Employment Cycles and Minimum Wages.
A Macro View*

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Abstract

We start from the hypothesis that Goodwin’s (1967) Classical growth cycle does not represent a process of social reproduction that can be considered as adequate and sustainable in a democratic society in the long-run, due to the degradation of part of the workforce it implies during periods of mass unemployment. The paper formulates on this background an unemployment benefit system and a minimum wage rule for the employed where this form of economic reproduction of capitalism with its workforce degradation is overcome by the protection of workforce skills and family structures through general base income payments, to be coupled with the obligation of life-long learning and social services provision. There is high labor mobility (concerning ‘hiring’ and ‘firing’), with fluctuations of employment made socially acceptable through the security aspect of this form of Goodwinian growth dynamics. We can show in this framework that minimum real wages provide extra stability to such dynamics by decreasing the amount of overshooting in employment and distribution they are otherwise subject to.

Keywords: Distributive growth cycles, minimum wage, basic income, flexicurity.

JEL classifications: E32, E64, H11.

*We thank Gangolf Groh for stimulating discussions, in particular on the minimum wage debate in Germany and its reflection in the Goodwin growth cycle approach. Of course, usual caveats apply.
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1 Introduction

At present there is a controversial debate in Germany, in parliament, in the media and also between economists, of whether minimum wages should be introduced in certain sectors or even throughout the economy. This debate was triggered by the rise of jobs with low salaries at or even below the subsistence level since the 1990s. It was also caused by the observation that the income of top managers has been rising drastically more than the average income of employees over the last decades. For example, according to Klesse and Voss (2007), the annual income of top managers of the largest 100 companies in Germany with total revenues exceeding 5 billion Euro have increased by the factor 8 over the last 30 years, while GDP has risen by the factor 4.5 and even exceeds the rise of the general level of earned income in that period. In order to raise the income of the lowest wage earners and in order to reduce the gap between the highest and lowest incomes, proponents call for a minimum wage and, some of them, even for an upper bound on wage revenues.

Opponents of such regulations by and large argue with the employment costs of such a regulation of the working of capitalism (see e.g. Sinn, 2007). Since they assume perfect competition (price taking behavior) where employers hire labor up to the point where real wages are equal to marginal products, a lower bound on the real (!) wage rate reduces employment and, thus, raises unemployment with all its negative effects for the economy (assuming that the marginal product at the current point of employment is below the minimum real wage rate). These are assumptions that certainly lack justification, in particular since it is obvious that the real world is not at all well-represented by a Walrasian economy with price-taking firms throughout. In the paper we therefore consider wage $w$ and price $p$ setting behavior and in addition a fixed proportions technology, for simplicity. We do not yet consider low-skilled labor in this framework, however, where money wage formation may be of a different type than in the case of normal labor supply. A minimum wage for this type of labor should however not alter the macro-behavior of the economy very much, so that we may conclude that the demonstrated gains in economic performance (less severe fluctuations in economic activity) will also apply to the low-skilled segment and will in addition increase the low skilled workers’ social acceptance of their situation, since they see that their case is not completely decoupled from the normal working of the economy. Their employers can then simply not exploit their weak bargaining power situation without limits, but instead have to search for changes in technology that raises the marginal product of these workers should it be below their minimum real wage level. Temporarily, employment in this segment may suffer from

\footnote{\textsuperscript{1}or monopolistic competition which does not modify the qualitative structure of the model in an essential way.}

\footnote{\textsuperscript{2}of a simple type, see however the appendix of the paper for a recent micro-founded extension of these wage-price dynamics.}

\footnote{\textsuperscript{3}in place of a neoclassical production function, where employment would be determined by the slightly more general formula $w/p = F_L(K,L^d)$ with no change in the general conclusions of the paper.}

\footnote{\textsuperscript{4}if they want to stay in their market.}
such minimum real wages. Yet, in the longer run the economy will function in a better way and, thus, also improve the situation of low-skilled workers again.

On the other hand, the ideas of some trade unions in Germany concerning minimum wages and other regulations of the labor market are to be considered as rather detrimental for the functioning of market economies. This concerns both a minimum real wage level that is above the steady state position of the economy as well as the number of working hours per week, since this is just another side of the same coin. In addition, we would like to emphasize however (form the perspective of the model of this paper), that proper minimum real wages should be accompanied by working regulations that allow flexible hiring and firing on the part of employers in the economy (as assumed by the model). Hence, employers should be able to react flexibly and fast to changes in the economic situation. We stress however that this demands for income security on the side of workers’ position and thus for a flexicurity type of economy as it is considered in Flaschel, Greiner, Luchtenberg and Nell (2008).

Another and different aspect to be pointed out is that social progress implies an evolution of societies that comprises more than just economic goals in a narrow sense. In particular, the question of whether and to what extent societies succeed in achieving human rights for their citizens should be a major concern for policy makers. This is a point that is often neglected by economists who only look at the economic sub-system of societies when enunciating policy recommendations.

