, the social entrepreneur can alternatively use additional donations to serve more or different individuals, or to improve the quality of the social good. The next proposition shows that, on the one hand, entrepreneurs react differently on variations in donations but, on the other hand, the classification of corner and interior solutions by level of inequity aversion remains unaffected.\(^{18}\)

Proposition 3: Given an increase in donations, entrepreneurs with $\alpha \in (0, \infty) \setminus \{1\}$ enlarge the quantity of served individuals $\left(\bar{n}^{D_n} - n(c^*, \bar{n}^{D_n}) > \bar{n}^* - n(c^*, \bar{n}^*)\right)$. In particular, weakly inequity-averse entrepreneurs ($\alpha \in (0,1)$) keep serving the wealthiest individuals $\left(n(c^*, \bar{n}^*) = n_{\text{min}}\right)$ at the social reference level $\left(c^*_D = c^* = b_{sr}\right)$ and expand their target group toward the next poorer individuals $\left(\bar{n}^{D_n} > \bar{n}^*\right)$. In contrast, highly inequity-averse entrepreneurs ($\alpha \in (1,\infty)$) still focus on the most needy individuals $\left(\bar{n}^{D_n} = \bar{n}^* = n_{\text{max}}\right)$, improve the social-good quality $\left(c^*_D > c^*\right)$ and serve the next wealthier applicants $\left(n(c^*, \bar{n}^*) < n(c^*, \bar{n}^*)\right)$.

Proof: See Appendix.

Intuitively, an increase in donations does not affect the entrepreneur’s marginal disutility of ex-ante inequity as obtained from equation (3). Hence, there is no effect on her decision on how to reduce this inequity optimally, i.e. the order of her marginal utilities of quality $c$.

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\(^{18}\) In the following the entrepreneur’s decision variables are superscripted by $D$ to account for the state of increased donations.
marginally poorest recipient $\pi$, and marginally wealthiest beneficiary $n(c, \pi)$ remains unchanged. Consequently, entrepreneurs with $\alpha \in (0,1)$ still have the highest marginal disutility for the lowest levels of inequity which incites them to serve the wealthiest individuals $(n(c^D, \pi^D) = n(c^*, \pi^*) = n_{min})$ at social reference quality $(c^D = c^* = b_{sr})$. These recipients now comprise the ex-ante target group and, additionally, the next poorer applicants $(\pi^D > \pi^*)$. Entrepreneurs with $\alpha \in (1, \infty)$, on the other hand, eliminate the maximum disutility of inequity, if they keep on serving the poorest individuals $(\pi^D = \pi^* = n_{max})$ at minimum quality. Additional donations are spent on serving the next wealthier applicants. However, these individuals are only willing to purchase the social good, if its quality is at least equal to their budget endowment. Hence, the entrepreneur, likewise, improves quality unless the wealthiest recipient is indifferent between the market and the social good $(c^D = b(n(c^D, \pi^D)))$. Consequently, the model predicts an increase in both the quantity of recipients and the social-good quality as reaction to an increase in third-party funds.

As a second variation, consider a general increase in input costs (in the following indexed by superscript I). Note that in section 2 we assumed perfectly competitive for-profit markets and identical quality-production technologies of for- and nonprofit firms. These assumptions imply that, for a constant quality, the increase in input costs equally increases the price of the market good. Additionally, it still holds that any individual owning a budget equal or below the quality level $c^I$ applies for the social good and individuals with $b(n) > c^I$ demand the market good. The increase in input costs is reflected by a change of two parameters. First, the social reference budget increases $(b_{sr} > b_{sr})$ because higher expenditures are required to purchase the corresponding consumption quality. Second, we assume that the total quantity of needy individuals enlarges by those people who are no longer able to afford the social reference consumption. As a result, the set of needy individuals is now characterized by $\mathcal{N}^I = [n^I_{min}, n^I_{max}] \subseteq \mathbb{R}_+$ with $n^I_{min} < n_{min}$ and $b_{sr} = b(n^I_{min})$.

