The Money-in-the-Utility Function Model
(MIU)

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January 11, 2007
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Introduction

The original Neoclassical Growth Model does not know “money”, neither as a medium of exchange that facilitates transactions, nor as an asset bearing zero nominal interest. However, when investigating monetary issues, we must introduce money into the economic framework in a way, that the individual agent has an incentive to hold a positive amount of money in the equilibrium.

In the New Keynesian Phillips Curve Model, money is considered as transaction costs and, therefore, dependent on the volume of trade. Logically, these transaction costs are decreasing in the stock of real money balances held by the economic agent.

A different approach to incorporating money into general equilibrium models is assigning utility to real money balances. Sidrauski (1967) was the first to come up with a Money-in-the-Utility Function.

In this model, not only the consumption of goods and leisure yields direct utility, but the possession of currency, as well. Since money itself has no intrinsic value and is only useful in reducing transaction costs, incorporating money in the utility function is not free of criticism. Nevertheless, according to Walsh [1], the savings in time spent to purchase consumption goods can equally be accounted for in the MIU approach. Obviously, what enters the utility function will not just be an amount of currency. Instead, it is the command over goods or the value of transaction services, expressed in terms of goods, that matters. This is modelled by relating the nominal stock of money $M$ to the price level $P$ and, thus, including real money balances $m = M / P$ in the representative economic agent’s utility function.

This seminar paper presents the Money-in-the-Utility Function model specified according to Walsh [1]. From the household’s constrained intertemporal maximization problem, the first order conditions are derived which enable us to determine the steady state relationships between consumption, labour supply, money and bonds balances, and the capital stock. The simulation of time paths requires setting up a loglinear version of the model in Matlab as seen in the Benchmark Model. We, finally, analyse the effect of a monetary shock on output, labour supply, inflation and the nominal interest rate by varying the three critical parameters that relate to money growth and the Money-in-the-Utility Function.
1 The Model

In contrast to our Benchmark Model, the utility of a representative household now depends on one additional variable: \( m_{t+1} \) are the real money balances at the end of period \( t \). Thus, the instantaneous utility function is given by

\[
u(c_t, m_{t+1}, n_t) = \frac{[ac_t^{1-b} + (1-a)m_{t+1}^{1-b}]}{1-\Phi} + \Psi (1-n_t)^{1-\eta}, \tag{1.1}\]

with \( 0 < a < 1, b, \eta, \Psi, \Phi > 0 \).

Accordingly, the household faces the following intertemporal maximization problem:

\[
\max E_o \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, m_{t+1}, n_t) \right] \text{ subject to } y_t + \tau_t + (1-\delta)k_t + \frac{(1+i_t)b_t}{\pi_t} + \frac{m_t}{\pi_t} = c_t + k_{t+1} + m_{t+1} + b_{t+1}, \tag{1.2}\]

i.e. the aggregate economy-wide budget constraint of the household sector in real terms.

The essential variables and parameters in equations (1.1) and (1.2) can be interpreted as:

- \( a \) - weight of consumption in the fractional utility function
- \( \beta \) - subjective rate of time discount
- \( c_t \) - consumption level of the household
- \( m_{t+1} = \frac{M_{t+1}}{P_t} \) - real money balances at the end of period \( t \)
- \( 1-n_t \) - leisure time for a time endowment of 1 and labour supply \( n_t \)
- \( \Phi \) - coefficient of relative risk aversion
- \( b \) - inverse of the elasticity of real money demand
- \( \eta \) - inverse of the intertemporal elasticity of substitution of leisure
- \( y_t \) - aggregate output
- \( \tau_t \) - lump-sum transfers or taxes if negative
- \( \delta \) - rate of depreciation of physical capital
- \( k_t \) - capital stock
- \( i_t \) - nominal interest rate
\[ b_{t+1} = \frac{B_{t+1}}{P_t} \] - real value of bonds in period \( t + 1 \)

\[ \pi_{t+1} = \frac{P_{t+1}}{P_t} \] - inflation factor in period \( t + 1 \)

Output is created according to a Cobb-Douglas production function of the form

\[ y_t = e^z_t k_t^\alpha n_t^{1-\alpha} . \]  

(1.4)

In the above equation, \( z_t \) represents a productivity shock evolving according to

\[ z_t = \rho e_{t-1} + e_t , \]  

(1.5)

where \( e_t \) is a serially uncorrelated process with zero mean.