In this paper we will show in a supply side framework that minimum (real) wages will be beneficial to the working of a modern capitalist economy (characterized by a high state of labor productivity and income per worker), at least in the longer run. First of all, we would stress, in solving this task, that such a topic can only be treated in a dynamic framework that includes the forces of growth and income distribution, and thus not by means of static arguments or even static and partial ones. Secondly, minimum wage levels must be chosen with some care and will in our analysis at most delay the rise in employment to a certain degree, but lead thereafter to economic and social outcomes that clearly dominate the situation of no minimum wages. Finally, also upper bounds in real wage evolution may be of help in such a context showing that the solutions to the mass unemployment problems involve an active participation of both capital and labor in the process of cyclical growth considered in this paper.

Section 2 provides a brief survey on what we would call the reserve army mechanism of modern capitalist economies. In section 3 we will introduce the Classical model of cyclical growth to be used in this paper as point of reference. Section 4 considers how general regulations concerning basic income needs, minimum wages, but also maximum wages modify (and improve) the growth dynamics of the Classical model. We will also discuss there a modern approach to wage negotiations that will improve the obtained growth cycles even further. Section 5 concludes.

\[^5\text{See the following analysis.}\]
2 Sustainable social evolution through an unrestricted reserve army mechanism?

We start this paper from the hypothesis that Goodwin’s (1967) Classical unemployment cycle model does not represent a process of social reproduction that can be considered as adequate and sustainable in a social and democratic society in the long-run, due to the degradation of part of the workforce it implies during its periods of mass unemployment. The paper formulates on this background an unemployment benefit system for the unemployed and a minimum wage rule for the employed where this form of cyclical growth and economic reproduction of capitalism with its workforce degradation is overcome by the protection of workforce skills and family structures through general base income payments coupled with the obligation of life-long learning and social services provision, here added to an economic reproduction process that is highly competitive and flexible and thus not of the type of the past Eastern socialism. Instead, there is high labor mobility (concerning ‘hiring’ and ‘firing’ in particular), whereby fluctuations of employment in the labor market of the economy (the private sector) are made socially acceptable through the security aspect of this revised form of Goodwinian growth cycle dynamics. We can show in this framework that minimum (and maximum) real wages provide extra stability to such dynamics by decreasing the amount of overshooting process of income distribution they are subject to.

Solow’s (1956) famous growth model is to a certain degree also of this flexicurity type, since competitive firms are always operating on their profit-maximizing activity level and since the labor market is assumed to always guarantee full employment. We have there employment flexibility coupled with income security, through the assumed behavior of firms and through the assumption of perfectly flexible money wages. The monetarist critique of Keynesianism and recent work by Blanchard and Katz (1999) in particular suggests however a wage Phillips curve which, when for example coupled with the assumption of myopic perfect foresight regarding the price inflation rate, implies a real wage Phillips curve where the growth rate of real wages depends positively on the employment rate and negatively on the level of the real wage rate. Adding such empirically supported real wage rigidity to the Solow model then gives rise to two laws of motion, for labor intensity and the real wage, a dynamical system which approaches the situation of the overshooting Goodwin growth cycle mechanism if factor substitution in production is sufficiently inelastic and if the Blanchard and Katz (1999) real wage error correction term in the Phillips curve is sufficiently weak (assumptions we shall start from in the following). Solow’s growth model thus becomes thereby a variant of the Classical distributive growth cycle and its overshooting reserve army mechanism, the adequacy of which for a democratic society and the preservation of a homogeneously skilled labor force is questioned in this paper. An empirical example of what is meant by this latter statement is provided by figure 1.
One insight that can be obtained from figure 1 for the UK 1855 – 1965 is that the Goodwin cycle must have been significantly shorter before 1914 (with larger fluctuations in employment during each business cycle), and that there has been a major change in it after 1945. This may be explained by significant changes in the adjustment processes of market economies for these two periods: primarily price adjustment before 1914 and primarily quantity adjustments after 1945. Based on data until 1965 one could have expected that the growth cycle had become obsolete (and maybe also the business cycle as it was claimed in the late 1960’s). Yet, extended by the data shown in figure 1, taken from Groth and Madsen (2007), it is now obvious that nothing of this sort took place in the UK economy. In fact, we see in figure 1 two periods of excessive over-employment (in the language of the theory of the NAIRU) which were followed by periods of dramatic underemployment, both started by periods of the more or less pronounced occurrence of stagflation.

Generating order and economic viability in market economies by large swings in the unemployment rate (mass unemployment with human degradation of part of the families that form the society), as shown above, is one way to make capitalism work, but it must surely be critically reflected with respect to its social consequences. Such a reproduction mechanism is not compatible in the long-run with an educated and democratic society supposed to provide equal opportunities to all of its citizens.

This situation must therefore be contrasted with an alternative social structure of accumulation and its labor market institutions which allows to combine the Solovian situation of a highly competitive market economy (free hiring and firing) with a human rights bill that includes the right (and the obligation) to do social work (including the preservation of workforce skills), and to get income from this work that at the least supports basic needs and basic happiness. By contrast, a laissez-faire capitalistic society that ruins family structures to a considerable degree (through alienated work, degrading
unemployment and education- and value-decomposing visual media) cannot be made compatible with a democratic society in the long-run, since it produces conflicts ranging from social segmentation to class clashes, racial clashes and more.

We will therefore argue in this paper that stable, though cyclical reproduction, with – despite real wage rigidities – all factors of production being in principle employed, is possible under the above regime of flexicurity capitalism and its economic and social institutions. The reader is referred to Flaschel, Greiner, Luchtenberg and Nell (2007), Flaschel, Greiner and Luchtenberg (2007) for more details on models of flexicurity capitalism and a model variant that enhances the model of the present paper significantly, in particular through detailed transfer payment schemes, credit relationships, Keynesian demand problems and also educational issues such as the principle of equal opportunities.