Given that the social entrepreneur does not change marginal production costs $(c^I = c)$, she is restricted to use qualitatively lower or less inputs per unit of the social good, which deteriorates its quality. Alternatively, she could increase $c^I$ to keep the quality constant, but this, according to nonprofit-condition (2), would imply a decrease in the quantity of served individuals. As proposition 4 shows, an increase in input costs leads to contrary reactions of social entrepreneurs depending on their level of inequity aversion.
**Proposition 4:** For weakly inequity-averse entrepreneurs \( (\alpha \in (0,1)) \) an increase in input costs leads to a provision of wealthier individuals \( \left( n(c^*, \bar{n}^*), n^*_I \right) = n^*_I \) at (unchanged) social reference quality \( (c^* = b^I_{sr}) \). In contrast, highly inequity-averse entrepreneurs \( (\alpha \in (1,\infty)) \) keep serving the status-quo target group \( \left( \bar{n}^* = n^*_I \right. \) and \( n(c^*, \bar{n}^*) = n(c^*, \bar{n}^*) \) at constant marginal costs \( (c^* = c^* = b(\bar{n}(c^*, n^*_I))) \), i.e. lower quality.

**Proof:** See Appendix.

Weakly inequity-averse entrepreneurs \( (\alpha \in (0,1)) \) show the highest marginal disutility of ex-ante inequity for marginal deviations of individual budgets \( b(n) \) from the social reference level. An increased budget \( b^I_{sr} \) required to consume the social-reference quality and a simultaneously enlarged quantity of needy individuals \( (n^*_I = n^*_I > n^*_I) \), thus, renders the initial choices of marginal costs \( (c^* = b^I_{sr}) \) and target group \( n(b^I_{sr}, n^*)_I = n^*_I \) suboptimal. The entrepreneur reacts by increasing marginal costs to \( b^I_{sr} \) and shifting the target group toward the ‘new’ wealthiest applicants \( n(b^I_{sr}, n^*) = n^*_I \). This way, she eliminates the fraction of inequity with the highest disutility. As figure 3 indicates, a complete shift in the target group occurs, if \( b^I_{sr} \) is such that donations are insufficient to allocate the good to more than the “new” applicants at social reference quality, i.e.

\[
D \leq \int_{n^*_I}^{n^*_I} [b^I_{sr} - b(n)] dn.
\]

No initially served individual is further considered by the entrepreneur.

In contrast, the marginal disutility of highly inequity-averse entrepreneurs \( (\alpha \in (1,\infty)) \) increases with the inequity level. As shown in section 2, they choose to serve the poorest individuals \( (\bar{n}^* = n^*_I) \) at minimum quality \( (c^* = b(\bar{n}(c^*, n^*_I))) \). Since an increase in input costs exerts no effect on the relative poverty of individuals, i.e. the individuals within the set \( n(c^*, n^*_I) \) are still poorest, the entrepreneur neither changes the target group \( (\bar{n}^* = \bar{n}^* = n^*_I \) and \( n(c^*, \bar{n}^*) = n(c^*, \bar{n}^*) \) nor the marginal production costs \( (c^* = c^* = b(\bar{n}(c^*, n^*_I))) \). However, quality necessarily drops due to increased input costs.
Additionally, figure 3 indicates that a weakly inequity-averse social entrepreneur not only changes the composition of recipients but also their quantity. The next proposition shows that this change unambiguously depends on the curvature of the budget function $b(n)$.

**Proposition 5**: Given a concave (convex) function of budget endowments $b(n)$, weakly inequity-averse entrepreneurs $(\alpha \in (0,1))$ increase (decrease) the quantity of served individuals, i.e. $n^* - n^*(c^*, \bar{n}^*) > \bar{n}^* - n^*(c^*, \bar{n}^*), \left( \bar{n}^* - n^*(c^*, \bar{n}^*) < \bar{n}^* - n^*(c^*, \bar{n}^*) \right)$, as a reaction to an increase in input costs. Given a linear budget function, they do not change the quantity of recipients.