Finally, the law of motion for the real money supply is

\[ m_{t+1} = \frac{\theta_t}{\pi_t} m_t , \]  

(1.6)

where \( \theta_t = M_{t+1}/M_t \) is the growth factor of the nominal stock of money. The deviation of this factor from its steady state is defined as \( u_t = \theta_t - \theta \) and can be described by a stochastic process

\[ u_t = \gamma u_{t-1} + \phi \varepsilon_{t-1} + \varepsilon_t , \]  

(1.7)

where \( \varepsilon_t \) is again serially uncorrelated with zero mean, and \( 0 < \gamma < 1 \).

2 Derivation of the First Order Conditions

As in the Benchmark model, we receive the FOCs that determine the households’ optimal levels of consumption, labour supply, money and bonds balances as well as the capital stock, by partially deriving the Lagrangean function

\[
L(c_t, m_t, n_t, b_t, k_t, \lambda_t) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{ac_t^{1-b} + (1-a)m_t^{1-b}}{1-\Phi} \right]^{1-\Phi} + \Psi \frac{(1-n_t)^{1-\eta}}{1-\eta} \right.
\]

\[
+ \lambda_t \left( e^z_t k_t^\alpha n_t^{1-\alpha} + \tau_t + (1-\delta)k_t + \frac{(1+i_t)b_t}{\pi_t} + m_t - c_t - k_{t+1} - m_{t+1} - b_{t+1} \right) \Bigg] \right. .
\]
Differentiating with respect to the 5 variables mentioned above and the Lagrange-multiplier $\lambda_i$ gives us:

$$\frac{\partial L}{\partial m_{t+1}} = \beta' \left[ (1-\Phi) \left( a c_t^{1-b} + (1-a) m_{t+1}^{b-b} \right) \right] + \beta' E_t \left[ \frac{\lambda_{t+1}}{\pi_{t+1}} \right] = 0$$

$$\frac{\partial L}{\partial c_t} = (1-\Phi) \left( a c_t^{1-b} + (1-a) m_{t+1}^{b-b} \right) a(1-b) c_t^{b-b} \lambda_i - \lambda_i = 0$$

$$\frac{\partial L}{\partial b_{t+1}} = \beta' E_t \left[ \frac{\lambda_{t+1}}{\pi_{t+1}} \right] - \beta' \lambda_i = 0$$

$$\frac{\partial L}{\partial n_t} = \Psi (1-\eta)(1-n_t)^{-q} (-1) + \lambda_i (1-\alpha) e^{x} k_t^{a} n_t^{1-a} = 0$$

$$\frac{\partial L}{\partial k_{t+1}} = \beta' E_t \left[ (1-\Phi) k_t^{a} n_t^{1-a} + (1-\delta) \right] + \beta' (-\lambda_i) = 0$$

$$\frac{\partial L}{\partial \lambda_i} = e^{x} k_t^{a} n_t^{1-a} + \tau_t + (1-\delta) k_t \left[ \frac{1+i_t}{\pi_t} + m_t \right] - c_t - k_t - m_{t+1} - b_{t+1} = 0.$$  