3 Classical growth dynamics

In this section we provide an extended version of the Goodwin (1967) growth cycle model of the interaction of income distribution and (un-)employment, as measured by the wage share and the employment rate, which serves as a baseline framework for our subsequent discussion of the role of base income payments for all unemployed members of the workforce and minimum wages for the employed part of the workforce. This Classical prototype model of fluctuating growth, based on a labor market and accumulation dynamic, was originally written in real terms. Goodwin (1972) however also suggested a nominal version of this model which separates wage from price inflation. We use its reformulation in Flaschel, Franke and Semmler (1997) for our discussion of Classical unemployment cycles and their modification through unemployment benefits and minimum wage payments.\(^6\) Let us start with the following formulation of this model. The growth rate of the money wage is given by:

\[
\hat{w} = \beta_w (e - \bar{e}) + \eta \pi^e, \quad \dot{w} = \hat{w}/w
\]

(1)

This is a nominal wage Phillips curve, here still linear and augmented with expected price inflation \(\pi^e\) when \(\eta > 0\) holds. We have \(\bar{e} \in (0, 1)\) where \(1 - \bar{e}\) denotes the NAIRU unemployment rate. In addition, we posit a dynamic markup pricing where \(v = wL/(pY)\) is the wage share.

\[
\dot{p} = \beta_p [Av^* - p] \Rightarrow \dot{p} = \beta_p [Av - 1]
\]

(2)

We assume for simplicity for the markup factor \(A = 1 + \mu, \mu > 0, Av^* = 1\). with \(v^*\) the steady state value of the share of wages in national income. There is therefore no inflation in the steady state. We allow adaptive or perfect expectations.

\[
\dot{\pi}^e = \beta_{\pi^e} (\hat{p} - \pi^e)
\]

(3)

\(^{6}\)To my knowledge, the first study of an unemployment benefit system in the context of the Goodwin growth cycle model was provided by Glombowski and Krüger (1984). They discuss in their paper various benefits systems and their stability implications in the context of fluctuating growth.
where $\beta_{\pi e} \in [0, \infty]$.

Goodwin’s accumulation equation reads on the basis of a linear technology with no technical change, i.e., on the basis of given input–output proportions $\bar{x} = Y/K = \text{const.}, \ y = Y/L = \text{const.}$, and its assumption of Say’s law in the simplest form $I \equiv S = (1 - v)Y$

$$\dot{K} = \ddot{x}(1 - v), \ \ddot{K} = \dot{K}/K$$

where it is assumed with respect to savings propensities that $s_c = 1; s_w = 0$ holds for the savings rates out of wages and out of profits. Using $Y/K = \text{const.}$ and $Y/L = \text{const.}$, we get from the definitional equation $e = L/L^*$,

$$\dot{e} = \dot{K} - n, \ \dot{e} = \hat{e}/e$$

with $n$ the growth rate of labor supply. Due to fixed coefficients in production we have $\ddot{y} = Y/L = 0$, and $\ddot{x} = Y/K = 0$. From the above we get:

$$\dot{v} = \dot{w} - \dot{p} - \ddot{y} = \beta_w(e - \bar{e}) + \eta \pi^e - \beta_p[Av - 1] \quad (4)$$

$$\dot{\bar{e}} = \ddot{x}(1 - v) - n \quad (5)$$

$$\ddot{\pi}^e = \beta_{\pi e}(\beta_p[Av - 1] - \pi^e) \quad (6)$$

Three autonomous differential equations in the variables $v$, $e$, $\pi^e$, are obtained. In contrast to the original Goodwin model our nominal extension of this model includes actual price and price expectations dynamics.

The interior steady state solution is

$$v^* = 1 - n/\bar{x}, \ e^* = \bar{e} \ \pi^{e*} = 0 = \hat{p}^* \text{ since } Av^* = 1,$$

We only claim here\(^7\) that we get local asymptotic stability at least for all $\eta$ with:

$$\eta < 1 + \frac{\bar{x}e^*v^*\beta_w}{(\beta_pAv^* + \beta_{\pi e})\beta_{\pi e}} =: \eta^H$$

We add the following remark. There occurs Hopf-bifurcation at $\eta^H$, i.e., in particular stability is lost in a cyclical fashion. Furthermore, for the perfect foresight case $\beta_{\pi e} \to \infty$, it follows that $\eta^H \to 1$. The same holds true for $\beta_p \to \infty$. An increase in $\beta_{\pi e}$, $\beta_p$ increases the instability range and an increase in $\beta_w$ reduces it. This is a striking difference in the role price and wage flexibilities play in such classical framework. For $\eta > 1$ it is also easy to determine the Hopf-bifurcation point with respect to the adjustment speed of inflationary expectations. This point is given by the uniquely determined positive solution of a quadratic equation for $\beta_{\pi e}$. The bifurcation from stability to instability is of the same type as before.