**Proof**: See Appendix.

From proposition 2, we know that the quantity of recipients is negatively correlated with the average subsidy margin required to serve the targeted individuals. Since the wealthiest recipient receives no subsidy independent of the input costs, this margin is only conditional on the distribution of individual budget endowments, i.e. the curvature of the $b(n)$-function. The average subsidy is thereby the smaller the more uniformly budgets are distributed. Given that $b(n)$ is concave, the dispersion is lowest among the highest budgets. Consequently, the target group is larger after input costs increased, because recipients are wealthier on average. However, the ex-post quantity is smaller if $b(n)$ is convex, which is exemplarily depicted in figure 3. Here, individual budgets are least uniformly distributed among the wealthiest applicants. Finally, due to the same reasoning, no differences occur if $b(n)$ is linear.
4. Conclusion

Our objective in this paper was to develop a positive model of a nonprofit entrepreneur’s allocation decision, which includes the selection of the target group and the quality of the social good, in the light of limited third-party funds. By assuming that a social entrepreneur’s decision is characterized by inequity aversion, we follow recent results of experimental economic research on social preferences. We demonstrate how this preference assumption conveys a better understanding of how the good’s quality, the quantity of recipients as well as their income distribution interact within the objective function of private nonprofit decision makers. Specifically, an improvement of service quality increases the consumption level of beneficiaries and, hence, reduces inequity. In contrast, an enlargement of the target group reduces the inequity for additional recipients. In both cases the entrepreneur benefits through a reduction of her disutility from inequity. Finally, the composition of recipients enters the decision calculus through the marginal disutility of inequity. With increasing (decreasing) marginal disutility the entrepreneur prefers to reduce a given amount of inequity of a poorer (wealthier) individual.

We find that weakly inequity-averse entrepreneurs choose to provide wealthier individuals at high social reference quality. In contrast, highly inequity-averse entrepreneurs care for the poorest individuals but offer minimum quality. These results allow for two explanations of the low quality of services to the very poor. First, the goods or services considered in these studies were provided by highly inequity-averse entrepreneurs and/or, second, they were supplied by weakly inequity-averse entrepreneurs applying a low subjective reference quality. Whether social entrepreneurs apply subjective reference levels or rather a societally standardized norm remains an empirical question.

As a further result, we show that the quantity of supplied individuals depends on the curvature of the budget function. Given convexity (concavity), highly (weakly) inequity-averse entrepreneurs serve the maximum number of needy people. Moreover, we find that entrepreneurs react differently with regard to variations in donations and input costs. Irrespective of the considered variation, entrepreneurs with low aversion never change the quality of the social good. In contrast, entrepreneurs with high aversion improve quality if additional funds are available, and they lower quality when inputs used for production become more expensive. Common to both types of decision makers is the provision of more individuals if donations increase. However, given a sufficiently high increase in input costs, highly inequity-averse
entrepreneurs do not change the target group while weakly inequity-averse entrepreneurs serve a completely different (viz. wealthier) group.

Our results yield implications for stakeholders of nonprofit organizations whose objectives are related to quality, quantity and the composition of recipients. More specifically, donors or governments aiming at maximizing the number of served individuals with given funds should fund entrepreneurs who focus on the poorest people, if the majority of needy individuals is relatively poor (suggesting a convex budget-function in the model). In contrast, stakeholders generally interested in minimizing the number of needy individuals, through a provision of maximum service quality, should support entrepreneurs serving less poor individuals. Those stakeholders do not even need to change their contribution if input costs increase.

Finally, the framework developed in this paper constitutes a basis for analyzing additional issues of social entrepreneurial behavior. Specifically, it merits further investigation of how the different allocation patterns change if stakeholders exert an influence on the social entrepreneur’s decision. Especially, so-called lead donors, typically granting a significant and often the largest part of the initial financial need of nonprofit organizations, might wish to regulate if the entrepreneurial behaviour inadequately reflects their own objectives.