The above equations are simplified, assuming that the inflation factor $\pi_i$, the nominal interest rate $i_t$, and the real interest factor $R_t = 1+r_t$ are related by the well known Fisher equation, namely $R_t = (1+i_t) / \pi_t$. We furthermore define: $X_t = ac_t^{1-b} + (1-a)m_{t+1}^{b-b}$. With this, the first order conditions can be rewritten as

$$\lambda_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\pi_{t+1}} \right] + (1-a) m_{t+1}^{b-b} X_t^{b-b} \quad (2.1)$$

$$\lambda_t = ac_t^{b-b} X_t^{b-b} \quad (2.2)$$

$$\lambda_t = \beta E_t \left[ \lambda_{t+1} R_{t+1} \right] \quad (2.3)$$

$$\frac{\Psi (1-n_t)^{-q}}{\lambda_t} = (1-\alpha) \frac{y_t}{n_t} \quad (2.4)$$

$$\lambda_t = \beta E_t \left[ \lambda_{t+1} \left( \alpha \frac{y_{t+1}}{k_{t+1}} + (1-\delta) \right) \right] \quad (2.5)$$

$$k_{t+1} = e^{x} k_t^{a} n_t^{1-a} + \tau_t + (1-\delta) k_t \left[ \frac{1+i_t}{\pi_t} + m_t \right] - c_t - m_{t+1} - b_{t+1}. \quad (2.6)$$
3 The Steady State

There are several essential assumptions concerning the model’s equilibrium. From the law of motion for real money supply it can easily be seen, that for \( m_{t+1} = m_t = m \) the inflation factor \( \pi \) must exactly equal the growth factor of the nominal stock of money \( \theta \), as constant per capita money holdings in the steady state require prices and the nominal stock of money to change at the same rate. Moreover, the nominal stock of money changes solely through the lump-sum transfers to or lump-sum taxes taken from the public. Therefore, the real value of these transfers equals \( (M_{t+1} - M_t)/P_t \) and steady-state \( \tau = m - m/\pi = [(\theta - 1)/\theta]m \).

In determining the equilibrium values of this model, use has also been made of the fact that we do not allow for any borrowing and lending in the steady state, and hence \( b = 0 \). Finally, we assume the non-appearance of productivity shocks, i.e. \( z = 0 \), which implies \( e^z = 1 \). Evaluated at the steady state, equations (2.1) to (2.6) become:

\[
\lambda \left( \frac{\pi - \beta}{\pi} \right) = (1 - a)m^{-b}X^{b - \Phi} \tag{3.1}
\]

\[
\lambda = ac^{-b}X^{1-b} \tag{3.2}
\]

\[
R = \frac{1}{\beta} \tag{3.3}
\]

\[
\Psi(1-n)^{\alpha} = (1-\alpha)^{\frac{y}{n}} \tag{3.4}
\]

\[
\frac{1}{\beta} = \alpha k^{\alpha-1}n^{\alpha} + (1-\delta) \tag{3.5}
\]

\[
k = k^\alpha n^{\alpha-1} + (1-\delta)k + c \tag{3.6}
\]

The FOC with respect to real bond holdings (3.3) determines the equilibrium value of the real interest factor \( R \). Solving equation (3.5) for \( y/k = k^{\alpha-1}n^{1-\alpha} \), we receive a production-to-capital ratio that depends on the parameters of the model, only. The steady-state consumption ratio results from dividing (3.6) by the level of physical capital. To calculate the equilibrium ratio of the real money balances, we solve equation (3.2) for \( X \), and substitute into (3.1), then simplifying and using the fact that \( \pi = \theta \). Finally, the labour supply ratio is received by dividing the steady-state production function through \( n \) and rearranging.
Table 1
Steady-State Values

<table>
<thead>
<tr>
<th></th>
<th>( \frac{y}{k} )</th>
<th>( \frac{c}{k} )</th>
<th>( \frac{m}{k} )</th>
<th>( \frac{n}{k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{\beta} )</td>
<td>( \frac{1-\beta(1-\delta)}{\alpha\beta} )</td>
<td>( \frac{y-\delta}{k} )</td>
<td>( \frac{c}{k} \left( \frac{1-\alpha}{\alpha} \right)^{\frac{1}{b}} \frac{\left( \frac{\theta}{\theta-\beta} \right)^{\frac{1}{b}}}{\left( \frac{y}{k} \right)^{\frac{1}{1-a}}} )</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 summarizes the steady-state values of the endogenous variables, expressed in terms of the model’s basic parameters. Apart from real money balances relative to the capital stock, none of the equilibrium ratios directly depends on the growth factor of the nominal money supply \( \theta \).