To prepare this system for a baseline analysis of the role of minimum wages in the Goodwin growth cycle framework let us go to the limit case $\beta_{\pi e} = \infty$: $\pi^e = \hat{p}$ which means myopic perfect foresight. The system then reduces to two differential equations:

$$\dot{v} = \beta_w(e - \bar{e}) - (1 - \eta)\beta_p[Av - 1] \quad (7)$$

$$\dot{\bar{e}} = \ddot{x}(1 - v) - n \quad (8)$$

\(^7\)See Flaschel, Franke and Semmler (1997, Ch.4) for details.
The above suggests that $\eta_H = 1$ is now the bifurcation value that separates nonexplosive from explosive cyclical behavior by a center type stability at $\eta = 1$. We would call the first case a consensus-driven economy and the second case an economy that is driven by dissent, since capital and labor here interact in a destabilizing manner by way of inconsistent income claims.

It is not difficult to prove these assertion from a local perspective. Yet, more general – since of a global nature – and also easy to apply is a proof which employs a Liapunov-function for the dynamics (1)–(2) around its steady state $e^* = \bar{e}$, $v^* = 1 - n/\bar{x}$. The function

$$H(v, e) = \int_{e^*}^{e} \beta_w(\bar{e} - e^*)/\bar{e} \, d\bar{e} - \int_{v^*}^{v} (x(1 - \bar{v}) - n)/\bar{v} \, d\bar{v}$$

defines such a Liapunov function. It has the following shape on the phase space for $(v, e) \in \mathbb{R}^2_+$:

![Figure 2: A Liapunov function for the myopic perfect foresight dynamics.](image)

Its level surfaces are the Goodwin (1967) growth cycles which here result from choosing $\eta = 1$, see figure 2. The Liapunov function is zero at the steady. It is well-defined and strictly positive elsewhere in the positive orthant $\mathbb{R}^2_+$. Moreover, its time derivative along the solution curves of the above dynamical system reads

$$\dot{H} = H_v \dot{v} + H_e \dot{e}$$

$$= - (\bar{x}(1 - v) - n)\dot{v} + \beta_w(e - \bar{e})\dot{e}$$

$$= + (\bar{x}(1 - v) - n)(1 - \eta)\beta_p[Av - 1]$$

which gives zero for $v = v^*$. For $v \neq v^*$ we on the other hand get:

$$\dot{H} < 0 \quad \text{if } \eta < 1$$

$$\dot{H} = 0 \quad \text{if } \eta = 1$$

$$\dot{H} > 0 \quad \text{if } \eta > 1$$
since \( x(1-v) - n \) and \( Av - 1 \) are of different sign to the left and to the right of \( v = v^* \).

The theorem 2 of Hirsch and Smale (1974, pp.195) is applicable, since \( \dot{H} = 0 \iff v = v^* \) (for \( \eta \neq 1 \)). We thereby get

- \( \eta < 1 \): a globally asymptotically stable dynamics in the invariant domain \( \{(u,v) \in \mathbb{R}^2, u,v > 0\} = \mathbb{R}^2_+ \)
- \( \eta > 1 \): a totally unstable dynamics in this same invariant domain
- \( \eta = 1 \): all trajectories in \( \mathbb{R}^2_+ \) are closed orbits, representing the original Goodwin growth cycles.

These results are intuitively plausible since the sign of the derivative \( \dot{H} \) simply expresses whether the trajectories of the dynamics in \( v, e \)-space are accompanied by declining or rising or constant magnitudes of their corresponding value \( H(v,e) \), see figure 2. Trajectories accompanied by a rising \( H(v,e) \) for example must therefore be explosive.

The above approach, when restricted to myopic perfect foresight, gives a neat and simple generalization of Goodwin’s (1967) Classical growth cycle model of the dynamic interaction of the employment rate and income shares in the conflict about income distribution. With respect to the closed orbit shown in figure 2, as well as any other such orbit, this conflict about income distribution points inside and thus produces convergence to the steady state for parameter values \( \eta < 1 \) and outside, implying divergence, for \( \eta > 1 \).

Two problems might arise in the latter situation. First, \((e,v) > (0,0)\) holds automatically along the orbits of the model, but we also have to ensure that \((e,v) \leq (1,1)\) holds true. Second, the case \( \eta > 1 \) is not yet viable, i.e., the dynamic is still incomplete. What, however, may determine outer bounds if economically implausible values are approached and thus instability prevails?

A possible solution for these problems is to use viability constraints as in Hick’s (1950) trade cycle model. We may posit

\[
\dot{e} = \begin{cases} 
\bar{x}(1-v) - n, & \text{if } e < 1 \quad \text{or } \dot{e} < 0 \\
0, & \text{if } e = 1 \quad \text{and } \dot{e} > 0 
\end{cases}
\]

enforced by forced consumption of capitalists such that \( s,\bar{x}(1-e) = n \) in the latter case. Here \( s_e \) is assumed as endogenous, since no worker will be found to operate additional machines.

On the other hand, or in addition, we could assume state dependent reaction functions \( \eta = \eta(v), \beta_p = \beta_p(v) \) of the wage share \( v \) such that

- \( \eta(v^*) > 1 \) local instability, but \( \eta(v) < 1 \) if \( v \) close to 1
- \( \beta_p(v) = 0 \) for \( v \leq v \leq v^* \)
- \( \beta_p(v) \to \infty \) as \( v \to 1 \) guaranteeing \( \dot{v} < 0 \) if \( v \) close to 1.