Appendix: Proof of Propositions 1-5

**Proof of Proposition 1:** For notational clarity, we temporarily expand the term $U(c,\bar{n},n(c,\bar{n}))$ to $U(c,\bar{n},n(c,\bar{n});\alpha)$ to emphasize the influence of the entrepreneur’s inequity aversion. However, we simplify the explicit notation by use of $U$.

By inserting user-fee revenues (1) into nonprofit-condition (2) and applying the implicit function theorem, one obtains the partial dependencies $d\bar{n}/dc = -[\bar{n} - n(c,\bar{n})]/[c - b(\bar{n})] < 0$, $d n(c,\bar{n})/dc = [\bar{n} - n(c,\bar{n})]/[c - b(n(c,\bar{n}))] > 0$ and $d n(c,\bar{n})/d\bar{n} = [c - b(\bar{n})]/[c - b(n(c,\bar{n}))] > 1$. Given that $n(c,\bar{n})$ is constant, the social entrepreneur increases $c$ at the cost of $\bar{n}$, or vice versa, if her total utility level is increased. She leaves both decision variables unchanged if the utility maximum is reached. Equivalent considerations apply for the pairwise variations of $c$ and $n(c,\bar{n})$, while keeping $\bar{n}$ constant, as well as $\bar{n}$ and $n(c,\bar{n})$, with $c$ constant.

Consider the variation of $c$ and $\bar{n}$ for a constant $n(c,\bar{n})$. The corresponding condition for marginal utilities can be written as
Specifically, the entrepreneur increases (decreases) $c$ and likewise decreases (increases) $\pi$ if (A1) holds with $> (<)$. Both variables are left unchanged if (A1) holds with equality. Inserting the partial derivatives into condition (A1) and rearranging it yields

\[
\alpha = \frac{b_{sr} - b(\pi)}{b_{sr} - b(\pi)} \left[ \frac{(b_{sr} - b(\pi))}{(b_{sr} - c)} \right]^{\alpha - 1} - \frac{(b_{sr} - c)}{b_{sr} - b(\pi)},
\]

As a first result, condition (A2) holds with equality for $\alpha = 0$ and $\alpha = 1$. Since any entrepreneur with $\alpha = 0$ draws no utility from and, hence, does not engage in the allocation of the social good, an interior utility maximum is solely given for $\alpha = 1$. Furthermore, the right term of condition (A2) is convexly increasing in $\alpha$. Combining the two results gives $\partial U/\partial c > [\partial U/\partial \pi] \cdot (d\pi/dc)$ if $\alpha \in (0, 1)$, $\partial U/\partial c = [\partial U/\partial \pi] \cdot (d\pi/dc)$ if $\alpha = 1$, and $\partial U/\partial c < [\partial U/\partial \pi] \cdot (d\pi/dc)$ if $\alpha \in (1, \infty)$.

The same reasoning applies to the pairwise variation of $c$ and $n(c, \pi)$ for a constant $\pi$. Formulating the condition on marginal utilities yields

\[
\alpha = \frac{b_{sr} - b(n(c, \pi))}{b_{sr} - b(n(c, \pi))} \left[ \frac{(b_{sr} - b(n(c, \pi)))}{(b_{sr} - c)} \right]^{\alpha - 1} - \frac{(b_{sr} - c)}{b_{sr} - b(n(c, \pi))},
\]

Again, condition (A4) holds with equality for $\alpha = 0$ and $\alpha = 1$ and its right term is convexly increasing in $\alpha$. Hence, $\partial U/\partial c > [\partial U/\partial n(c, \pi)] \cdot [d_n(c, \pi)/dc]$ if $\alpha \in (0, 1)$, $\partial U/\partial c = [\partial U/\partial n(c, \pi)] \cdot [d_n(c, \pi)/dc]$ if $\alpha = 1$, and $\partial U/\partial c < [\partial U/\partial n(c, \pi)] \cdot [d_n(c, \pi)/dc]$ if $\alpha \in (1, \infty)$.