4 Loglinearising the Model

We introduce the following loglinear variables:

\[
\begin{align*}
\hat{m}_t &:= \frac{m_t - m}{m}, \\
\hat{c}_t &:= \frac{c_t - c}{c}, \\
\hat{n}_t &:= \frac{n_t - n}{n}, \\
\hat{\lambda}_t &:= \frac{\lambda_t - \lambda}{\lambda}, \\
\hat{i}_t &:= \frac{i_t - i}{i}, \\
\hat{k}_t &:= \frac{k_t - k}{k}, \\
\hat{R}_t &:= \frac{R_t - R}{R}, \\
\hat{\pi}_t &:= \frac{\pi_t - \pi}{\pi}, \\
\hat{y}_t &:= \frac{y_t - y}{y}, \\
\hat{z}_t &:= \frac{z_t - z}{z}, \\
\hat{\theta}_t &:= \frac{\theta_t - \theta}{\theta} = \frac{u_t}{\theta}
\end{align*}
\]

In addition to the first order conditions (2.1) to (2.6), we have to linearize the Fisher relation, the model’s production function and the law of motion for the real money supply. In total, this gives us a set of nine equations that have already been simplified or written in an appropriate form:

\[
\begin{align*}
\frac{1-a}{a} \left( \frac{m_{t+1}}{c_t} \right)^{-b} &= \frac{i_{t+1}}{1+i_{t+1}}, \\
\lambda_t &= ac_t^{-b} X_t^{1-b}, \\
\dot{\theta}_t &= \beta E_t \lambda_{t+1} R_{t+1}, \\
E_t R_{t+1} &= E_t \frac{1+i_{t+1}}{\pi_{t+1}}, \\
\frac{\Psi(1-n_t)^{-q}}{\lambda_t} &= (1-\alpha) \frac{y_t}{n_t}, \\
y_t + (1-\delta)k_t &= c_t + k_{t+1}
\end{align*}
\]
\[ y_t = e^{\delta t} k_t^a n_t^{1-a} \quad (4.7) \]
\[ m_{t+1} = \frac{\theta}{\pi_t} m_t \quad (4.8) \]
\[ R_t = \alpha \frac{y_t}{k_t} + 1 - \delta \quad (4.9) \]

Defining \( \xi_1 = (1-a)(b-\Phi)X^{-1}m^{1-b} \), \( \xi_2 = a(b-\Phi)X^{-1}c^{1-b} - b \) and \( \xi_3 = 1 + \eta \frac{n}{1-n} \), the resulting system of loglinear equations consists of

\[ E_t \hat{m}_{t+1} = \hat{c}_t - \left( \frac{1}{b} \right) E_t \hat{i}_{t+1} \quad (4.1)' \]
\[ \hat{\lambda}_t = \xi E_t \hat{m}_{t+1} + \xi_2 \hat{c}_t \quad (4.2)' \]
\[ \hat{\lambda}_t = E_t \left[ \hat{\lambda}_{t+1} + \hat{R}_{t+1} \right] \quad (4.3)' \]
\[ E_t \hat{R}_{t+1} = E_t \left[ \hat{\pi}_{t+1} - \hat{\pi}_{t+1} \right] \quad (4.4)' \]
\[ \hat{\lambda}_t = \xi \hat{n}_t - \hat{\gamma}_t \quad (4.5)' \]
\[ E_t \hat{k}_{t+1} = (1-\delta) \hat{k}_t + \left( \frac{\nu}{k} \right) \hat{y}_t - \left( \frac{c}{k} \right) \hat{c}_t \quad (4.6)' \]
\[ \hat{y}_t = a \hat{k}_t + (1-\alpha) \hat{n}_t + \hat{z}_t \quad (4.7)' \]
\[ E_t \hat{m}_{t+1} = \hat{m}_t - \hat{\pi}_t + \hat{\theta}_t \quad (4.8)' \]
\[ \hat{R}_t = \alpha \beta \left( \frac{\nu}{k} \right) (\hat{y}_t - \hat{k}_t) \quad (4.9)' \]