These assumptions state that sufficiently large wage shares – caused by local instability – will speed up inflation while at the same time the pressure from nominal wage increases
is released. Furthermore, and for simplicity, the occurrence of deflation is restricted to a certain domain below the steady state. Assumptions of this kind give rise to a phase-portrait of the following type

![Phase-Portrait Diagram](image)

Figure 3: Viability constraints for the explosive Classical growth cycle.

This not only solves the first but also the second problem, since \((0, 1) \times (0, 1]\) is made on invariant set by these assumptions, which cannot be left by any trajectory that starts in it. Furthermore, the steady state is unstable by the choice of the \(\eta\) and the \(\beta_p\)-function.

By the Poincaré-Bendixson one can then demonstrate existence of a stable limit cycle in \((0, 1) \times (0, 1]\). This result is intuitively clear from the above figure, since the dynamics point outward at the steady state while the depicted box is at the same time an invariant set of the dynamics where the trajectories cannot escape from.

This situation provides an example of what we would call a dissent-driven economy. We have local instability due to too excessive wage claims, oriented at a target that is higher than actual inflation (in an economy where labor productivity growth is still excluded). Due to the explosive nature of the dynamics the full employment ceiling \(e = 1\) will sooner or later be approached and will give rise to significant increases in the wage share which continue and generate increasing price inflation also when the employment rate is falling again (but still higher than the NAIRU rate of employment). This period of stagflation will be followed by a – depending on downward wage flexibility – long period of stagnation where there is rising mass unemployment, degradation of skills in particular of the long-term unemployed, decay in family structures due to the lack of basic income provisions and more. Of course, such a process can in principle be avoided if the parameter \(\eta\) of price inflation targeting is less than one, i.e., if there is some consensus concerning the interaction between labor and capital and their unions that pays tribute to the otherwise severely overshooting interactions between employment and income distribution where each party strictly pursues its own interest when in a strong position on the market for labor. The basic message of this approach to macroeconomic dynamics
is however that order is generated in this economy by large swings in unemployment (the reserve army mechanism) with severe social consequences that are rarely considered in standard economic reasonings on the interaction of real wages and the unemployment rate.

4 Hiring and firing, social security and restricted reserve army fluctuations

We assume for the following the prevalence of a balance between a consensus and a dissent driven economy, i.e., the limit case $\eta = 1$ which implies the closed orbit structure of the original Goodwin (1967) growth cycle model. The question then is whether such an economy (where there is inflation targeting without misperceptions and downward pressure in wage negotiations, but still significantly overshooting in unemployment and income distribution) can be further improved by allowing for unemployment compensations and also for minimum wages (and later on also for maximum wages), viewed as expressing certain compromises in the interaction between capital and labor.

4.1 Human rights: Basic income and minimum wages

1 Everyone has the right to work, to free choice of employment, to just and favorable conditions of work and to protection against unemployment.

2 Everyone, without any discrimination, has the right to equal pay for equal work.

3 Everyone who works has the right to just and favorable remuneration ensuring for himself and his family an existence worthy of human dignity, and supplemented, if necessary, by other means of social protection.

4 Everyone has the right to form and to join trade unions for the protection of his interests.


In this section we want to show that the quoted article 23 from the United Nations’ declaration of Human Rights does not only represent a normative statement, but can also be justified from the economic point of view in the context of analysis of the process of capital accumulation. We believe that capitalism is a very robust system of resource allocation and income distribution that can adjust to many social restrictions if these restrictions are justified from a normative point of view. For more detailed discussions of such an approach, the reader is referred to Bowles, Gordon and Weisskopf’s (1983) work ‘Beyond the Waste Land’ and in particular to their chapter on ‘an economic bill of rights’.

In this paper, we now augment the analysis of the working of the reserve army mechanism in a capitalist economy of the preceding section by two fundamental human rights:
the right for basic (of course: real) income when unemployed and the right that the wages of the employed should not fall below a certain real minimum level. Of course, there are also obligations connected with the formulations of these rights which concern the need of skill preservation when unemployed and the provision of adequate social services for the considered society. In this paper our focus is however on the macroeconomic sustainability of these minimum restrictions on the working of a capitalist economy and not on the detailed analysis on how such a system can work on the microlevel. We will argue that the social costs of reproduction mechanisms as they are shown in figures 2 and 3 are much higher than what will be the result under the above minimum restrictions on the working of a capitalist economy in a democratic society and that it is the duty of capital as well as labor to provide the necessary behavior such that these restrictions can be realized not only theoretically, but also – at least – in actual (advanced) capitalist democracies.

4.2 Capital’s and labor’s responsibility: Minimum wages and basic income needs

The dynamical system to be reformulated from these points of view reads:

\[
\begin{align*}
\dot{v} &= \beta w(e - \bar{e}) \\
\dot{e} &= \bar{x}(1 - v) - n
\end{align*}
\]  

We modify this system by way of assuming that a fraction measured by \( \tau \) of the wage income of the employed must be provided as means for unemployment insurance and the restriction that the real wage of the employed can at most fall to the level \( \omega_{\text{min}} \). The basic income of the unemployed is then derived by assuming that their ‘real wage’ is a certain fraction of this minimum real wage and given by \( \bar{\omega} \). The supply of labor of the unemployed is \((1 - e)L\) and is assumed to go into activities that concern skill preservation or social services. Since labor productivity \( y \) is a given magnitude the above assumptions can of course be equally represented by constraints \( v_{\text{min}}, \bar{v} \).