Finally, consider the pairwise variation of $\pi$ and $n(c, \pi)$ for a constant $c$. Here, the condition on marginal utilities is written as

\[
\alpha = \frac{b_{sr} - b(\pi)}{b_{sr} - b(\pi)} \left[ \frac{(b_{sr} - b(\pi))}{(b_{sr} - c)} \right]^{\alpha - 1} - \frac{(b_{sr} - c)}{b_{sr} - b(\pi)}.
\]
or, equivalently,

\[ x(\varphi, \alpha) := \frac{[b_{sr} - b(n(c, \bar{n})) + \varphi]^\alpha - (b_{sr} - c)^\alpha}{c - b(n(c, \bar{n})) + \varphi} \geq \frac{[b_{sr} - b(n(c, \bar{n}))]^\alpha - (b_{sr} - c)^\alpha}{c - b(n(c, \bar{n}))}, \]

with \( \varphi := b(n(c, \bar{n})) - b(\bar{n}) > 0 \) and

\[ \frac{\partial x(\varphi, \alpha)}{\partial \varphi} = \frac{[b_{sr} - b(n(c, \bar{n})) + \varphi]^{\alpha-1}}{(c - b(n(c, \bar{n})) + \varphi)^2} \cdot \dot{x}(\alpha) > 0, \]

with \( \dot{x}(\alpha) := \alpha \cdot [c - b(n(c, \bar{n})) + \varphi] - [b_{sr} - b(n(c, \bar{n})) + \varphi] + (b_{sr} - c) \cdot \left( \frac{b_{sr} - c}{b_{sr} - b(n(c, \bar{n})) + \varphi} \right)^{\alpha-1} \).

For \( \alpha = 0 \) and \( \alpha = 1 \), condition (A6) holds with equality and \( \dot{x}(\alpha) = 0 \) and, hence, \( \partial x(\varphi, \alpha)/\partial \varphi = 0 \). For \( \alpha \neq \{0, 1\} \), \( \partial x(\varphi, \alpha)/\partial \varphi \) and \( d\dot{x}(\alpha)/d\alpha \) are indeterminate. However, since \( d^2 \dot{x}(\alpha)/d\alpha^2 > 0 \), it follows that \( \partial x(\varphi, \alpha)/\partial \varphi < 0 \) and, hence, \( \partial U/\partial \bar{n} < \left[ \partial U/\partial n(c, \bar{n}) \right] \cdot (d n(c, \bar{n})/d\bar{n}) \) if \( \alpha \in (0,1) \). \( \partial U/\partial \bar{n} = \left[ \partial U/\partial n(c, \bar{n}) \right] \cdot (d n(c, \bar{n})/d\bar{n}) \) if \( \alpha = 1 \). Finally, \( \partial x(\varphi, \alpha)/\partial \varphi > 0 \) and \( \partial U/\partial \bar{n} > \left[ \partial U/\partial n(c, \bar{n}) \right] \cdot (d n(c, \bar{n})/d\bar{n}) \) if \( \alpha \in (1, \infty) \).

The results of the pairwise comparisons show that, for any given \( \alpha \), the ordering of marginal utilities is independent of the levels of \( c, \bar{n}, \) and \( n(c, \bar{n}) \). Hence, with exception of the special case \( \alpha = 1 \), the social entrepreneur directly or indirectly chooses the maximum levels of those two variables that show the highest marginal utility. Thus, combining the previous results, one obtains

\[ \frac{\partial U}{\partial c} > \left| \frac{\partial U}{\partial \bar{n}} \cdot \frac{d \bar{n}}{d c} \right| \quad \text{and} \quad \frac{\partial U}{\partial \bar{n}} < \left| \frac{\partial U}{\partial n(c, \bar{n})} \cdot \frac{d n(c, \bar{n})}{d \bar{n}} \right| \quad \text{if} \quad \alpha \in (0,1), \]