For a more convenient use of the system in the following calculations, we will differentiate between control, state and costate variables. Therefore, we set up the vector of controls as

\[ \mathbf{u}_t := (\hat{c}_t, \hat{n}_t, \hat{y}_t, \hat{R}_t). \]

Two of the variables have predetermined conditions and comprise the vector of states:

\[ \mathbf{x}_t := (\hat{k}_t, \hat{m}_t) \]

Since we will dispose of \( \hat{i}_t \) later on using the Fisher equation, the vector of costates only contains the two remaining variables.

\[ \mathbf{\lambda}_t := (\hat{\lambda}_t, \hat{\pi}_t) \]

Finally, productivity and monetary growth shocks are summarized in the vector

\[ \mathbf{z}_t := (\hat{z}_t, u_t). \]
Substituting (4.8)' into (4.2)', we receive 4 equations without expectations, namely (4.2)', (4.5)', (4.7)' and (4.9)'). Solving for the vector of controls, the loglinear system that relates to the matrix notation

\[ \mathbf{C}_u \mathbf{u}_t = \mathbf{C}_x \begin{pmatrix} \mathbf{x}_t \\ \lambda_t \end{pmatrix} + \mathbf{C}_z \mathbf{z}_t \]

can be written in extensive form as

\[ -\xi z \hat{c}_t = \xi x \hat{m}_t - \hat{\lambda}_t - \xi \hat{r}_t + \frac{\xi}{\theta} u_t, \]

\[ \xi z \hat{n}_t - \hat{y}_t = \hat{\lambda}_t, \]

\[ (\alpha - 1) \hat{n}_t + \hat{y}_t = \alpha \hat{k}_t + \hat{z}_t, \]

\[ \alpha \beta \frac{y}{k} \hat{y}_t - \hat{R}_t = \alpha \beta \frac{y}{k} \hat{k}_t. \]

As mentioned before, we can drop the nominal interest rate’s percentage deviation from the steady state, if we substitute (4.4)' into (4.1)'. Accordingly, we end up with 4 equations that contain expectations. They correspond to the matrix notation

\[ \mathbf{D}_u \mathbf{E}_t \begin{pmatrix} \mathbf{x}_{t+1} \\ \lambda_{t+1} \end{pmatrix} + \mathbf{F}_u \begin{pmatrix} \mathbf{x}_t \\ \lambda_t \end{pmatrix} = \mathbf{D}_u \mathbf{E}_t \mathbf{u}_{t+1} + \mathbf{F}_u \mathbf{u}_t + \mathbf{D}_z \mathbf{E}_t \mathbf{z}_{t+1} + \mathbf{F}_z \mathbf{z}_t. \]

Solving for the state and costate variables, (4.1)', (4.3)', (4.6)' and (4.8)' can be rewritten as

\[ \hat{m}_{t+1} + \left( \frac{1}{b} \right) E, \hat{\lambda}_{t+1} = \hat{c}_t - \left( \frac{1}{b} \right) E, \hat{R}_{t+1} \]

\[ \hat{\lambda}_t - E, \hat{\lambda}_{t+1} = \hat{R}_{t+1} \]

\[ (1 - \delta) \hat{k}_t - E, \hat{k}_{t+1} = \left( \frac{c}{k} \right) \hat{c}_t - \left( \frac{y}{k} \right) \hat{y}_t \]

\[ E, \hat{m}_{t+1} - \hat{m}_t + \hat{\lambda}_t = \frac{1}{\theta} u_t. \]