We thus assume for the above dynamics that \( \bar{v} < v_{\text{min}} < v^{*} \) and \( v_{\text{min}} \leq v \) holds true at all points in time (since minimum wages must of course lie below their steady state value). The only modification that this implies for the above dynamics is that they are now augmented by \( \dot{v} = 0 \) in the cases where \( v < v_{\text{min}} \) applies in the original Goodwin growth cycle dynamics.

We consider the viability of the assumed transfer payments structure first. Reserves for unemployment benefits are represented by the symbol \( R \). Their rate of change is on the basis of the above assumptions given by

\[
\dot{R} = \tau \omega e L - \bar{\omega}(1 - e)L
\]

where \( L \) is the total labor supply. Transferred to intensive form magnitudes this gives

\[
\dot{R}/K = \tau \omega \bar{x}/\bar{y} - \bar{\omega}(l - \bar{x}/\bar{y}) = \tau v \bar{x} - \bar{v}(\bar{y}l - \bar{x})
\]
For the dynamic of the intensive form variable \( r = \frac{R}{K} \) we get from these equations:

\[
\dot{r} = \frac{\dot{R}}{K} - \frac{\dot{K}}{r}, \quad i.e. \quad (11)
\]

\[
\dot{r} = \tau \bar{x} - \bar{v}(\bar{y}l - \bar{x}) - (\bar{x}(1 - v))r
\]

(12)

For the steady state value of \( r \) this gives

\[
r^* = \frac{\tau v^* \bar{x} - \bar{v}(\bar{y}l^* - \bar{x})}{n}, \quad i.e.
\]

\[
\left(\frac{R}{L}\right)^* = \frac{r^*}{l^*} = \frac{(\tau \omega^* + \bar{\omega})e - \bar{\omega}}{n}
\]

Assuming for example the parameter values \( n = 0.02, \tau = 0.15, \bar{\omega} = 0.5\omega^* \) and as minimum for the actual employment rate \( \bar{e} = 0.8 \) gives for \( \left(\frac{R}{L}\right)^* \) the value \( \bar{\omega} \) which means that the steady state reserves for unemployment benefits per worker – at an unemployment rate of 20% – are just equal to the basic income wage, while steady state employment is \( \bar{e} \) and steady state real wages are given by \( \omega^* = (1 - \frac{n}{2})\bar{y} \). At least for the steady state we therefore have that the economy is reproducible at base income wages \( 0.5\omega^* \), with no role to play for the minimum wage \( \omega_{min} \in (\bar{\omega}, \omega^*) \).

The question now however is how the dynamics of the original Goodwin model are modified in the large through the assumption of a minimum wage rate for the employed workers. Figure 4 shows what is happening in the growth cycle dynamics if a minimum wage restriction is added to the model. We stress with respect to this figure that the base income real wage does not matter for it, since it only concerns the redistribution of income between employed and unemployed workers (who both have a propensity to spend equal to one).

![Figure 4: The distributive cycle with a minimum wage restriction](image-url)
The smallest cycle in figure first of all shows that nothing is changed if the minimum real wage is less than the lowest real wage along this cycle. The minimum wage restriction then simply is not a binding one. If however, as shown by the largest cycle the minimum wage bound is hit, the economy will move along this boundary upwards (since profitability is above the steady state profit rate) until the NAIRU rate of employment is reached. From there on real wages will be rising again along the cycle that is just tangent to the minimum wage restriction. The result therefore is that all larger cycles will be dampened towards this boundary case (around the grey area in figure 4). Minimum real wages therefore make the fluctuations the economy is subject to less severe, reduce among others stagflationary periods, and diminish the volatility in the employment rate in the longer run.\(^8\)

This is clearly an economically more desirable situation, since excessive fluctuations of the employment rate are avoided now, and this positive judgment the further strengthened, since all social consequences of unemployment are avoided through the transfer payments underlying this tamed operation of the Classical reserve army mechanism. Moreover, increasing minimum real wages moderately will improve this situation further while a return to a cold turkey strategy of no minimum wages at all may be the faster solution to end the depression, but one that reintroduces larger fluctuations in the employment rate and income distribution with all their social consequences.

Instead of pursuing such a radical strategy this paper would propose a further reflection of the strategies that will make the distributive cycle even less severe and maybe also convergent to the steady state of the economy. The addition of Blanchard and Katz (1999) error correction may be a candidate here,\(^9\), neoclassical smooth factor substitution adds another stabilizing mechanism,\(^10\) and any dialogue between workers’ union and capitalists’ unions can also be of help. The advantage of the Goodwin approach to cyclical growth is that it is not biased against capitalist interest, since it entails that workers’ union bear responsibility for overshooting wage share and unemployment rates in the prosperity phase of the cycle.\(^11\) Minimum wages come to the help of workers’ unions in stagnant phases by avoiding more severe unemployment situations. Their responsibility however is to provide a similar mechanisms for the situation to the right of figure 4, see the next section, where prudent wage policies (and supporting fiscal and monetary policies) have to be found that avoid the occurrence of severe stagflation and its twin evils. We add finally that a payment of base income out of profits will make the Goodwin cycle an explosive one, while combined wages – as they are suggested in Germany – may make the recovery along the vertical line to the left of figure 4 faster, since they increase profitability, but of course put an extra burden on the transfer payments that is requested from the employed workforce.