\[ \frac{\partial U}{\partial c} = \left| \frac{\partial U}{\partial \bar{n}} \cdot \frac{d \bar{n}}{d c} \right| = \left| \frac{\partial U}{\partial n(c, \bar{n})} \cdot \frac{d n(c, \bar{n})}{d c} \right| \quad \text{if} \quad \alpha = 1, \quad \text{and} \]

\[ \frac{\partial U}{\partial \bar{n}} > \left| \frac{\partial U}{\partial n(c, \bar{n})} \cdot \frac{d n(c, \bar{n})}{d \bar{n}} \right| \quad \text{and} \quad \frac{\partial U}{\partial c} < \left| \frac{\partial U}{\partial n(c, \bar{n})} \cdot \frac{d n(c, \bar{n})}{d c} \right| \quad \text{if} \quad \alpha \in (1, \infty). \]

Consequently, \( c^* = b_{sr} \) and \( n(c^*, \bar{n}^*) = n_{\text{min}} \) if \( \alpha \in (0,1) \), \( c^* \) and \( \bar{n}^* \) can adopt any values that satisfy nonprofit-condition (2) if \( \alpha = 1 \), and \( c^* = b(n(c^*, \bar{n}^*)) \) and \( \bar{n}^* = n_{\text{max}} \) if \( \alpha \in (1, \infty) \). Q.e.d.
**Proof of Proposition 2**: Let \( l \) index the optimal choices for \( \alpha \in (0,1) \) and \( h \) for \( \alpha \in (1,\infty) \). The maximum quantity of recipients is given if the average subsidy margin to served individuals, \( c - \int_{n(c,\bar{n})}^{\pi} b(n)dn / [\bar{n} - n(c,\bar{n})] \), is minimal. Since \( db(n)/dn < 0 \) and \( c^* \) is constant for all \( n \in [n(c^*,\bar{n}^*),\bar{n}] \), a minimum average margin implies non-subsidization of the marginally wealthiest recipient, i.e.

\[
(A7) \quad c^* - b(a(c^*,\bar{n}^*)) = 0 ,
\]

which is, following the proof of proposition 1, fulfilled for \( \alpha \neq 1 \). Furthermore, for any two pairs \( c_i^*, \bar{n}_i^* \) and \( c_j^*, \bar{n}_j^* \) fulfilling (A7) and with \( c_i^* > c_j^* \) and for all \( \mu \in (0,n_{max} - n(c_j^*,\bar{n}_j^*)) \), it holds that

\[
(A8) \quad c_i^* - b(a(c_i^*,\bar{n}_i^* + \mu)) > c_j^* - b(n(c_j^*,\bar{n}_j^* + \mu)) \quad \text{if} \quad d^2b(n)/dn^2 > 0 .
\]

Consequently, if \( d^2b(n)/dn^2 > 0 \), then the average individual subsidy margin is minimal for the choices \( c_h^* \) and \( \bar{n}_h^* (= n_{max}) \) which implies the maximum quantity of served individuals \( n_{max} - n(c_h^*,n_{max}) \). In contrast, if \( d^2b(n)/dn^2 < 0 \) then the choices \( c_i^* (= b_{sr}) \) and \( \bar{n}_i^* \) imply the maximum quantity of recipients \( \bar{n}_i^* - n(b_{sr},\bar{n}_i^*) \). Finally, if \( d^2b(n)/dn^2 = 0 \), then we have \( n_{max} - n(c_h^*,n_{max}) = \bar{n}_i^* - n(b_{sr},\bar{n}_i^*) \). Q.e.d.