Finally, \( \mathbf{z}_t = \mathbf{P} \mathbf{z}_{t-1} + \mathbf{E}_t \) contains the loglinearized technological and monetary shocks:

\[ \hat{z}_t = \rho \hat{e}_{t-1} + e_t, \]

\[ u_t = \gamma v_{t-1} + \phi \hat{e}_{t-1} + \epsilon_t. \]
5 Calibration and Analysis

5.1 Choice of Parameter Values

For the numerical analysis in Matlab, the values of the model’s parameters were chosen as described in Walsh [1] and, therefore, deviate from those in the Benchmark Model. Table 2 summarizes all parameter values that remained constant throughout the simulations.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Baseline Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>( \delta )</td>
</tr>
<tr>
<td>0.36</td>
<td>0.019</td>
</tr>
</tbody>
</table>

The outstanding parameters \( \gamma \), \( \phi \) and \( \Phi \) were alternated to investigate the effects of a money growth shock on output, labour supply, inflation and the nominal interest rate.

Note that \( \Psi \) is not a freely chosen parameter but determined by the steady-state relation

\[
\frac{n^\phi}{(1-n)^\eta} = \frac{1-\alpha}{\Psi} \left( \frac{\gamma}{k} \right) \left( \frac{\epsilon}{k} \right) \left( \frac{k}{n} \right)^{\phi} \left( 1 + \left( \frac{a}{1-a} \right)^{\frac{\phi}{\beta}} \left( \frac{\theta - \beta}{\theta} \right)^{\frac{b-\phi}{b}} \right).
\]  

(5.1)

Assuming an exogenously given steady-state labour supply of 1/3 which corresponds to 8 daily working hours of every individual, we have to fix \( \Psi \) at approximately 1.343.

Furthermore, equation (5.1) can be assessed to derive the influence of money growth on the equilibrium labour supply. While the left hand side is increasing in \( n \), the effect of \( \theta \) on the right hand side depends on the sign of \( b - \Phi \). If this is positive (negative), faster money growth decreases (increases) the r. h. s. and the steady state level of employment falls (rises).

\[
\Psi = \frac{1-\alpha}{\gamma} \left( \frac{\epsilon}{k} \right) \left( \frac{k}{n} \right)^{\phi} \left( 1 + \left( \frac{a}{1-a} \right)^{\frac{\phi}{\beta}} \left( \frac{\theta - \beta}{\theta} \right)^{\frac{b-\phi}{b}} \right).
\]  

More thoroughly, if consumption and money are Edgeworth complements, (5.2) > 0. The reduction in real money holdings due to higher inflation will decrease the marginal utility of consumption in this case, such that households expand their leisure time at the cost of labour supply. The opposite effects can be observed for Edgeworth substitutes, i.e. when \( b - \Phi < 0 \).

Applying the above parameter values to the steady-state ratios from Table 1, we receive the results stated in Table 3 and with \( n = 1/3 \), we can also determine the equilibrium values of our variables as reported in Table 4.
Table 3
Steady-State Values at Baseline Parameter Values

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>y/k</td>
<td>c/k</td>
<td>m/k</td>
<td>n/k</td>
</tr>
<tr>
<td>1.011</td>
<td>0.084</td>
<td>0.065</td>
<td>0.089</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Table 4
Steady-State Values of Variables

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>y</td>
<td>c</td>
<td>m</td>
<td>n</td>
</tr>
<tr>
<td>16.082</td>
<td>1.346</td>
<td>1.040</td>
<td>1.432</td>
<td>1/3</td>
</tr>
</tbody>
</table>

5.2 Simulation Results and Economic Interpretation

The simulation of time paths with Matlab required adjusting three of the original programs from the seminar, namely “Ramsey4d.m”, “SolveLA.m” and “RBCRun4d.m”. For output, consumption, labour supply, real interest factor, nominal interest rate and inflation factor, we calculated standard deviations $s_x$, crosscorrelations with output $r_{xy}$, and autocorrelations $r_x$.