\(^8\)These results also hold in the case of a single limit cycle version of the Goodwin model.
\(^9\)see the appendix 1 and Flaschel, Tavani, Taylor and Teuber (2007).
\(^10\)see Flaschel (1993)
\(^11\)See Wörgötter (1986) for the details of such an observation.
4.3 Union’s responsibility: Upper bounds for real wage increases

One may ask how the lower flower to real wage payments is in fact monitored in a society where wage negotiations are about money wages and not about real wages and are subject to collective bargaining (tariff autonomy). The answer to this question is however on the theoretical level not a difficult one, since it only demands that wages have to increase exactly with price inflation when minimum real wages are reached and as long as employment is below the NAIRU. The problem may of course be to reach agreement between capital and labor on the management of wage inflation in this phase of the distributive cycle, here primarily concerning capital, since labor is in a weak position.

A compensation that can be offered by labor is that a similar rule is applied when labor is in a strong position, i.e., when the maximum real wage shown in figure 5 has been reached. Wage inflation is then higher than price inflation (since the real wage is increasing) and it demands now for a compromise primarily from the side of workers’ unions to accept that there will be only inflationary compensation until again the NAIRU level $\bar{e}$ has been reached (now from above). If such an agreement can be reached between capital and labor we get what is shown in figure 5 and thus a further improvement in the cyclical behavior that is generated by the wage-price Phillips curve mechanism and the pace of capital accumulation this implies.

The choice of the correct levels of minimum (and maximum) wages may however run into problems when set to close to the unobserved steady state level. Though this may dampen, on the one hand, the fluctuations in the rate of employment further if it really stays below $\omega^*$ it will, on the other hand, lead to disastrous consequences if set above the steady state level, since profits are then not sufficient to maintain even the current level.
of the employment rate which will fall without limit then if this choice of the minimum wage level is not revised. It may therefore be wise to use both the minimum with sense of proportion and look for help from the maximum real wage level in order to tailor the fluctuations in growth and employment in the best achievable way.

4.4 Automatic stabilizers: Blanchard and Katz error correction terms

In the appendix to this paper we provide a sketch of Blanchard and Katz’s (1999) microfoundation of the wage Phillips curve as it was discussed in section 2, but not fully used so far. This microfounded type of Phillips curve extends the PC shown in equation (1) in the following way (in the case $\eta = 1$, see the appendix for details):

$$\dot{w} = \beta_{we}(e - \bar{e}) - \beta_{wv}(v - v^*) + \hat{p}$$

In terms of the appendix the parameter $\beta_{wv}$ is given by $1 - \theta \lambda$, and thus strictly positive if labor productivity plays a role both in the formulation of reservation wages as well as the real wages targeted by unions in their wage negotiations.

Making use again of the Liapunov function in section 3:

$$H(v, e) = \int_{\bar{e}}^{e} \beta_{w}(\hat{e} - e^*)/\hat{e} \ d\hat{e} - \int_{v^*}^{v} (x(1 - \hat{v}) - n)/\hat{v} \ d\hat{v}$$

we get with respect to the above extended Phillips curve (which implies $\hat{v} = \beta_{we}(e - \bar{e}) - \beta_{wv}(v - v^*)$) the result:

$$\dot{H} = H_v \hat{v} + H_e \hat{e}$$

$$\dot{H} = -(\bar{x}(1 - v) - n)\hat{v} + \beta_{w}(e - \bar{e})\hat{e}$$

$$\dot{H} = -(\bar{x}(1 - v) - n)(-\beta_{wv}(v - v^*))$$

$$\dot{H} = (\bar{x}(v^* - v)\beta_{wv}(v - v^*) = -\bar{x}\beta_{wv}(v - v^*)^2 \leq 0$$

The unrestricted Goodwin growth cycle is therefore now globally convergent to the steady state of the economy in the case $\eta = 1$ by the arguments we used in section 3 for the case $\eta < 1$. Since the cycles that so far resulted from either minimum or maximum real wages are tangent to these restrictions we get from the above that they are only needed once to restrict the unrestricted excessive cycle to them. Thereafter such bounds are no longer necessary, since the next cycle remains inside of these bounds and converges to the steady state eventually. We thus get from the microfounded and estimated wage PC of Blanchard and Katz (1999) type, at least for Europe as their study is concerned, that minimum and maximum wages will dampen the fluctuations of the unrestricted reserve army mechanism significantly and make it thereafter convergent to its long run equilibrium position.
5 Conclusions

In this paper we departed from the conventional discussion of the impact of minimum wage legislation, which is only partial in nature, by considering the macroeconomic effects of such legislation or agreements between capital and labor. We think that sector specific rules concerning minimum wages can only be discussed against the background of such macrofoundations where the medium- and long-run consequences of minimum wages are the focus of interest and not so much the short-run adjustment problems such a legislation may cause.