**Proof of Proposition 3**: From the proof of proposition 1, the order of the marginal utilities of \( c, \bar{n}, \) and \( n(c,\bar{n}) \), as given in (A2), (A4), and (A6), is uniquely determined by \( \alpha \), and consequently independent of \( D \). Thus, for \( \alpha \in (0,1) \) an increase in \( D \) leads to \( c^{D*} = c^* = b_{sr} \) and \( n(c^{D*},\bar{n}^{D*}) = n_{\min}^* = n(\bar{n}^*,\bar{n})^* \). Given these values, nonprofit-condition (2) is fulfilled if \( \bar{n}^{D*} > \bar{n}^* \) which implies \( \bar{n}^{D*} - n(c^{D*},\bar{n}^{D*}) > \bar{n}^* - n(c^*,\bar{n})^* \). In contrast, for \( \alpha \in (1,\infty) \) the entrepreneur chooses \( \bar{n}^{D*} = \bar{n}^* = n_{max} \) and \( c^{D*} = b(a(c^{D*},\bar{n}^{D*})) \) which implies \( c^{D*} > c^* \) and \( n(c^{D*},\bar{n}^{D*}) < n(\bar{n}^*,\bar{n})^* \) and, hence, \( \bar{n}^{D*} - n(c^{D*},\bar{n}^{D*}) > \bar{n}^* - n(c^*,\bar{n})^* \). Q.e.d.

**Proof of Proposition 4**: In the proof of proposition 1 we showed that the order of the marginal utilities of \( c, \bar{n}, \) and \( n(c,\bar{n}) \), as given in equations (A2), (A4), and (A6), is uniquely deter-
mined by $\alpha$, and hence independent of $b_{sr}$. Thus, for $\alpha \in (0,1)$ an increase in input costs, i.e. an increase in $b_{sr}$, leads to $c^{l*} = b^{l*}_{sr}$ and $\frac{n(c^{l*}, \bar{n}^{I*})}{n} = n_{min}^{I}$. In contrast, for $\alpha \in (1,\infty)$ we obtain $\bar{n}^{I*} = n_{max}$ and $c^{l*} = c^{*}$, which implies a decrease in social-good quality. \textbf{Q.e.d.}

\textbf{Proof of Proposition 5:} The proof of proposition 2 shows that the quantity of recipients is negatively correlated to the average subsidy margin $c - \left[ \frac{\bar{n}}{\underline{n}(c, \bar{n})} b(n)dn \right]$ to served individuals. Since, according to proposition 4, condition (A7) is still fulfilled after input costs rise, i.e. $c^{l*} - b(\underline{n}(c^{l*}, \bar{n}^{I*})) = 0$, differences in the average subsidy margin between the two states are uniquely determined by the sign of $d^{2}b(n)/dn^{2}$. With $c^{*} = c^{l*}$, $\bar{n}_{i}^* = \bar{n}^{I*}$, $c^{*} = c^{*}$, and $\bar{n}_{j}^* = \bar{n}^{*}$ and, hence, $\mu \in \left(0, n_{max} - \frac{n^{I*} - n^{*}}{n} \right)$, it follows by condition (A8) that if $d^{2}b(n)/dn^{2} < 0$ then the average individual subsidy margin is smaller for the choices $c^{l*}$ and $\bar{n}^{I*}$ which implies $\bar{n}^{l*} - \underline{n}(c^{l*}, \bar{n}^{I*}) > \bar{n}^{*} - \underline{n}(c^{*}, \bar{n}^{*})$. In contrast, if $d^{2}b(n)/dn^{2} > 0$ then $\bar{n}^{l*} - \underline{n}(c^{l*}, \bar{n}^{I*}) < \bar{n}^{*} - \underline{n}(c^{*}, \bar{n}^{*})$. Finally, if $d^{2}b(n)/dn^{2} = 0$ then we have $\bar{n}^{l*} - \underline{n}(c^{l*}, \bar{n}^{I*}) = \bar{n}^{*} - \underline{n}(c^{*}, \bar{n}^{*})$. \textbf{Q.e.d.}

\textbf{References}