Table 5
Implied Contemporaneous Correlations

<table>
<thead>
<tr>
<th></th>
<th>$\Phi = 2$</th>
<th></th>
<th>$\Phi = 3$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$s_x$</td>
<td>$r_{xy}$</td>
<td>$r_x$</td>
<td>$s_x$</td>
<td>$r_{xy}$</td>
</tr>
<tr>
<td>y'</td>
<td>1.129</td>
<td>1.000</td>
<td>0.659</td>
<td>1.044</td>
<td>1.000</td>
</tr>
<tr>
<td>c</td>
<td>0.284</td>
<td>0.949</td>
<td>0.709</td>
<td>0.226</td>
<td>0.956</td>
</tr>
<tr>
<td>n</td>
<td>0.414</td>
<td>0.957</td>
<td>0.659</td>
<td>0.299</td>
<td>0.896</td>
</tr>
<tr>
<td>R</td>
<td>0.035</td>
<td>0.975</td>
<td>0.655</td>
<td>0.032</td>
<td>0.973</td>
</tr>
<tr>
<td>i</td>
<td>0.862</td>
<td>-0.074</td>
<td>-0.092</td>
<td>0.864</td>
<td>-0.055</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.865</td>
<td>-0.112</td>
<td>-0.090</td>
<td>0.867</td>
<td>-0.090</td>
</tr>
</tbody>
</table>

Table 5 compares the model’s properties for two different values of the inverse of the elasticity of substitution, one below and one above $b = 2.56$. We further assume all deviations of the nominal money growth factor from its steady state $\theta = 1.0125$ to be purely random, i.e. $\gamma = \phi = 0$. Remember that the marginal utility of consumption is increasing (decreasing) in the household’s real money holdings for $\Phi = 2$ ($\Phi = 3$).

While the calculations in Matlab reproduced the results in the reference literature, in general, our statistics for the nominal interest rate have to be treated with caution. As nominal interest is linearly linked to inflation through the Fisher equation (4.4)', we unfortunately observe an implausible bias of the values displayed for $i$ towards those of $\pi$. In contrast to Table 5, Walsh [1] finds a very low standard deviation and an almost perfect positive correlation of $i$ with output.
There are at least two possible sources to this considerable divergence in the results. One is the fact that working with a different algorithm in this seminar paper required the adjustment of several of the original model’s time indices just to be able to simulate time paths in Matlab. On the other hand, we treated real interest and inflation as factors but nominal interest as a rate, while Walsh [1] exclusively employs rates. Especially in the derivation of the loglinear equations, however, the book can be criticised for being fairly inaccurate in the treatment of those rates.

To study how the model responds to different time-series properties of the money supply process, we now drop the white noise assumption for deviations from $\theta$. Instead, we follow the book and adopt $\gamma = 0.5$ as a plausible estimator for the autoregressive coefficient of money growth.

Table 6 summarises the model’s statistics for $\phi = 0.15$, 0 and -0.15, while the parameter of relative risk aversion $\Phi$ is kept fixed at its benchmark value 2. The key effect of altering $\phi$ is on the behaviour of inflation and nominal interest. While $y$ and $\pi$ are positively correlated, when money growth increases in response to a positive productivity shock that raises output and reduces prices, the correlation becomes increasingly negative for a coefficient equal to or smaller than 0. Unfortunately, we face the same problem for the nominal interest rate as in Table 5. Walsh [1] calculates a significantly lower standard deviation of $i$. Moreover, the book plausibly suggests that nominal interest and output exhibit a higher positive correlation for $\phi = 0.15$ than in our findings and are negatively correlated, only if the response of money growth to a productivity shock is strictly negative.

Table 6 also shows that the effects of $\phi$ on the standard deviations of output, consumption, labour supply and real interest are hardly noticeable. Although monetary policy matters in our MIU model, its influence on the real economy seems to be quite small.