We conclude however from what has been shown in this paper that the introduction of a general level of minimum (or maximum) real wages into a supply-side macro model of fluctuating growth does not do much harm even in the shorter run to capital accumulation and employment as described through this model type, and does definitely and significantly improve the performance of the implied cyclical growth path in the course of time. We have not only got less severe fluctuations in the unemployment rate than in the unrestricted case (where there is an unlimited working of the wage-price spiral and the reserve army mechanism), but can also avoid the social consequences of mass unemployment through basic income payments – and an employer of last resort, if meaningful activities of the unemployed are added to the reformulated social structure of accumulation. An educated society, in which the principle of equal opportunities holds in its schooling system, may be a very important ingredient in the working of such a social structure, where partial workforce degradation is avoided by meaningful qualification processes of the unemployed and also life-long learning of the employed, see Flaschel, Greiner and Luchtenberg (2007) for further details on such a scenario.

![Figure 6: G7 Distributive Cycles 1870–2004: WS=wage share, ER=employment rate](image)

Of course, there may be obstacles on the way towards such a social structure of accumulation, given by the factual sclerosis of existing social structures (degraded long-term unemployed persons, segmented labor markets, degrading job offers and more). Globalization may also represent a big challenge for our reformulated Goodwin growth cycle dynamics, concerning international competition for traded commodities and services,
workforce migration, outsourcing and more. This however essentially demands that the baseline flexicurity system discussed in this paper needs further refinements along the lines proposed in the quoted work. What is shown in figure 6 for the G7 countries, based on yearly data,\textsuperscript{12} is however not the ideal we should strive for in the future gestation of the institutions of capitalist market economies.

\textsuperscript{12}obtained from Groth and Madsen (2007).
References


Appendix: Wage dynamics. Theoretical foundations

This subsection builds on the paper by Blanchard and Katz (1999) and briefly summarizes their theoretical motivation of a money-wage Phillips curve which is closely related to our dynamic equation (1). Blanchard and Katz assume – following the suggestions of standard models of wage setting – that real wage expectations of workers, \( \omega^r = w_t - p_t^l \), are basically determined by the reservation wage, \( \bar{\omega}_t \), current labor productivity, \( y_t - l_t^d \), and the rate of unemployment, \( U_t^l \):

\[
\omega_t^r = \theta \bar{\omega}_t + (1 - \theta)(y_t - l_t^d) - \beta_w U_t^l.
\]

Expected real wages are thus a Cobb-Douglas average of the reservation wage and output per worker, but are departing from this normal level of expectations by the state of the demand pressure on the labor market. The reservation wage in turn is determined as a Cobb-Douglas average of past real wages, \( \omega_{t-1} = w_{t-1} - p_{t-1} \), and current labor productivity, augmented by a factor \( a < 0 \):

\[
\bar{\omega}_t = a + \lambda \omega_{t-1} + (1 - \lambda)(y_t - l_t^d)
\]

Inserting the second into the first equation results in

\[
\omega_t^r = \theta a + \theta \lambda \bar{\omega}_{t-1} + (1 - \theta \lambda)(y_t - l_t^d) - \beta_w U_t^l,
\]

which gives after some rearrangements

\[
\Delta w_t = p_t^r - p_{t-1} + \theta a - (1 - \theta \lambda)[(w_{t-1} - p_{t-1}) - (y_t - l_t^d)] - \beta_w U_t^l
\]

\[
= \Delta p_t^r + \theta a - (1 - \theta \lambda)u_{t-1} + (1 - \theta \lambda)(\Delta y_t - \Delta l_t^d) - \beta_w U_t^l
\]

where \( \Delta p_t^r \) denotes the expected rate of inflation, \( u_{t-1} \) the past (log) wage share and \( \Delta y_t - \Delta l_t^d \) the current growth rate of labor productivity. This is the growth law for nominal wages that flows from the theoretical models referred to in Blanchard and Katz (1999).

In this paper, we proposed to operationalize this theoretical approach to money-wage inflation by replacing the short-run cost push term \( \Delta p_t^r \) by the weighted average \( \kappa_w \Delta p_t^r + (1 - \kappa_w)\pi_t \), where \( \Delta p_t^r \) is determined by myopic perfect foresight. Thus, temporary changes in the correctly anticipated rate of inflation do not have full impact on temporary wage inflation, which is also driven by lagged inflation rates via the inflationary climate variable \( \pi_t \). Adding inertia to the theory of wage inflation introduced a distinction between the temporary and persistent cost effects to this equation. Furthermore we have that \( \Delta y_t - \Delta l_t^d = n_z \) due to the assumed fixed proportions technology. Altogether, we end up with an equation for wage inflation of the type presented in section 2, though now with a specific interpretation of the model’s parameters from the perspective of efficiency wage or bargaining models.\(^\text{14}\)

\(^{13}\)In this section, lower case letters (including \( w \) and \( p \)) indicate logarithms.

\(^{14}\)Note that the parameter in front of \( u_{t-1} \) can now not be interpreted as a speed of adjustment coefficient. Note furthermore that Blanchard and Katz (1999) assume that, in the steady state, the wage share is determined by the firms’ markup \( \mu = -\mu \) (both in logs) to be discussed in the next subsection. Therefore the NAIRU can be determined endogenously on the labor market by \( U_t^l = \beta_w^{-1} [\theta a - (1 - \theta \lambda)\mu - \theta \lambda(\Delta y_t - \Delta l_t^d)] \). The NAIRU of their model therefore depends on both labor and goods market characteristics in contrast to the NAIRU levels for labor and capital employed in our approach.