<table>
<thead>
<tr>
<th></th>
<th>$\phi = 0.15$</th>
<th>$\phi = 0.0$</th>
<th>$\phi = -0.15$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$s_x$</td>
<td>$r_{xy}$</td>
<td>$r_x$</td>
</tr>
<tr>
<td>$y$</td>
<td>1.128</td>
<td>1.000</td>
<td>0.664</td>
</tr>
<tr>
<td>$c$</td>
<td>0.284</td>
<td>0.948</td>
<td>0.715</td>
</tr>
<tr>
<td>$n$</td>
<td>0.412</td>
<td>0.956</td>
<td>0.663</td>
</tr>
<tr>
<td>$R$</td>
<td>0.034</td>
<td>0.974</td>
<td>0.659</td>
</tr>
<tr>
<td>$i$</td>
<td>1.046</td>
<td>0.087</td>
<td>0.238</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.045</td>
<td>0.056</td>
<td>0.232</td>
</tr>
</tbody>
</table>
When a positive technology shock leads to higher expected money growth, real money holdings fall, which, in the case of $\Phi = 2 < b$, decreases the marginal utility of consumption and the labour supply, as households substitute towards leisure. Conversely, when $\phi = -0.15$, a respective shock causes expected inflation to fall, such that employment and output will be slightly higher afterwards.

We have seen that a monetary shock $\varepsilon$, only affects the labour supply, when deviations from the equilibrium growth rate of the nominal money supply are not white noise, i.e. when $\gamma \neq 0$ or $\phi \neq 0$. If the money growth process exhibits serial correlation ($\gamma > 0$), a positive money growth shock will lead to higher expected money growth that decreases real money balances and, when $\Phi = 2$, the marginal utility of consumption. This, in turn, gives the agent an incentive to expand leisure and output drops. Analogously, a respective shock increases labour supply and output in the case of $\Phi > b$.

As tables are not convenient to demonstrate the reaction of the model’s variables to monetary shocks, we make use of two graphics from Walsh [1]. Figure 1 depicts the response of output and labour supply to a positive money shock for serial correlation coefficients of 0.5 and 0.8.

![Figure 1: Output and Labour Responses to a Money Growth Shock](image)
While the deviations from the steady-state levels subsequent to a monetary shock are always small in absolute terms, their magnitude obviously depends on the degree of persistence in the money growth process.

Figure 2 shows how the response of the nominal interest rate and inflation is affected by the choice of $\gamma$. In contrast to what is usually expected from policy actions that accelerate money growth, a positive monetary shock increases the nominal interest rate. As we assume perfectly flexible prices in the MIU model, the price level will jump immediately in response to the shock. This decreases the real money supply consistently with a fall in the real money demand resulting from a higher nominal interest rate. Therefore, the main effect of money growth rate shocks is to increase expected inflation and raise the nominal interest rate. We do not observe a so-called liquidity effect that would arise, if an increase in the nominal money supply increased the real money supply, as well, due to rigidity of prices in the economy.

Figure 2
Nominal Interest Rate and Inflation Response to a Money Growth Shock
Conclusion

Sidrauski’s Money-in-the-Utility Function represents one approach to model the demand for money in an economy. Its crucial assumption is that holding real money balances yields direct utility. Setting up the model in loglinear form enabled us to perform a time series analysis in Matlab. Although some of the computational results deviated from those in the reference literature, the key effects of the variations in the money growth process could be observed.
To sum up, the size of the coefficient of relative risk aversion $\Phi$ determines whether the marginal utility of consumption increases or decreases in real money balances and, thus, how employment and output respond to a monetary growth shock. Varying $\phi$, i.e. the manner in which a technology shock affects the money growth factor, leads to significant changes in the correlation of inflation and nominal interest with output. Finally, the magnitude of the effect on the model’s variables and the time an economy needs to converge back to its steady state after a monetary shock, increase with the vigour of serial correlation within the money growth process $\gamma$. 
Bibliography

[1] Carl E. Walsh; 2003; Monetary Theory and Policy; The MIT Press; Massachusetts Institute of Technology